

# Uncertain Supply Chain Management

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## A two-stage reverse supply chain model for pricing remanufactured products under collection policy and promotional incentives: A game theory approach

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### ABSTRACT

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The efficient management of reverse supply chains, particularly the collection and remanufacturing of defective products, plays a critical role in reducing production costs and determining the final pricing of remanufactured products. While existing research extensively explores warranty policies and maintenance services to enhance customer satisfaction and profitability, the integration of vehicle routing for product collection and sustainability advertising strategies remains underexplored. Addressing this gap, this study introduces a comprehensive two-stage reverse supply chain model that captures the interactions between manufacturers (MFRs) and remanufacturers (RMFRs) through a Stackelberg game framework. Methods: The proposed model incorporates interactive production constraints, vehicle routing problem (VRP) for optimizing collection logistics, and sustainability advertising to influence consumer behavior towards remanufactured products. Utilizing mixed nonlinear programming (MINLP) and nonlinear programming (NLP) techniques, the model simultaneously optimizes pricing strategies, collection efforts, and advertising investments for both MFRs and RMFRs. Numerical analyses are conducted to solve the optimization problems, accompanied by sensitivity analyses to evaluate the impact of key parameters such as production costs, defect rates, and routing constraints. The numerical results demonstrate that increases in production costs for MFRs lead to higher selling prices, thereby reducing their profit margins and negatively impacting RMFR profitability due to decreased demand for remanufactured products. Sensitivity analysis reveals that higher defect rates ( $\alpha \geq 0.8$ ) significantly diminish overall supply chain profitability by lowering customer acceptance of RMPs. Additionally, expanding the allowable vehicle routing distance  $L$  effectively reduces collection costs, enhancing RMFR profits and enabling greater investment in sustainability advertising. The study shows that the integration of VRP and advertising strategies proves crucial in balancing cost efficiencies and market competitiveness, ultimately fostering a more sustainable and profitable reverse supply chain.

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## 1. Introduction

The rapid advancement of technology coupled with increasing consumer demand for diverse products has significantly shortened product lifecycles. This trend has led to a surge in industrial waste and the accelerated depletion of natural resources. In response, numerous countries have enacted stringent environmental regulations targeting manufacturing companies to mitigate these adverse effects. To comply with these regulations and address the growing emphasis on sustainable production processes and products, many companies have adopted remanufacturing practices. Notable organizations such as Kodak, IBM, and Xerox have established specialized facilities dedicated to the remanufacturing or reproduction of products at the end of their lifecycle or usage period. Remanufacturing involves transforming wasted or defective products into like-new items, offering substantial economic and environmental benefits over producing brand-new products. This approach not only curtails industrial waste but also conserves natural resources, aligning with broader sustainability objectives. Furthermore, the

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integration of remanufacturing within supply chains contributes to the development of closed-loop systems, which are pivotal for advancing the Green Economy.

Despite the recognized benefits, challenges persist in optimizing remanufacturing processes, particularly in the logistics and transportation phases of handling defective or used products. Most previous studies focus on pricing, product quality, and manufacturing characteristics, while only a few consider the logistical complexities of remanufacturing. This study aims to bridge this gap by employing the vehicle routing problem to assess the logistics of defective product transportation, while also considering pricing strategies and the perspectives of remanufacturers. Additionally, the study introduces a two-stage remanufacturing supply chain model that incorporates interactive production constraints between manufacturers and remanufacturers, utilizing a Stackelberg game framework to analyze decision-making processes.

## 2. Literature Review

Remanufacturing plays a crucial role in sustainable production by transforming end-of-life (EOL) and end-of-use (EOU) products into valuable resources. (Gan et al., 2017) highlight the economic and environmental advantages of remanufactured products, noting their contribution to waste reduction and resource conservation. This aligns with the principles of the Circular Economy, which emphasizes the continuous use of resources through recycling and remanufacturing processes (Kurilova-Palisaitiene & Sundin, 2021).

### *Closed-Loop Supply Chains*

Closed-loop supply chains (CLSC) integrate forward and reverse logistics to manage product flows effectively. Factors Affecting the Re-Manufacturing of a Product (Manninen et al., 2018) discuss how CLSC models convert EOL product components into resources for other industries, thereby reducing waste and supporting the Green Economy. (Zhou et al., 2017) examined a three-level CLSC involving retailers, manufacturers, and suppliers, finding that higher return rates enhance system performance by mitigating the bullwhip effect and reducing inventory dispersion.

### *Mathematical Modeling in Remanufacturing*

A substantial body of research has focused on mathematical modeling to optimize remanufacturing processes. (Agrawal, 2018) proposed a comprehensive framework encompassing five critical stages of remanufacturing: disassembly, inspection, cleaning and repairing, assembly, and final testing. Emphasizing quality and pricing, (Agrawal, 2018) underscored their importance in the success of remanufactured products. Similarly, (Liu et al., 2020) highlighted the role of pricing and warranty coverage in stimulating customer demand, suggesting that optimal strategies are contingent upon warranty duration and the production cost ratio.

(Maleki et al., 2017) utilized an M/M/1/k queuing system to model remanufacturing facilities dealing with incompatible product types and independent stations, incorporating decision variables related to contracting strategy and pricing. (Gan et al., 2017) identified pricing and sales channel management as critical challenges for companies offering both new and remanufactured products. They developed a decision-making model that favors direct channels for remanufactured goods, thereby enhancing supply chain profitability by aligning with consumer preferences.

### *Pricing Strategies and Market Dynamics*

(Zhou & Gupta, 2019) explored the complexities of maintaining multiple product generations in the market, emphasizing the need for nuanced pricing strategies for both new and remanufactured products. (de Vicente Bittar, 2018) discussed the challenges Original Equipment Manufacturers (OEMs) face in balancing consumer perceptions regarding the performance equivalence of new versus remanufactured products. (Sun et al., 2020) further expanded on this by examining competition differentiation, focusing on price sensitivity and quality perception in consumer decision-making processes.

### *Logistics and Transportation in Remanufacturing*

Although logistics and transportation play a key role in remanufacturing—significantly impacting the timely and cost-efficient collection and transport of defective or used products—there is a noticeable gap in the literature addressing these aspects. Only a few studies have considered the effect of transportation and logistics on remanufacturing operations. For instance, (Dowlatshahi, 2000) identified transportation as a critical operational factor in reverse logistics, emphasizing its importance for successful implementation. (Ullah, 2023) developed a model demonstrating that increased reverse transportation distances negatively impact remanufacturing rates and increase total emissions, highlighting the environmental implications of logistics decisions.

To address transportation optimization in remanufacturing logistics, some studies have applied the Vehicle Routing Problem (VRP). (Rahman et al., 2023) developed a VRP tool for e-waste collection, optimizing routes for heterogeneous vehicle fleets, while (Tee & Cruz, 2022) extended VRP models by incorporating collection point location decisions and wait times, providing a flexible approach for plastic waste management. Additionally, (Babazadeh & Torabi, 2018) and (Vahdani, 2015) proposed

optimization models for reverse logistics within closed-loop supply chains, utilizing robust and fuzzy-stochastic programming methods to handle uncertainties in reverse supply chain operations. These studies underscore the necessity of efficient logistics management in enhancing the sustainability and profitability of remanufacturing supply chains.

### ***Technological Advancements in Remanufacturing***

Technological innovations have significantly impacted remanufacturing processes. (Gupta & Lambert, 2007) discussed the integration of sensors in products to provide detailed lifecycle information, thereby reducing uncertainties in disassembly yields. (Ilgin & Gupta, 2011) further explored how sensor data can estimate the remaining useful life of components, facilitating informed decisions regarding optimal end-of-life timing without necessitating extensive disassembly or initial inspections.

### ***Consumer Acceptance and Market Performance***

(Zhu & Wang, 2021) addressed the low acceptance rate of remanufactured products, linking it to overall remanufacturing performance. They proposed optimizing pricing and production decisions to enhance acceptance rates. (Ho et al., 2018) examined hybrid production systems that simultaneously produce new and remanufactured products, incorporating customer segmentation and competitive pricing strategies to navigate demand uncertainties and competitive market landscapes.

### ***Integrated Models and Decision-Making Frameworks***

(Liu et al., 2018) emphasized the importance of managing remanufactured products amidst economic benefits and heightened global sustainability awareness. They presented a model to determine optimal production and pricing strategies for monopolistic manufacturers, considering factors such as collection and inspection costs. Their convex programming approach identified optimal policies for producing new products, remanufacturing, or adopting a mixed strategy.

Moreover, integrated models that consider multiple stages and decision-making processes within CLSC have been developed. For example, (Pouralikhani et al., 2013) focused on a multi-period model for managing used products in a green supply chain, incorporating strategic network design and tactical material flow decisions. These studies collectively underscore the necessity of incorporating uncertainty and robust decision-making strategies in the design and optimization of CLSC models.

Despite extensive research on remanufacturing, closed-loop supply chains, and associated mathematical models, there remains a scarcity of studies focusing on the transportation and logistics phases specific to defective or used products intended for recycling and remanufacturing. This study aims to fill this gap by applying the vehicle routing problem to evaluate the logistics of defective product transportation and by considering return flows from manufacturers for remanufacturing purposes (Hong & Zhang, 2019).

The literature sheds light on significant advancements in remanufacturing processes, closed-loop supply chain management, and optimization modeling. However, the integration of logistics and transportation considerations within remanufacturing supply chains remains underexplored. By addressing this gap, the current study contributes to a more comprehensive understanding of remanufacturing logistics, offering valuable insights for enhancing the efficiency and sustainability of supply chains in alignment with Circular Economy principles.

## **3. Methodology**

In this article, a two-stage reverse supply chain model is introduced, while considering the interactive production constraints between the manufacturer and remanufacturer. Utilizing these constraints, a two-stage Stackelberg game is developed to examine interactive decisions between the two firms. The model investigates pricing for products previously sold by a manufacturer and then collected, recycled, and remanufactured by the remanufacturing company.

Due to production imperfections, a fraction  $\alpha$  of the manufacturer's products are defective. The remanufacturer collects both used products from customers—specifically, those at the end of their life cycle (EOL) or end of use (EOU)—and defective products from the manufacturer. These collected items are then remanufactured and sold back to the manufacturer in the second period.

To optimize transportation costs, collection costs are analyzed using the vehicle routing problem (VRP). The collected products undergo quality inspection, and the necessary remanufacturing operations are identified. The final pricing of remanufactured products accounts for all costs from collection to sale, comparing it to the price of similar new products. Promotional policies, including sustainability advertising, are considered to improve the sales process and attract customers.

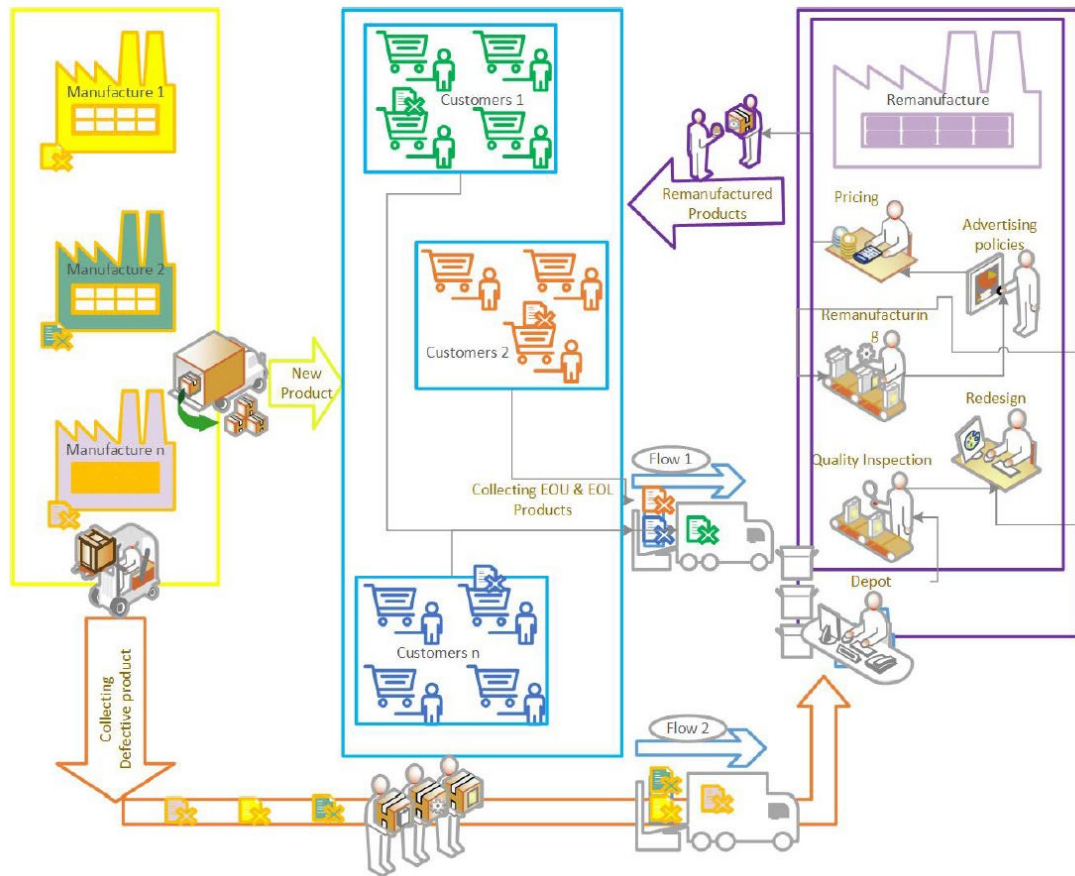


Fig. 1. Manufactured and Remanufactured product flow in a Closed-Loop Supply Chain model

### Close-loop supply chain Model (CLSC)

The closed supply chain model comprises a manufacturer (MFR) and a remanufacturer (RMFR). In the first period, the manufacturer produces new products and sells them to customers (see Figure 1).

The model is based on the following assumptions:

- I. A monopolistic market structure exists with a single manufacturer operating in the market.
- II. The manufacturer's products have an inherent defect rate ( $\alpha$ ).
- III. The remanufacturer is responsible for collecting defective products.
- IV. The remanufacturer engages in marketing strategies to promote remanufactured products.
- V. Customer locations in the collection network are known.
- VI. Transportation time between two points is constant.
- VII. Transfer costs of defective products from the manufacturer are fixed.
- VIII. The remanufacturer bears the cost of collecting defective products from customers.

Table 1 summarizes the parameters and decision variables used in the model.

**Table 1**  
Summary of Parameters and Decision Variables

Notation	Description
<b>Parameters</b>	
$V$	Utility value of a new product
$\theta$	Discount factor representing perceived quality of remanufactured products
$\delta$	Discount rate ( $0 < \delta < 1$ )
$\alpha$	Defect rate of the manufacturer's products ( $0 < \alpha < 1$ )
$c_n$	Unit production cost of new products
$c_r$	Unit cost of remanufactured products for the manufacturer
$c_{cr}$	Unit cost of remanufacturing for the remanufacturer
$k$	Cost coefficient for collection effort

Notation	Description
$b$	Effectiveness coefficient of the remanufacturer's collection effort
$\gamma$	Recycling rate of used products ( $0 < \gamma \leq 1$ )
$c_{rp}$	Unit cost associated with recycling efforts due to advertising
$d$	Effectiveness coefficient of environmental advertising on collection rate
$\beta$	Cost coefficient for environmental advertising efforts
$L$	Maximum allowable route length in VRP
Decision Variables	
$p_1$	Price of new products in the first period
$p_n$	Price of new products in the second period
$p_r$	Price of remanufactured products in the second period
$\tau$	Intensity of environmental advertising efforts
$\chi$	Collection effort level by the remanufacturer
$q_1$	Demand for new products in the first period
$q_n$	Demand for new products in the second period
$q_r$	Demand for remanufactured products in the second period
$q_{cr}$	Quantity of remanufactured products collected
$x_{ij}^z$	Binary variable indicating movement from node $i$ to node $j$ in route $z$

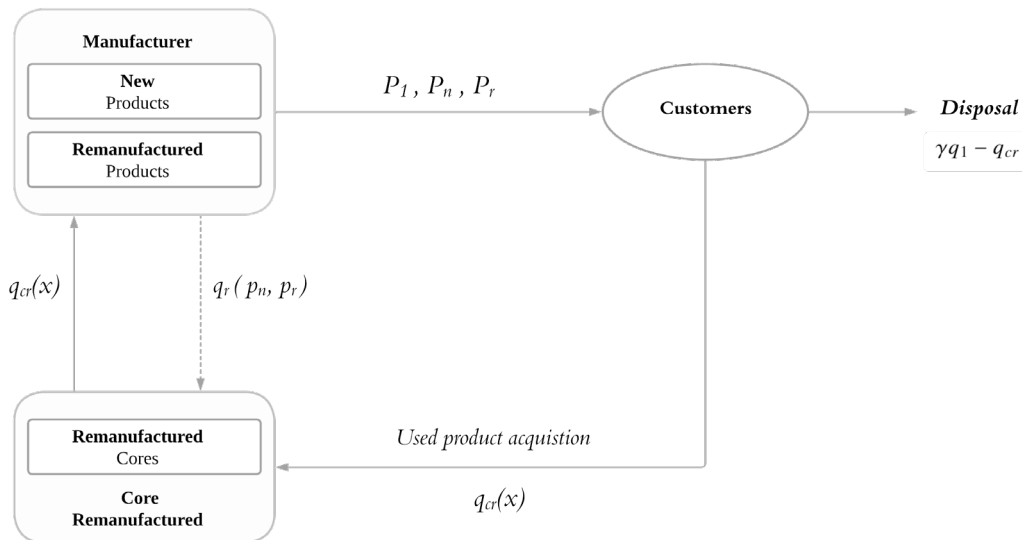


Fig. 2. Basic model

**Demand Functions**

Customer demand is influenced by product price and perceived utility. Let  $V$  represent the utility value of a new product. The perceived utility of a remanufactured product is  $\theta V$ , where  $\theta$  ( $0 < \theta < 1$ ) reflects the discount factor due to reduced perceived quality. A higher discount rate indicates a more negative customer perception of remanufactured products.

The customer utility functions are defined as:

$$u_1 = V - p_1, \quad (\text{Utility of new product in stage 1}) \tag{1}$$

$$u_n = V - p_n, \quad (\text{Utility of new product in stage 2}) \tag{2}$$

$$u_r = \theta V - p_r, \quad (\text{Utility of remanufactured product in stage 2}) \tag{3}$$

Customers will purchase a product if their utility is positive ( $u > 0$ ). In the second stage, customers will buy new products if  $u_n \geq u_r$  and remanufactured products if  $u_r > u_n$ . Therefore, the demand functions are derived as:

$$q_1 = 1 - p_1, \quad (\text{Demand for new product in stage 1}) \tag{4}$$

$$q_n = \frac{1 - \theta - p_n + p_r}{1 - \theta}, \quad (\text{Demand for new product in stage 2}) \quad (5)$$

$$q_r = \frac{\theta p_n - p_r}{(1 - \theta)\theta}, \quad (\text{Demand for remanufactured product in stage 2}) \quad (6)$$

These demand functions ensure that the total market demand is satisfied and that customers choose between new and remanufactured products based on their utilities. Specifically, if  $p_r > \theta p_n$ , then  $q_r = 0$ , indicating that no customers will purchase remanufactured products due to their negative utility.

### **Manufacturer's Decision-Making Problem**

The manufacturer (MFR) sets the production plan by deciding on prices  $p_1$ ,  $p_n$ , and  $p_r$ . The variable  $p_{cr}$  represents the price the manufacturer pays to buy back remanufactured products from the remanufacturer's core (RMFR). The costs of producing each unit of new and remanufactured products are denoted by  $c_n$  and  $c_r$ , respectively. The symbol  $\delta$  stands for the discount factor, and  $q_{cr}$  indicates the quantity of products that are recycled. The manufacturer's profit function is:

$$f_M = (p_1 - c_n)q_1 + \delta(p_n - p_r - c_r)q_r \quad (7)$$

subject to:

$$q_r \leq q_{cr} \quad (8)$$

$$0 \leq p_1 \leq 1, \quad 0 \leq p_n \leq 1, \quad 0 \leq p_r \leq 1. \quad (9)$$

### **Remanufacturer's Decision-Making Problem**

The remanufacturer makes a profit as shown in Eq. (10). In this equation,  $\chi$  ( $0 \leq \chi \leq 1$ ) represents the effort the remanufacturer puts into recycling used products. Let  $k$  be the cost of this effort,  $c_{cr}$  the cost per remanufactured product, and  $\gamma$  ( $0 \leq \gamma \leq 1$ ) the percentage of used products that get recycled. The remanufacturer's profit function is:

$$f_{RM} = \delta((p_{cr} - c_{cr})q_{cr} - k\chi^2). \quad (10)$$

subject to:

$$q_{cr} \leq \min\{\gamma q_1, q_r\}. \quad (11)$$

In Eq. (11), it is hypothesized that the quantity  $q_{cr}$  of used products recycled by the RMFR is  $q_{cr} = b\chi$ , considered as a linear function (Hong & Zhang, 2019). Furthermore,  $k\chi^2$  represents the cost of efforts made by the RMFR for recycling, assumed to be a quadratic function. The constraint  $q_{cr} \leq \gamma q_1$  ensures that the recycled products do not exceed the amount of recyclable used products in the market.

In the model by (Hong & Zhang, 2019), the manufacturer acts as the Stackelberg leader, optimizing their prices first. Subsequently, the RMFR, as the follower, optimizes their policies.

This section presents a mathematical optimization of the model based on the initial defect rate, the vehicle routing problem, and a sustainable advertising strategy.

### **Initial defect rate**

In the proposed model, it is assumed that a fraction of the initially produced products, denoted by  $0 < \alpha < 1$ , are defective. These defective items are then sent to the RMFR for correction or recycling (Equation 12).

### **Optimized Manufacturer's Decision-Making function based on initial defect rate**

The manufacturer's profit function is modified to account for defective products as follows:

$$f_M = (p_1 - c_n)q_1(1 - \alpha) + \delta(p_n - c_n)q_n + \delta(p_r - p_{cr} - c_r)q_r. \quad (12)$$

Subject to constraints (8) and (9).

According to Eq. (12), the manufacturer sells an amount equal to  $q_1(1 - \alpha)$  of its production. The defective products ( $\alpha q_1$ ) are sent to the remanufacturer for remanufacturing.

**Optimized remanufacturer’s Decision-Making function based on initial defect rate**

The remanufacturer’s profit function remains similar to Eq. (10), but now includes the defective products from the manufacturer:

$$f_{RM} = \delta((p_{cr} - c_{cr})q_{cr} - k\chi^2). \tag{13}$$

subject to:

$$q_{cr} \leq \min\{\gamma q_1, q_r\}, \tag{14}$$

where  $q_{cr} = b\chi + \alpha q_1$ . In addition to collecting the remanufactured products at the end of the first period, the remanufacturer must also remanufacture the defective products from the initial production.

**Vehicle Routing Problem (VRP)**

The process of product collection was optimized using the concept of relative collection costs, representing the expenses incurred by the remanufacturer (RMFR) in collecting defective products, relative to the efficiency of the optimized collection routes determined by the vehicle routing problem (VRP). The manufacturer (MFR) sells  $q_1(1 - \alpha)$  of its products to customers at the beginning of the first period. It is assumed that the costs associated with collecting defective products from customers at the end of the first period, as well as the costs of collecting defective products produced at the start of the period, are borne by the remanufacturer (RMFR). Since the RMFR makes no effort to acquire the initially defective products (which are given by the MFR), it is logical for the RMFR to bear the collection costs. Consequently, the decision-making problem for the RMFR is expanded as follows.

**Optimized Remanufacturer’s Decision-Making function based on VRP**

The remanufacturer’s profit function now incorporates the relative costs associated with collecting defective products, modeled through a Vehicle Routing Problem (VRP) (Equation 15):

$$f_{RM} = \delta \left( (p_{cr} - c_{cr})q_{cr} - \left( 1 - \frac{\sum_{i=1}^n \sum_{j=1}^n c_{ij}}{\sum_{i=1}^n \sum_{j=1}^n \sum_{z=1}^Z c_{ij}^z x_{ij}^z} \right) \chi^2 \right), \tag{15}$$

Subject to:

$$q_{cr} \leq \min\{\gamma q_1, q_r\}, \tag{16}$$

$$\sum_{i=1}^n \sum_{z=1}^Z x_{ij}^z = 1, \quad \forall j = 1, 2, \dots, n, \tag{17}$$

$$\sum_{j=1}^n \sum_{z=1}^Z x_{ij}^z = 1, \quad \forall i = 1, 2, \dots, n, \tag{18}$$

$$\sum_{i=1}^n x_{ip}^z - \sum_{j=1}^n x_{jp}^z = 0, \quad \forall p = 1, 2, \dots, n, \quad \forall z = 1, 2, \dots, Z, \tag{19}$$

$$\sum_{i=0}^n \sum_{j=0}^n c_{ij} x_{ij}^z \leq L, \quad \forall z = 1, 2, \dots, Z, \tag{20}$$

$$\sum_{j=1}^n x_{0j}^z \leq 1, \quad \forall z = 1, 2, \dots, Z, \quad (21)$$

$$\sum_{i=1}^n x_{i0}^z \leq 1, \quad \forall z = 1, 2, \dots, Z, \quad (22)$$

$$x_{ij}^z \in \{0, 1\}, \quad \forall i = 1, 2, \dots, n, \quad \forall j = 1, 2, \dots, n, \quad \forall z = 1, 2, \dots, Z. \quad (23)$$

In Equation (15), the term  $\left(1 - \frac{\sum_{i=1}^n \sum_{j=1}^n c_{ij}}{\sum_{i=1}^n \sum_{j=1}^n \sum_{z=1}^Z c_{ij}^z x_{ij}^z}\right) \chi^2$  represents the relative cost the remanufacturer incurs to collect defective items from customers, equivalent to the effort cost  $k\chi^2$ . The VRP constraints (17) – (23) ensure proper routing and collection efficiency, where  $L$  denotes the maximum allowable distance for vehicle routing.

### **Sustainability Advertising**

In order to enhance the performance of the reverse supply chain model, this section explores sustainability advertising policies aimed at encouraging environmentally friendly behaviors among customers. By promoting greater use of recycled products and reducing pollution from discarded defective products, these advertisements can foster environmental sustainability. This advertising can be conducted by either the MFR or RMFR. It is assumed that the manufacturer, as the leader, sets the prices of the products ( $p_1, p_n, p_r$ ) and the Sustainability advertising strategy ( $\tau$ ).

#### **Optimized Manufacturer's Decision-Making function based on sustainability advertising**

The manufacturer's profit function is further extended to account for advertising expenses aimed at improving customer perception of remanufactured products:

$$f_M = (p_1 - c_n)q_1(1 - \alpha) + \delta[(p_n - c_n)q_n + (p_r - p_{cr} - c_r)q_r - \beta\tau^2], \quad (24)$$

Subject to constraints (8) and (9).

In Equation (24),  $\beta\tau^2$  denotes the costs associated with advertising efforts, where  $\tau$  is the intensity of advertising and  $\beta$  is the cost coefficient representing the rate of advertising expenditure. This particular formulation of the advertising effect draws on the approach used in the study by (Hong et al., 2015).

#### **Optimized Remanufacturer's Decision-Making function based on sustainability advertising**

The remanufacturer's profit function is similarly extended to incorporate the costs of recycling products influenced by advertising efforts. The updated profit function is:

$$f_{RM} = \delta \left( (p_{cr} - c_{cr})q_{cr} - \left(1 - \frac{\sum_{i=1}^n \sum_{j=1}^n c_{ij}}{\sum_{i=1}^n \sum_{j=1}^n \sum_{z=1}^Z c_{ij}^z x_{ij}^z}\right) \chi^2 - c_{rp}d\tau \right) \quad (25)$$

Subject to constraints (16) and (17) – (23).

In Equation (25),  $c_{rp}d\tau$  denotes the cost associated with recycling products due to advertising efforts, and  $d$  represents the effectiveness coefficient of advertising. The quantity of collected recycled products is given by  $q_{cr} = b\chi + \alpha q_1 + d\tau$ .

## **4. Model solution Approach**

The interaction between the manufacturer and the remanufacturer is modeled as a two-stage Stackelberg game:

- I. **Leader (Manufacturer):** The manufacturer sets the prices  $p_1, p_n, p_r$  and determines the level of environmental advertising  $\tau$  to maximize profit  $f_M$ .
- II. **Follower (Remanufacturer):** Observing the manufacturer's decisions, the remanufacturer sets the collection effort  $\chi$  to maximize profit  $f_{RM}$ .

### **Mixed-integer non-linear programming (MINLP) Solution**



Due to the complexity of the model, numerical methods and optimization algorithms are employed to solve the profit maximization problems for both the manufacturer and the remanufacturer. The solution procedure involves the following steps (backwards induction):

1. **Remanufacturer’s Optimization:** For given prices  $p_1, p_n, p_r$  and advertising effort  $\tau$ , solve the remanufacturer’s profit maximization problem to determine the optimal collection effort  $\chi$ .
2. **Manufacturer’s Optimization:** Substitute the optimal  $\chi$  into the manufacturer’s profit function and solve for the optimal prices  $p_1^*, p_n^*, p_r^*$ , and advertising effort  $\tau^*$ .
3. **Iterative Process:** Repeat the above steps iteratively until convergence is achieved, indicating that the Stackelberg equilibrium has been found.
4. **Vehicle Routing Problem (VRP):** At each iteration, solve the VRP to update the transportation costs and incorporate them into the remanufacturer’s profit function.

This iterative approach ensures that both the manufacturer and the remanufacturer reach an equilibrium where neither can unilaterally improve their profit by changing their strategies.

**Vehicle Routing Problem (VRP)**

The Vehicle Routing Problem (VRP) is integrated into the remanufacturer’s decision-making process to optimize the collection of defective products from customers. The objective is to minimize the total transportation cost while ensuring that all defective products are collected within the allowable distance  $L$ , as it reflect in equation (15) to do so in a single tour (i.e.,  $\sum_{i=1}^n \sum_{j=1}^n c_{ij}$ ). The constraints (17) – (21) ensure that each customer is visited exactly once, vehicles do not exceed the maximum distance  $L$ , and routing is feasible.

**Sustainable Strategy**

The advertising strategy  $\tau$  plays a crucial role in influencing customer perceptions and, consequently, demand for remanufactured products. By investing in advertising, the manufacturer can enhance the perceived utility of remanufactured products ( $\theta$ ) and increase customer willingness to purchase them, thereby potentially increasing  $q_r$ .

**5. Numerical Analyses**

This section presents a comprehensive evaluation of the proposed research model through a series of numerical examples. The analysis commences with an illustrative numerical problem, followed by a detailed description of the computational environment employed. Subsequently, a sensitivity analysis is conducted on the key parameters influencing the model, culminating in the interpretation of the obtained results.

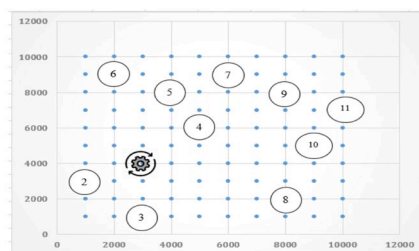
**Problem Illustration**

Consider a manufacturer that distributes products to 10 customer regions located within a specific geographical area. A remanufacturer collects defective products from customers and the manufacturer, remanufactures them, and sells them back to the manufacturer. Fig. 3 illustrates the geographical distribution of customers in relation to the manufacturer. (see distance matrix Table 3A).

**Table 2**

Model Parameters

$\alpha$	$c_n$	$\delta$	$\theta$	$c_r$	$\gamma$	$\beta$	$c_{rp}$	$d$	$b$	$L$
0.05	0.3	0.5	0.8	0.2	0.8	0.05	0.008	0.15	0.055	17000



**Fig. 3.** Geographical distribution of customers and the manufacturer

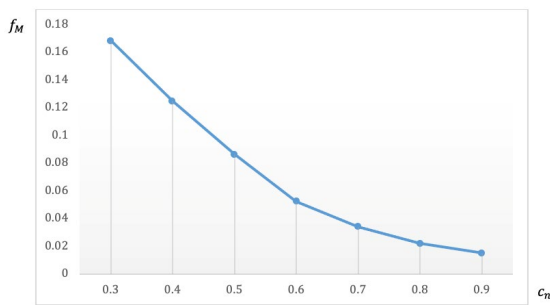
The numerical analyses were conducted using a system equipped with 8 GB of RAM and a dual-core Core i5 CPU. The routing information pertinent to the problem was initially processed using GAMS software and subsequently interfaced with MATLAB for advanced computational tasks.

**Basic model Sensitivity Analysis**

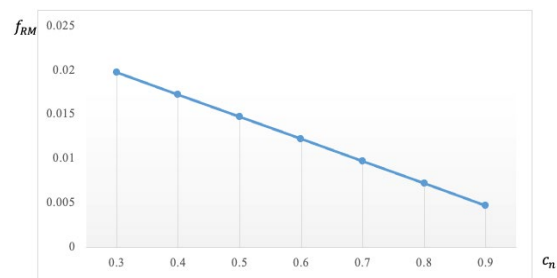
A sensitivity analysis was performed to examine the impact of key parameters on the model’s performance. Specifically, the analysis focused on the cost of production per unit ( $c_n$ ) and the cost of re-manufacturing per unit ( $c_r$ ).

**Impact of Production Cost ( $c_n$ )**

Fig. 4a illustrated the model under varying production costs. As observed, an increase in  $c_n$  leads to a reduction in the Manufacturer (MFR) profit, see Table 4A.



(a) Impact of unit production cost ( $c_n$ ) on the profitability of the manufacturer.

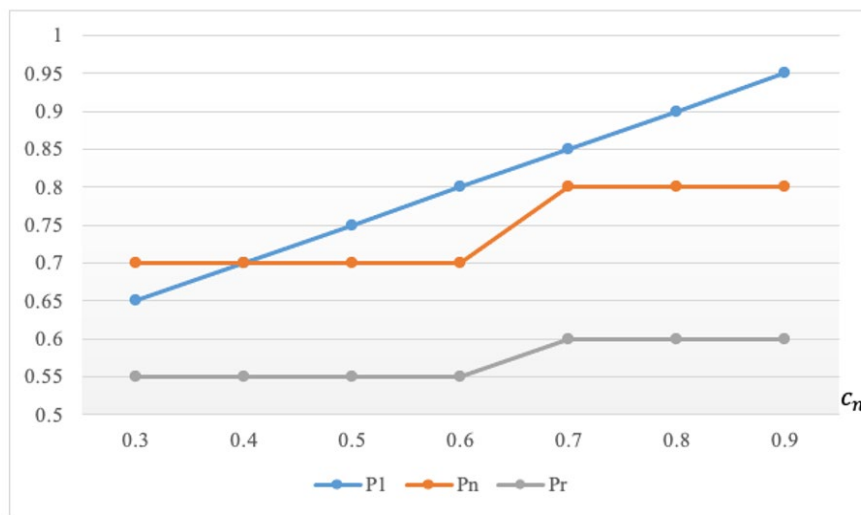


(b) Impact of unit production cost ( $c_n$ ) on the profitability of the remanufacturer.

**Fig. 4.** Comparison of manufacturer and remanufacturer profitability under varying unit production costs ( $c_n$ ).

An increase in the unit production cost ( $c_n$ ) necessitates a corresponding escalation in the product’s selling price ( $p_1$ ), thereby diminishing the manufacturer’s profit margins. This pricing adjustment adversely impacts the remanufacturer by elevating the cost of remanufactured products ( $p_r$ ), which leads to a lower adoption rate among consumers and reduced profit margins for the remanufacturer, Fig. 4b .

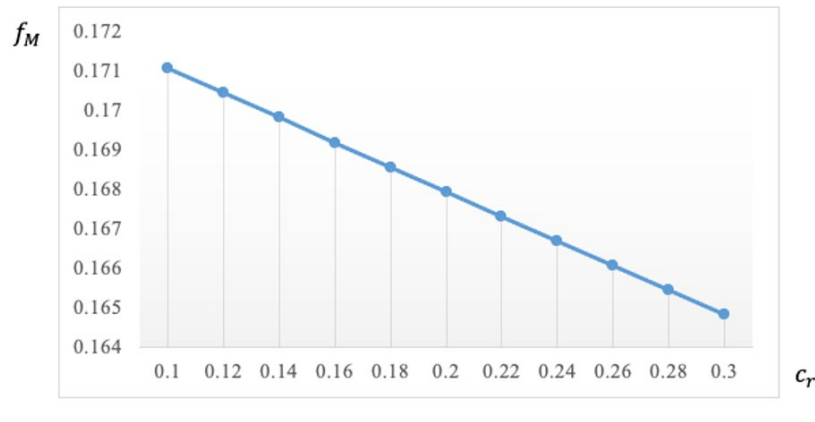
Furthermore, Fig. 5 illustrates that an increase in  $c_n$  results in higher selling prices during both the initial ( $p_n$ ) and subsequent periods ( $p_1$ ). This rise in prices forces the remanufacturer to purchase products at higher costs ( $p_r$ ), thereby increasing the overall cost of remanufactured products (see Table 5A).



**Fig. 5.** Variation in prices of new and remanufactured products across periods in response to changes in production cost ( $c_n$ ).

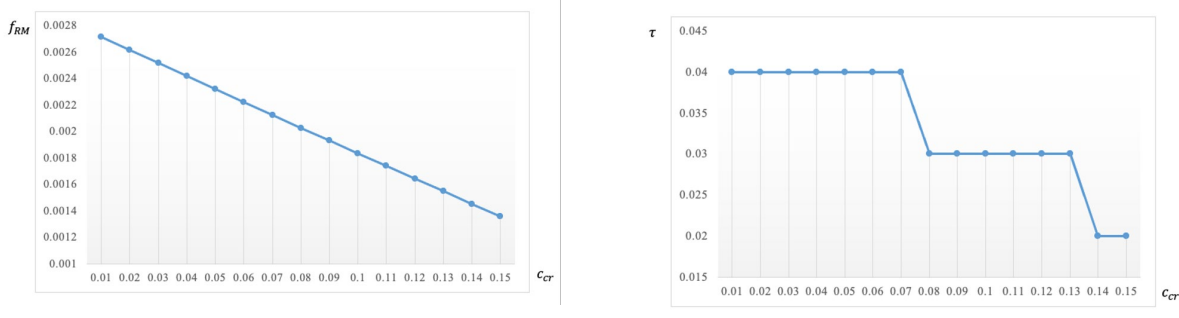
**Impact of Re-manufacturing Cost ( $c_r$ )**

Fig. 6 presents the model’s responses to varying re-manufacturing costs. An increase in  $c_r$  results in decreased revenue for the MFR, as it becomes unfeasible to adjust the price of remanufactured products ( $p_r$ ) under competitive market conditions. This reduction in revenue subsequently lowers the MFR’s profit margins, (see Table 5A).



**Fig. 6.** Impact of re-manufacturing cost ( $c_r$ ) on the profitability of the MFR.

Additionally, Fig. 7a shows the effects of fluctuations in re-manufacturing costs ( $c_{cr}$ ) determined by the manufacturer. As  $c_{cr}$  increases, the remanufacturer’s ability to maintain profitable margins diminishes, see detail Table 6A.



**(a)** Effect of re-manufacturing cost ( $c_{cr}$ ) on the profitability of remanufacturers.

**(b)** The strategic effort of remanufacturers’ sustainable advertising efforts in response to changes in re-manufacturing cost ( $c_{cr}$ ).

**Fig. 7.** Comparison of remanufacturers’ profitability and sustainable advertising efforts under varying re-manufacturing costs ( $c_{cr}$ ).

Moreover, Fig. 7b demonstrates that an increase in the cost of supplying remanufactured products ( $c_{cr}$ ) leads to a reduction in the remanufacturer’s motivation for environmental advertising, as profit margins are already compromised due to higher supply costs.

The sensitivity analyses on basic model reveal that both production and re-manufacturing costs significantly influence the profitability of the manufacturer and remanufacturer. Increases in  $c_n$  and  $c_r$  lead to higher selling prices, reduced profit margins, and decreased incentives for environmental advertising among remanufacturers. These findings underscore the importance of cost management in maintaining a sustainable and profitable supply chain.

**6. Optimization model Sensitivity Analysis**

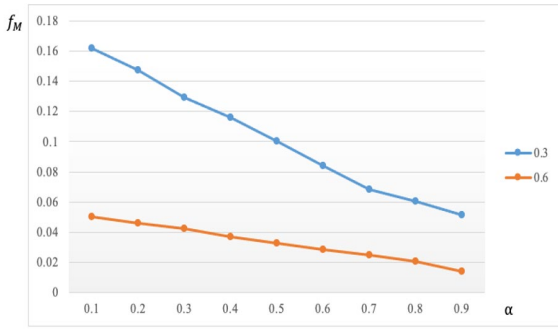
In this section, a sensitivity analysis is performed on key parameters associated with the innovations addressing the research problem. Specifically, the impact of the manufacturer’s initial production defect rate ( $\alpha$ ) and the allowable limit of vehicle movement during the routing of defective product collections from customers ( $L$ ) is examined.

### Sensitivity Analysis on the Initial Production Defect Rate of the MFR

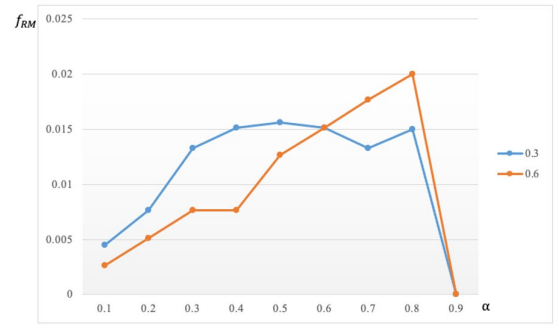
The parameter  $\alpha$  is analyzed to understand its influence on the optimal outcomes for both the Manufacturer (MFR) and the Remanufacturer (RMFR) profit functions. The parameter  $\alpha$  is varied within the set  $\{0.1, 0.2, \dots, 0.9\}$  across different states of  $c_n$ ,  $c_r$ , and  $c_{cr}$ . The results, detailed in Appendix Table 7A, illustrate how changes in  $\alpha$  affect in two levels  $c_n \in \{0.3, 0.6\}$ .

As the initial defect rate  $\alpha$  increases, the costs associated with modifying defective products escalate, leading to a decline in the MFR's performance and a consequent reduction in profit margins. This relationship is visually represented in Figure 8a, where higher defect rates correlate with decreased profits for the MFR.

Figure 8b further elucidates the effect of  $\alpha$  on the RMFR's profit. Initially, as  $\alpha$  increases, the demand for remanufactured products rises, enhancing RMFR's profitability. However, beyond a threshold ( $\alpha \geq 0.8$ ), customer demand sharply declines due to dissatisfaction with high defect rates, rendering remanufacturing economically unviable and reducing RMFR's profits to zero.



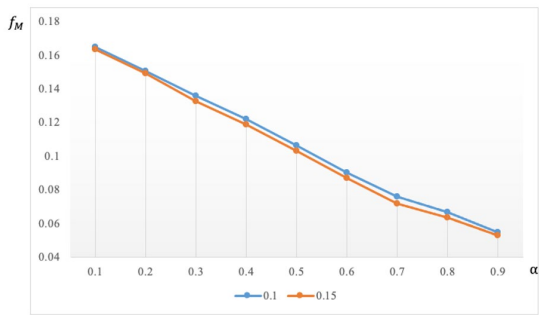
(a) Impact of varying  $\alpha$  on the MFR's profit for different values of  $c_n \in \{0.3, 0.6\}$ .



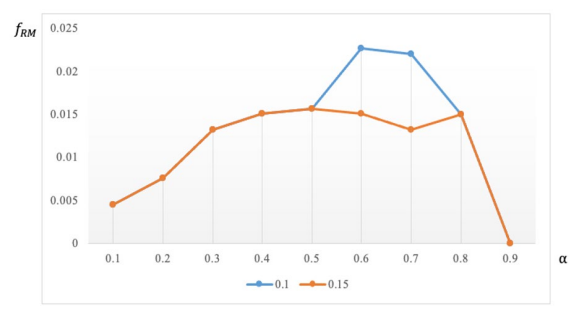
(b) Impact of varying  $\alpha$  on the RMFR's profit for different values of  $c_n \in \{0.3, 0.6\}$ .

**Fig. 8.** Comparison of MFR and RMFR profits under varying  $\alpha$  and remanufacturing costs  $c_n$ .

Table 7A and Table 8A provide a comprehensive overview of the model's responses to changes in  $\alpha$  under varying cost conditions ( $c_n \in \{0.3, 0.6\}$  and  $c_r \in \{0.1, 0.15\}$ ). The analysis indicates that an increase in waste not only diminishes the MFR's profit margins but also adversely affects the RMFR, especially when remanufacturing costs are elevated. This dynamic is further illustrated in Figure 9a and b.



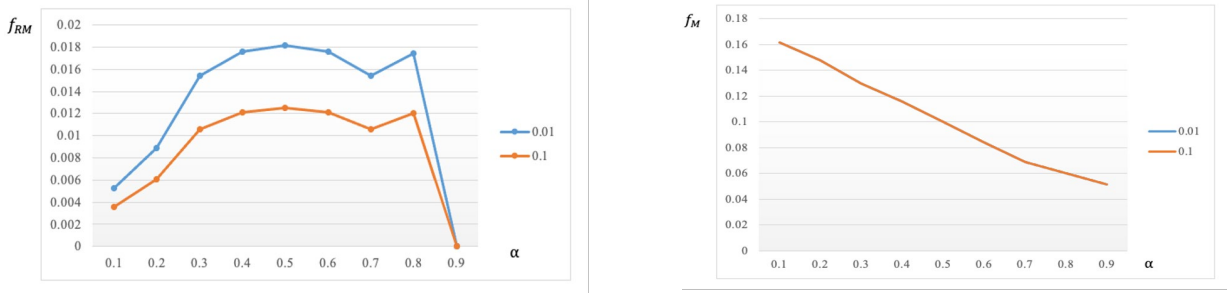
(a) Impact of varying  $\alpha$  on the MFR's profit for different values of  $c_r \in \{0.1, 0.15\}$ .



(b) Impact of varying  $\alpha$  on the RMFR's profit for different values of  $c_r \in \{0.1, 0.15\}$ .

**Fig. 9.** Comparison of MFR and RMFR profits under varying  $\alpha$  and remanufacturing costs  $c_r$ .

Further analysis, illustrated in Fig. 10a, demonstrates that the profitability of the RMFR is considerably more sensitive to lower remanufacturing costs ( $c_{cr} = 0.01$ ) compared to higher costs ( $c_{cr} = 0.1$ ). Specifically, as the level of waste increases—referenced in Table 7A and Table 8A—the manufacturer's profit ( $f_M$ ) declines, while the demand for remanufactured products (RMPs) initially rises and subsequently falls. Moreover, Table 9A reveals that changes in the defect rate ( $\alpha$ ) have minimal impact on the manufacturer's profit when remanufacturing costs vary, as depicted in Figure 10b and Figure 10b. This insensitivity is attributed to the manufacturer's fixed supply price for RMPs. In contrast, lower remanufacturing costs ( $c_{cr} = 0.01$ ) make remanufacturing operations more attractive and viable for the RMFR, thereby sustaining profitability despite variations in  $\alpha$ . This indicates that reducing remanufacturing costs not only enhances the RMFR's profit margins but also supports the overall sustainability and viability of remanufacturing within the supply chain.



(a) Impact of varying  $\alpha$  on the RMFR's profit for different remanufacturing costs  $c_{cr} \in \{0.1, 0.01\}$ .

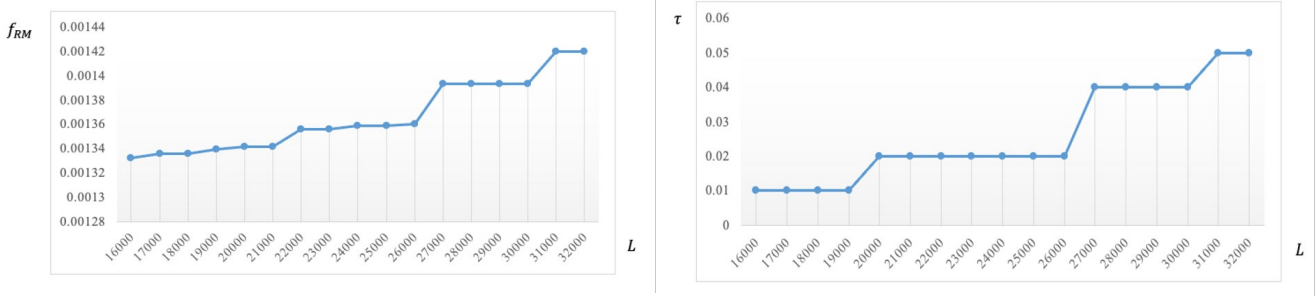
(b) Impact of varying  $\alpha$  on the MFR's profit for different remanufacturing costs  $c_{cr} \in \{0.1, 0.01\}$ .

**Fig. 10.** Comparison of RMFR and MFR profits under varying  $\alpha$  and remanufacturing costs  $c_{cr}$ .

**Sensitivity Analysis on Vehicle Routing problem**

This subsection examines the sensitivity of the model to the parameter  $L$ , which defines the maximum allowable vehicle movement range during the routing of defective product collections. The parameter  $L$  is varied from 16,000 to 32,000, and the corresponding impacts on the RMFR's profit and routing efficiency are summarized in Appendix Table 10A.

As  $L$  increases, the RMFR's profitability rises significantly. This improvement is attributed to more efficient routing, which reduces the number of sub-networks formed within the collection network. By allowing vehicles to cover longer distances, the routing solution becomes closer to the ideal, thereby minimizing the relative remanufacturing costs. Mathematically, this cost reduction is represented by the term  $\left(1 - \frac{\sum_{i=1}^n \sum_{j=1}^n c_{ij}}{\sum_{i=1}^n \sum_{j=1}^n \sum_{z=1}^Z c_{ij}^z x_{ij}^z}\right)$ . A higher  $L$  leads to a smaller value of this term, indicating lower remanufacturing costs. This relationship is depicted in Figure 11a, where increased vehicle range  $L$  results in enhanced RMFR profits.



(a) Changes in the re-manufacturer's profit as a result of changes in parameter  $L$ .

(b) Impact of varying  $L$  on the RMFR's environmental advertising efforts.

**Fig. 11.** Comparison of the re-manufacturer's profit and environmental advertising efforts under varying parameter  $L$ .

Fig. 11b illustrates the variations in the level of environmental advertising efforts in relation to changes in the parameter  $L$ . As evident, as the range of motion of the vehicle increases and the costs of the remanufacturer's route decrease, it becomes more feasible for the remanufacturer to invest in environmental advertising policies, leading to an increase in their efforts. To examine the routing structure of vehicle traffic among customers, As depicted in Figure 12A and Figure 21A, an increase in the value of  $L$  results in an expansion of sub-network formations, leading to a decrease in the collection costs of remanufactured products (RMPs) by the RMFR. Consequently, the MFR is able to enhance their profit margins, as reflected in Table 10A.

**7. Conclusion**

This study presents a comprehensive two-stage reverse supply chain model that captures the intricate interactions between manufacturers (MFRs) and remanufacturers (RMFRs) within a Stackelberg game framework. By integrating production constraints, vehicle routing optimization, and sustainability advertising strategies, the model provides valuable insights into the pricing and profitability dynamics of remanufactured products (RMPs).

Our numerical analyses reveal several key findings:

- *Impact of Production Costs:* An increase in the manufacturer's production cost per unit ( $c_n$ ) necessitates a corresponding rise in the selling price of new products ( $p_1$ ). This escalation reduces the manufacturer's profit margins and adversely affects the remanufacturer by increasing the cost of RMPs ( $p_r$ ), thereby diminishing consumer demand and the remanufacturer's profitability.
- *Defect Rate Sensitivity:* Higher defect rates ( $\alpha$ ) initially enhance the remanufacturer's demand for RMPs, boosting profitability. However, beyond a critical threshold ( $\alpha \geq 0.8$ ), excessive defects lead to significant customer dissatisfaction, resulting in a sharp decline in demand for RMPs and rendering remanufacturing economically unviable.
- *Vehicle Routing problem:* Expanding the allowable vehicle movement range ( $L$ ) in the vehicle routing problem (VRP) significantly enhances RMFR profitability by optimizing collection routes and reducing overall collection costs. This efficiency gain not only increases the remanufacturer's profit margins but also allows for greater investment in sustainability advertising, further strengthening market competitiveness.
- *Sustainability Advertising:* Investment in sustainability advertising ( $\tau$ ) plays a crucial role in influencing consumer perceptions and demand for RMPs. However, increased remanufacturing costs ( $c_{cr}$ ) can constrain advertising efforts, highlighting the need for cost-effective marketing strategies to sustain environmental initiatives.

The integration of VRP and sustainability advertising within the reverse supply chain model underscores the importance of logistical efficiency and proactive marketing in achieving a sustainable and profitable supply chain. By addressing both operational and strategic dimensions, the model offers a holistic approach to managing remanufacturing processes.

*Implications for Practice:* Manufacturers and remanufacturers can leverage the insights from this study to optimize pricing strategies, manage production and remanufacturing costs, and design effective advertising campaigns. Specifically, enhancing vehicle routing efficiency and strategically investing in sustainability advertising can mitigate the adverse effects of rising production costs and high defect rates, thereby sustaining profitability.

*Future Research Directions:* While this study provides a robust foundation, future research could extend the model to accommodate multiple manufacturers and a diverse range of products, thereby introducing competitive dynamics within the remanufacturing sector. Additionally, exploring consumer behavior variations related to risk perceptions of RMPs and the impact of independent sales channels for remanufacturers would enrich our understanding of market segmentation and distribution strategies. Moreover, detailed case studies in diverse industrial settings should be conducted to empirically validate the model, test its underlying assumptions, and refine its applicability to real-world reverse supply chain operations.

In conclusion, this research contributes significantly to the literature on reverse supply chain management by elucidating the multifaceted factors that influence the profitability and sustainability of remanufacturing operations. The findings advocate for a balanced approach that harmonizes cost management, logistical optimization, and strategic marketing to foster a resilient and environmentally responsible supply chain.

## Acknowledgments

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## Data Availability Statement:

Data are contained within the article.

## Conflicts of Interest

The authors declare no conflict of interest.

## Appendix

### Result Details

**Table 3A**

The distance between customer centers and each other as well as RMFR.

	2	3	4	5	6	7	8	9	10	11
1	3162.3	4000.0	2828.4	4123.1	5099.0	5831.0	5385.2	6403.1	6082.8	7615.8
2	0.0	4242.6	5831.0	6403.1	6324.6	8485.3	8062.3	9434.0	9219.5	10770.3
3	-	0.0	6324.6	8062.3	9055.4	9486.8	5385.2	9434.0	7810.2	9899.5
4	-	-	0.0	2236.1	4242.6	3162.3	5000.0	3605.6	4123.1	5099.0
5	-	-	-	0.0	2236.1	2236.1	7211.1	4000.0	5831.0	6082.8
6	-	-	-	-	0.0	4000.0	9219.5	6082.8	8062.3	8246.2
7	-	-	-	-	-	0.0	7280.1	2236.1	5000.0	4472.1
8	-	-	-	-	-	-	0.0	6000.0	3162.3	5385.2
9	-	-	-	-	-	-	-	0.0	3162.3	2236.1
10	-	-	-	-	-	-	-	-	0.0	2236.1

**Table 4A**

Results of solving the model with respect to parameter changes  $c_n$

$c_n$	$f_M$	$q_r$	$q_n$	$q_1$	$p_1$	$p_n$	$p_r$	$f_{RM}$	$\tau$	$q_{cr}$
0.3	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002319	0.04	0.0197
0.4	0.124563	0.0625	0.25	0.3	0.7	0.7	0.55	0.002006	0.04	0.0172
0.5	0.085938	0.0625	0.25	0.25	0.75	0.7	0.55	0.001694	0.04	0.0147
0.6	0.052063	0.0625	0.25	0.2	0.8	0.7	0.55	0.001381	0.04	0.0122
0.7	0.033875	0.25	0	0.15	0.85	0.8	0.6	0.001069	0.04	0.0097
0.8	0.022	0.25	0	0.1	0.9	0.8	0.6	0.000756	0.04	0.0072
0.9	0.014875	0.25	0	0.05	0.95	0.8	0.6	0.000444	0.04	0.0047

**Table 5A**

Impact of the cost of re-manufacturing each unit of product by the manufacturer

$c_r$	$f_M$	$q_r$	$q_n$	$q_1$	$p_1$	$p_n$	$p_r$	$f_{RM}$	$\tau$	$q_{cr}$
0.1	0.171 063	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.12	0.170 438	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.14	0.169 813	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.16	0.166 188	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.18	0.168 563	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.2	0.167 938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.22	0.167 313	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.24	0.166 688	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.26	0.166 063	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.28	0.165 438	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197
0.3	0.164 813	0.0625	0.25	0.35	0.65	0.7	0.55	0.002 319	0.04	0.0197

**Table 6A**

Impact of the cost of re-manufacturing the product by the remanufacturer( $c_{cr}$ ).

$c_{cr}$	$f_M$	$q_r$	$q_n$	$q_1$	$p_1$	$p_n$	$p_r$	$f_{RM}$	$\tau$	$q_{cr}$
0.01	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002713	0.04	0.0197
0.02	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002614	0.04	0.0197
0.03	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002516	0.04	0.0197
0.04	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002417	0.04	0.0197
0.05	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002319	0.04	0.0197
0.06	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002222	0.04	0.0197
0.07	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002122	0.04	0.0197
0.08	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.002026	0.03	0.01915
0.09	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.00193	0.03	0.01915
0.1	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.001834	0.03	0.01915
0.11	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.001738	0.03	0.01915
0.12	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.001643	0.03	0.01915
0.13	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.001547	0.03	0.01915
0.14	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.001452	0.02	0.0186
0.15	0.167938	0.0625	0.25	0.35	0.65	0.7	0.55	0.001359	0.02	0.0186

**Table 7A**

Impact of variations in  $\alpha$  for different values of  $c_n$

$c_n$	$\alpha$	$f_M$	$q_r$	$q_n$	$q_1$	$p_1$	$p_n$	$p_r$	$f_{RM}$	$\tau$	$q_{cr}$
0.3	0.1	0.161813	0.0625	0.25	0.35	0.65	0.7	0.55	0.004506	0.04	0.0372
0.3	0.2	0.147563	0.0625	0.25	0.3	0.7	0.7	0.55	0.007631	0.04	0.0622
0.3	0.3	0.12995	0.125	0.25	0.35	0.65	0.65	0.5	0.013256	0.04	0.1072
0.3	0.4	0.11575	0.125	0.25	0.3	0.7	0.65	0.5	0.015131	0.04	0.1222
0.3	0.5	0.1	0.125	0.25	0.25	0.75	0.65	0.5	0.015625	0	0.125
0.3	0.6	0.08375	0.125	0.25	0.2	0.8	0.65	0.5	0.015131	0.04	0.1222
0.3	0.7	0.0685	0.125	0.25	0.15	0.85	0.65	0.5	0.013256	0.04	0.1072
0.3	0.8	0.06025	0.125	0.25	0.15	0.85	0.65	0.5	0.015	0	0.12
0.3	0.9	0.051563	0.0625	0.25	0	1	0.7	0.55	0	0	0
0.6	0.1	0.050063	0.0625	0.25	0.2	0.8	0.7	0.55	0.002631	0.04	0.0222
0.6	0.2	0.046063	0.0625	0.25	0.2	0.8	0.7	0.55	0.005131	0.04	0.0422
0.6	0.3	0.042063	0.0625	0.25	0.2	0.8	0.7	0.55	0.007631	0.04	0.0622
0.6	0.4	0.036563	0.0625	0.25	0.15	0.85	0.7	0.55	0.007631	0.04	0.0622
0.6	0.5	0.0325	0.25	0	0.2	0.8	0.8	0.6	0.012631	0.04	0.1022
0.6	0.6	0.0285	0.25	0	0.2	0.8	0.8	0.6	0.015131	0.04	0.1222
0.6	0.7	0.0245	0.25	0	0.2	0.8	0.8	0.6	0.017631	0.04	0.1422
0.6	0.8	0.0205	0.25	0	0.2	0.8	0.8	0.6	0.02	0	0.16
0.6	0.9	0.014063	0.0625	0.25	0	1	0.7	0.55	0	0	0



**Table 8A**

Impact of  $\alpha$  for different values of  $c_r$

$c_r$	$\alpha$	$f_M$	$q_r$	$q_n$	$q_l$	$p_1$	$p_n$	$p_r$	$f_{RM}$	$\tau$	$q_{cr}$
0.10	0.1	0.164938	0.0625	0.25	0.35	0.65	0.7	0.55	0.004506	0.04	0.0372
0.10	0.2	0.150688	0.0625	0.25	0.3	0.7	0.7	0.55	0.007631	0.04	0.0622
0.10	0.3	0.13575	0.125	0.25	0.35	0.65	0.65	0.5	0.013256	0.04	0.1072
0.10	0.4	0.122	0.125	0.25	0.3	0.7	0.65	0.5	0.015131	0.04	0.1222
0.10	0.5	0.10625	0.125	0.25	0.25	0.75	0.65	0.5	0.015625	0	0.125
0.10	0.6	0.090188	0.1875	0.25	0.3	0.7	0.6	0.45	0.022631	0.04	0.1822
0.10	0.7	0.075938	0.1875	0.25	0.25	0.75	0.6	0.45	0.022006	0.04	0.1772
0.10	0.8	0.0665	0.125	0.25	0.15	0.85	0.65	0.5	0.015	0	0.12
0.10	0.9	0.054688	0.0625	0.25	0	1	0.7	0.55	0	0	0
0.15	0.1	0.163375	0.0625	0.25	0.35	0.65	0.7	0.55	0.004506	0.04	0.0372
0.15	0.2	0.149125	0.0625	0.25	0.3	0.7	0.7	0.55	0.007631	0.04	0.0622
0.15	0.3	0.132625	0.125	0.25	0.35	0.65	0.65	0.5	0.013256	0.04	0.1072
0.15	0.4	0.118875	0.125	0.25	0.3	0.7	0.65	0.5	0.015131	0.04	0.1222
0.15	0.5	0.103125	0.125	0.25	0.25	0.75	0.65	0.5	0.015625	0	0.125
0.15	0.6	0.086875	0.125	0.25	0.2	0.8	0.65	0.5	0.015131	0.04	0.1222
0.15	0.7	0.071625	0.125	0.25	0.15	0.85	0.65	0.5	0.013256	0.04	0.1072
0.15	0.8	0.063375	0.125	0.25	0.15	0.85	0.65	0.5	0.015	0	0.12
0.15	0.9	0.053125	0.0625	0.25	0	1	0.7	0.55	0	0	0

**Table 9A**

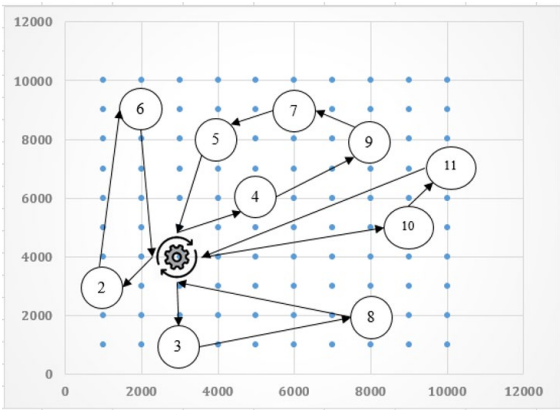
Impact of  $\alpha$  and for different amounts of re-manufactured cost by the remanufacturer

$c_{cr}$	$\alpha$	$f_M$	$q_r$	$q_n$	$q_l$	$p_1$	$p_n$	$p_r$	$f_{RM}$	$\tau$	$q_{cr}$
0.01	0.1	0.161813	0.0625	0.25	0.35	0.65	0.7	0.55	0.00525	0.04	0.0372
0.01	0.2	0.147563	0.0625	0.25	0.3	0.7	0.7	0.55	0.008875	0.04	0.0622
0.01	0.3	0.1295	0.125	0.25	0.35	0.65	0.65	0.5	0.0154	0.04	0.1072
0.01	0.4	0.11575	0.125	0.25	0.3	0.7	0.65	0.5	0.017575	0.04	0.1222
0.01	0.5	0.1	0.125	0.25	0.25	0.75	0.65	0.5	0.018125	0	0.125
0.01	0.6	0.08375	0.125	0.25	0.2	0.8	0.65	0.5	0.017575	0.04	0.1222
0.01	0.7	0.0685	0.125	0.25	0.15	0.85	0.65	0.5	0.0154	0.04	0.1072
0.01	0.8	0.06025	0.125	0.25	0.15	0.85	0.65	0.5	0.0174	0	0.12
0.01	0.9	0.051563	0.0625	0.25	0	1	0.7	0.55	0	0	0
0.10	0.1	0.161813	0.0625	0.25	0.35	0.65	0.7	0.55	0.003584	0.03	0.03665
0.10	0.2	0.147563	0.0625	0.25	0.3	0.7	0.7	0.55	0.006084	0.03	0.06165
0.10	0.3	0.1295	0.125	0.25	0.35	0.65	0.65	0.5	0.010584	0.03	0.10665
0.10	0.4	0.11575	0.125	0.25	0.3	0.7	0.65	0.5	0.012084	0.03	0.12165
0.10	0.5	0.1	0.125	0.25	0.25	0.75	0.65	0.5	0.0125	0	0.125
0.10	0.6	0.08375	0.125	0.25	0.2	0.8	0.65	0.5	0.012084	0.03	0.12165
0.10	0.7	0.0685	0.125	0.25	0.15	0.85	0.65	0.5	0.010584	0.03	0.10665
0.10	0.8	0.06025	0.125	0.25	0.15	0.85	0.65	0.5	0.012	0	0.12
0.10	0.9	0.051563	0.0625	0.25	0	1	0.7	0.55	0	0	0

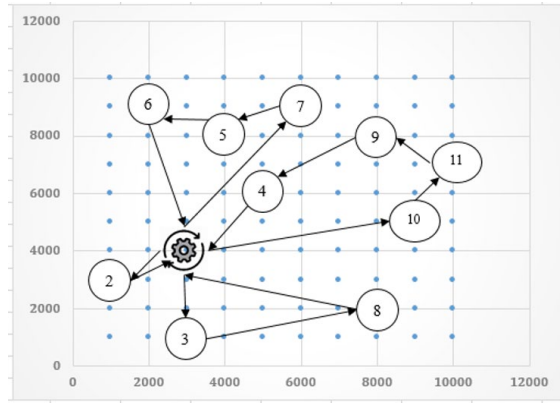
**Table 10A**

Results of model sensitivity analysis on the parameter L

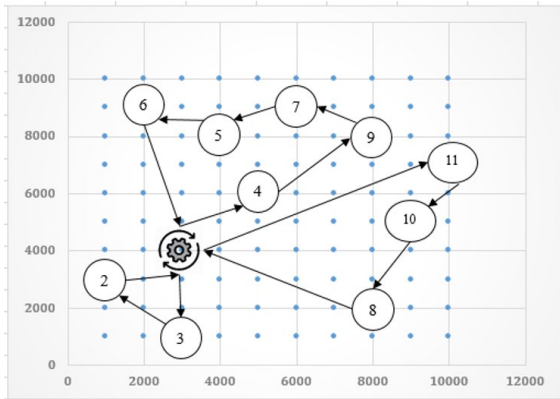
$L$	$f_M$	$f_{RM}$	$\tau$	$q_{cr}$	Vehicle motion routing structure
16000	0.167938	0.001332	0.01	0.01805	[1 4 9 7 5 1; 1 10 11 1; 1 3 8 1; 1 2 6 1]
17000	0.167938	0.001335	0.01	0.01805	[1 10 11 9 4 1; 1 2 1; 1 3 8 1; 1 7 5 6 1]
18000	0.167938	0.001335	0.01	0.01805	[1 10 11 9 4 1; 1 2 1; 1 3 8 1; 1 7 5 6 1]
19000	0.167938	0.001339	0.01	0.01805	[1 3 2 1; 1 4 9 7 5 6 1; 1 11 10 8 1]
20000	0.167938	0.001342	0.02	0.0186	[1 2 3 1; 1 6 5 7 1; 1 8 10 11 9 4 1]
21000	0.167938	0.001342	0.02	0.0186	[1 2 3 1; 1 6 5 7 1; 1 8 10 11 9 4 1]
22000	0.167938	0.001356	0.02	0.0186	[1 3 2 6 5 1; 1 8 10 11 9 7 4 1]
23000	0.167938	0.001356	0.02	0.0186	[1 3 2 6 5 1; 1 8 10 11 9 7 4 1]
24000	0.167938	0.001359	0.02	0.0186	[1 2 3 8 1; 1 6 5 7 9 11 10 4 1]
25000	0.167938	0.001359	0.02	0.0186	[1 4 6 3 2 1; 1 9 10 11 7 5 6 1]
26000	0.167938	0.00136	0.02	0.0186	[1 3 8 10 11 9 7 4 1; 1 2 6 5 1]
27000	0.167938	0.001393	0.04	0.0197	[1 3 2 1; 1 4 5 6 7 9 11 10 8 1]
28000	0.167938	0.001393	0.04	0.0197	[1 3 2 1; 1 4 5 6 7 9 11 10 8 1]
29000	0.167938	0.001393	0.04	0.0197	[1 3 2 1; 1 4 5 6 7 9 11 10 8 1]
30000	0.167938	0.001393	0.04	0.0197	[1 3 2 1; 1 4 5 6 7 9 11 10 8 1]
31000	0.167938	0.00142	0.05	0.02025	[1 2 1; 1 3 8 10 11 9 7 6 5 4 1]
32000	0.167938	0.00142	0.05	0.02025	[1 2 1; 1 3 8 10 11 9 7 6 5 4 1]



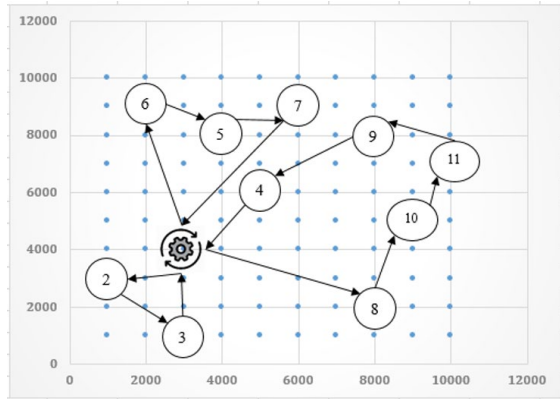
**Fig. 12A.** Sub-tours Formation - L = 16000



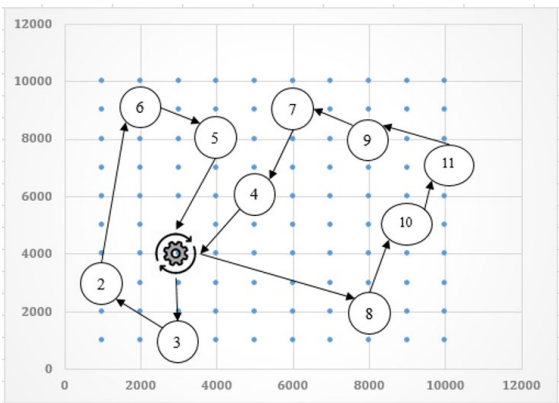
**Fig. 13A.** Sub-tours Formation - L = 18000 and L = 17000



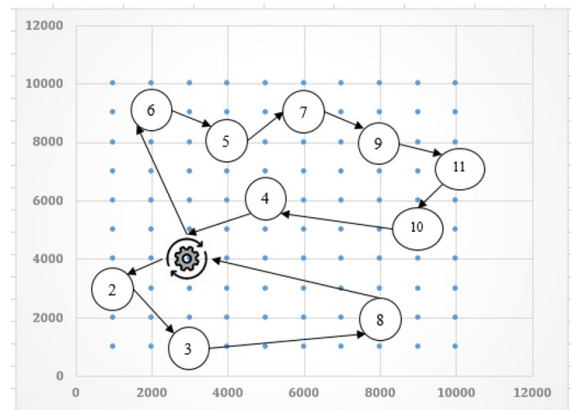
**Fig. 14A.** Sub-tours Formation - L = 19000



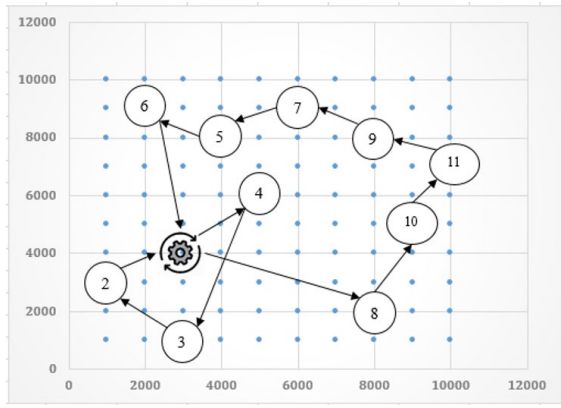
**Fig. 15A.** Sub-tours Formation - L = 20000 and L = 25000



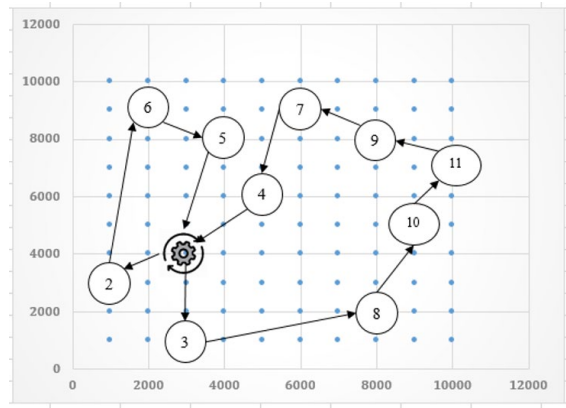
**Fig. 16A.** Sub-tours Formation - L=22000 and Sub-tours Formation - L=23000



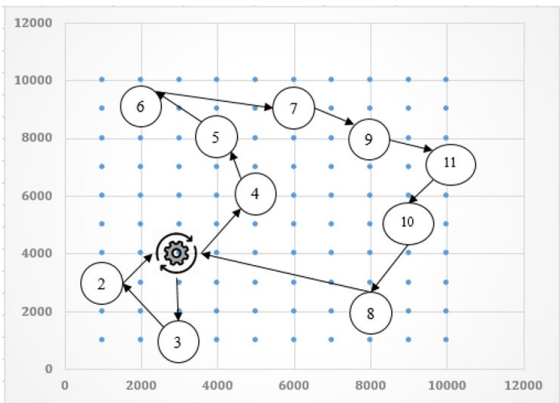
**Fig. 17a.** Sub-tours Formation - L=24000



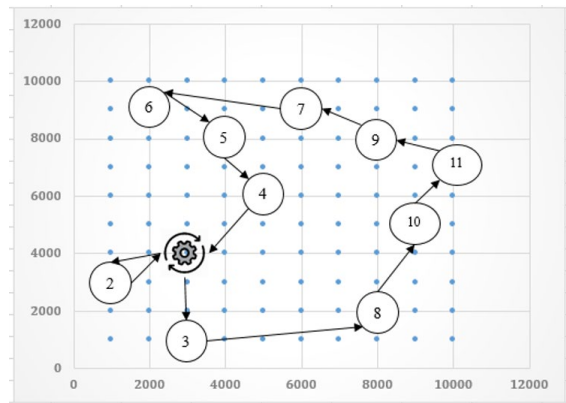
**Fig. 18A.** Sub-tours Formation - L=25000



**Fig. 19A.** Sub-tours Formation - L=26000



**Fig. 20A.** Sub-tours Formation - L=27000 and L=30000



**Fig. 21A.** Sub-tours Formation - L=31000 and L=32000



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