Two-warehouse production policy for different demands under volume flexibility

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ABSTRACT

In this paper, an inventory model of two-warehouse is considered, which evaluates the impact of a reduction rate in the selling price with volume flexibility. In real life, there are many products, which may decay or deteriorate or become obsolete. Therefore, one alternative is to clear the stock by selling a large amount of items at reduced prices. Taking this concept into account, this paper considers a fixed demand rate at the beginning of planning until the certain time point occurs, while demand is assumed to follow the pattern of nonlinear and non-decreasing power function of the reduction rate. The total cost function includes warehouse and rented warehouse holding costs, set up cost and the production cost. Numerical illustrations are given to exemplify the model and the proposed model is solved using a Genetic Algorithm (GA) with sensitivity analysis.

1. Introduction

During the past few decades, production firms and business enterprises have executed broad information systems in order to improve their performances but in many scenarios, the yields come out to be much less than anticipated (Sharma, 2012). Earlier, the researchers observed that in order to comprehend maximum functioning improvements, it is necessary to get well-time data about customer’s demand. Functioning procedures like inventory management and accumulation arrangement have to be valuable to improve the firms’ or enterprises’ inventory performance. The key objective of firms is to meet demand punctually by providing high quality services. Keeping this point in mind, shops or companies do their best to store inventories in their own warehouse (OW) or rented warehouse (RW), in case of necessary. In general, when suppliers offer price discounts for bulk purchases, or when the considered items are seasonal products such as the harvest output, or for stock-dependent demand, the administrator may procure more goods than the capacity of its own warehouse. Therefore, the surplus inventories are stored in a rented warehouse.
The inventory costs including the holding and the deterioration expenditures in the RW are generally higher than those in the OW due to the extra cost of safeguarding and material holding. The studies by Hartely (1976), Sarma (1983, 1987), Goswami and Chaudhuri (1992), Bhunia and Maiti (1994), Benkerouf (1997), shed light at these topics. Bhunia and Maiti (1998) established a deterministic inventory model with two warehouses by considering various deteriorations in both warehouses. Lee and Ma (2000) worked on a two-warehouse model and developed a heuristic solution of equal production cycle times with a general time-dependent demand function over a finite planning horizon. Zhou (2003) extended the model of Bhunia and Maiti (1998) allowing shortages where stock is transported from each RW to OW, continuously.

Yang (2004) observed that the RW works as a Central Warehousing Facility that usually provides better preservation facility than the OW resulting in a lower deterioration rate for the goods. In order to decrease inventory expenditures, it is reasonable to store goods in an OW before the RW and to clear the stock in the RW before the OW. Yu et al. (2005) studied a production-inventory model based on deteriorating item with imperfect quality and partial back-ordering. Zhou and Yang (2005) explored a two warehouse inventory model with stock-dependent demand rate considering that the own warehouse had limited capacity and the rented warehouse had unlimited capacity. They assumed the transportation cost for transferring items from RW to OW dependent on the supplied quantity. Teng and Chang (2005) examined an economic production quantity (EPQ) model for deteriorating items with demand depending on stock-level and price.

Ghosh and Chaudhuri (2006) worked on an economic order quantity (EOQ) model for quadratic demand assuming time-dependent deterioration rate. Chung and Huang (2007) established a two-warehouse inventory model for deteriorating items under a permissible delay in payments assuming the same deterioration rates of the two warehouses. Dey et al. (2008) explored a finite time horizon inventory problem for a deteriorating item having two separate warehouses with interval-valued lead-time under inflation and a time value of money. Niu and Xie (2008) customized Pakkala and Achary’s model (last-in-first-out) where inventory in the RW was stored last but would be consumed before those in the OW.

Hsieh et al. (2008) examined a deterministic inventory model for deteriorating items involving two warehouses and assuming the inventory costs in the RW to be higher than those in the OW. Mishra and Mishra (2008) developed an EOQ model with the objective of analysis and computing the unit price for deteriorating items under the ideal competition as a significant market constitution. Rong et al. (2008) presented an optimization inventory policy for a deteriorating item with imprecise lead-time, partially/fully backlogged shortages and price dependent demand under a two-warehouse system. Lee and Hsu (2009) extended the Lee and Ma (2000)’s model by considering variable production cycle times instead of equal production cycle times. Chung et al. (2009) extended a two-warehouse inventory model with an imperfect quality production processes. Liao and Huang (2010) explored an order-level inventory model for deteriorating items with two-storage facilities and trade credit.

Hariga (2011) suggested an EOQ model with multi-warehouses where both owned and rented warehouses had limited stock capacity. He assumed that the inventory manager could negotiate either a fixed or flexible space rental contract and also have access to spot markets to acquire more space, if needed. Liang and Zhou (2011) examined a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payments. They considered the greater holding costs for rented warehouse in comparison to own warehouse while RW’s superior preservation resulting in a lower rate of deterioration for the goods than in the own warehouse. Sana (2010) discussed a multi-item EOQ model for deteriorating and ameliorating items under capacity constraint with a time varying demand also subjected to organizational schemes like advertising. Sana et al. (2011) developed a two-warehouse ordering quantity model considering pricing decision. Sett et al. (2012) considered a two-warehouse production model for quadratically increasing demand and time varying deterioration.
Practically, each item has a lifetime after which its value (quality) decreases. The lifetime is unfixed and depends on the quality of the item and also on preservation storage space. For instance, when deterioration occurs in textile shops, management enhances the sale of the commodities. Simultaneously, management offers a reduced selling price to motivate customers to buy more. Taking into account in this concept, a structural EPQ model is developed in this chapter that estimates the influence of a reduction in the selling price when the capacity of OW is limited and RW is used, if needed. The demand rate is fixed \( d_1 \) up to time \( \mu \); and after time \( \mu \) the demand rate \( d_2 \) dependent on its reduction rate \( r \). The demand rate \( d_2(r) \) has an exponential trend that can be estimated/fitted using a curve fitting method. The associated profit maximization objective function is solved using GA. All possible cases of the model are formulated and then numerically illustrated. Finally, conclusions are drawn from the proposed models.

In this paper, the demand rate \( d_2(r) \) is incorporated with EPQ model considering two warehouses and volume flexibility. This combination has not been studied methodically in existing literature. The rest of the chapter is organized as follows: section 2 provides the fundamental assumptions and notations, section 3 describes the formulation of the model, section 4 provides numerical examples, and conclusions are given in section 5.

2. Assumptions and Notations

2.1. Assumptions

(i) The model is developed for a single item.
(ii) The replenishment rate is infinite but replenishment size is finite.
(iii) The lead time is zero.
(iv) No shortages are permitted.
(v) Own Warehouse (OW) and Rented Warehouse (RW) are considered.
(vi) The time horizon is infinite.

3. Model formulation

The production starts at \( t = 0 \) with a rate \( P \) and due to the combined effect of production and demand, inventory level increases up to \( W \) till time \( t = t_1 \) in OW. After that the inventory continues to store at RW till the production stops at \( t = t_2 \). The inventory level in RW is depleted gradually due to
demand and that stock is cleared up to \( t=t_3 \). Further, the inventory stored at OW starts depleting and reaches zero at \( t=t_4 \). The unit production cost is

\[ C_p = R + \frac{G}{P}, \]

where \( R \) is the material cost per unit item, \( g \) is the total labor/energy cost per unit time of a production system which is equally distributed over the unit time. So \( \frac{G}{P} \) decrease with \( P \) increase in short run process, total cost and average cost of production decrease with increase of production rate \( P \).

The demand for items \( (d_i) \) is met during the time span \([0, \mu]\), \( \mu \) is the expected time after which manufacturer starts giving discount on selling price. Here, \( \mu \) is measured using an appropriate distribution such as normal distribution, and uniform distribution, over time, according to the quality, the marketplace climate and the age of the commodity. The management seeks to clear stock to minimize loss. To enhance these sales, reductions are proposed on the selling price. It can be justified that the demand \( d_2 \) is a monotonic increasing function of the reduction rate \( r \). Here, the demand rate \( d_2 \) is assumed to be \( d_2(r) = ab^r, \ a>0, b>1, \ 0 < r < 1 \), where \( d_2(r) \) is an exponential monotonic increasing function of \( r \). For convenience, \( d_2 \) is used rather than \( d_2(r) \) throughout this chapter. In the proposed model, the following cases may arise:

**Case 1** \( 0 \leq \mu \leq t_1 \)

![Graphical representation of the system in case 1](image)

The system is governed by the following differential equations in case 1.

\[ \frac{dI_1(t)}{dt} = P - d_1, \quad 0 \leq t \leq \mu \]  
\[ \frac{dI_2(t)}{dt} = P - d_2, \quad \mu \leq t \leq t_1 \]  
\[ \frac{dI_3(t)}{dt} = P - d_2, \quad t_1 \leq t \leq t_2 \]  
\[ \frac{dI_4(t)}{dt} = -d_2, \quad t_2 \leq t \leq t_3 \]  
\[ \frac{dI_5(t)}{dt} = 0, \quad t_1 \leq t \leq t_3 \]  
\[ \frac{dI_6(t)}{dt} = -d_2, \quad t_3 \leq t \leq t_4 \]  

The above equations can be solved by using the boundary conditions \( I_{11}(0) = 0, \ I_{12}(t_1) = W, \ I_{2}(t_1) = 0, \ I_3(t_3) = 0, \ I_4(t_3) = W \) and \( I_5(t_4) = 0 \), respectively. The solution of the Eqs.(1-6) are given below:

\[ I_{11}(t) = (P - d_1)t, \quad 0 \leq t \leq \mu \]
\[ I_{12}(t) = (P - d_2)(t - t_1) + W, \quad \mu \leq t \leq t_1 \]  
(8)

\[ I_{2}(t) = (P - d_2)(t - t_1), \quad t_1 \leq t \leq t_2 \]  
(9)

\[ I_{3}(t) = -d_2(t - t_3), \quad t_2 \leq t \leq t_3 \]  
(10)

\[ I_{4}(t) = W, \quad t_1 \leq t \leq t_3 \]  
(11)

\[ I_{5}(t) = -d_2(t - t_4), \quad t_3 \leq t \leq t_4 \]  
(12)

Since \( I_{11}(\mu) = I_{12}(\mu) \Rightarrow \mu = \frac{W + d_3 t_1}{d_2 - d_1} \)
\[ I_2(t_2) = I_3(t_2) \Rightarrow t_1 = \frac{P t_2 - d_3 t_3}{P} \]  
(13)

\[ I_5(t_3) = W \Rightarrow t_5 = \frac{d_2 t_4 - W}{d_2} \]  
(14)

Total relevant cost of the system = Set-up cost + Production cost + Holding costs in (OW and RW)

\[
TC_1 = C_s + C_p Pt_2 + h_o \left\{ \frac{(P - d_1)}{2} \mu^2 - \frac{(P - d_2)}{2} (t_1 - \mu)^2 + W (t_1 + t_3) + \frac{d_2}{2} (t_4 - t_3)^2 \right\} \\
+ h_r \left\{ \frac{(P - d_2)}{2} (t_2 - t_1)^2 + \frac{d_2}{2} (t_3 - t_2)^2 \right\}

\]  
(15)

**Case 2** \( t_1 \leq \mu \leq t_2 \)

\[ \frac{dI_1(t)}{dt} = P - d_1, \quad 0 \leq t \leq t_1 \]  
(16)

\[ \frac{dI_{21}(t)}{dt} = P - d_1, \quad t_1 \leq t \leq \mu \]  
(17)

\[ \frac{dI_{22}(t)}{dt} = P - d_2, \quad \mu \leq t \leq t_2 \]  
(18)

\[ \frac{dI_3(t)}{dt} = -d_2, \quad t_2 \leq t \leq t_3 \]  
(19)

\[ \frac{dI_5(t)}{dt} = -d_2, \quad t_3 \leq t \leq t_4 \]  
(20)

\( I_4(t) \) is represented by Eq. (6).

**Fig. 2.** Graphical representation of the system in case 2

The system is governed by the following differential equations in case 2.
The above equations can be solved by using the boundary conditions $I_1(0) = 0$, $I_{21}(t_1) = 0$, $I_{21}(\mu) = I_{22}(\mu)$, $I_3(t_3) = 0$ and $I_3(t_4) = 0$, respectively. The solution of the eq.(16)-(20) are given below:

\begin{align*}
I_1(t) &= (P - d_1) t, \quad 0 \leq t \leq t_1 \\
I_{21}(t) &= (P - d_1)(t - t_1), \quad t_1 \leq t \leq \mu \\
I_{22}(t) &= (P - d_2) t - d_1(\mu - t_1) + d_2 \mu, \quad \mu \leq t \leq t_2 \\
I_3(t) &= -d_2 (t - t_3), \quad t_2 \leq t \leq t_3 \\
I_5(t) &= -d_2 (t - t_4), \quad t_3 \leq t \leq t_4
\end{align*}

Since $I_3(t_2) = I_{22}(t_2) \Rightarrow \mu = \frac{d_3 t_1 + d_2 (t_2 - t_3)}{d_1 - P}$

\begin{align*}
I_3(t_3) &= W \Rightarrow t_3 = \frac{d_3 t_1 - W}{d_2} \\
I_3(t_4) &= W \Rightarrow t_4 = \frac{W}{P - d_1}
\end{align*}

\begin{align*}
TC_2 &= C_s + C Pt_2 + h_0 \left\{ \frac{(P - d_1)}{2} t_1^2 + W (t_3 - t_1) - \frac{d_2}{2} (t_4 - t_3)^2 \right\} \\
&\quad + h_1 \left\{ \frac{(P - d_1)}{2} (t_1 - \mu)^2 + \frac{(P - d_2)}{2} (t_2^2 - \mu^2) + \{ d_2 \mu - d_1 (\mu - t_1) \} (t_2 - \mu) + \frac{d_2}{2} (t_3 - t_2)^2 \right\}
\end{align*}

Case 3 $t_2 \leq \mu \leq t_3$

\[\text{Fig. 3. Graphical representation of the system in case 3}\]

The system is governed by the following differential equations in case 3.

\begin{align*}
\frac{dI_1(t)}{dt} &= P - d_1, \quad 0 \leq t \leq t_1 \\
\frac{dI_{21}(t)}{dt} &= P - d_1, \quad t_1 \leq t \leq t_2 \\
\frac{dI_{31}(t)}{dt} &= -d_2, \quad t_2 \leq t \leq \mu \\
\frac{dI_{32}(t)}{dt} &= -d_2, \quad \mu \leq t \leq t_3
\end{align*}
\[
\frac{dI_5(t)}{dt} = -d_2, \quad t_3 \leq t \leq t_4
\]  

(34)

\(I_4(t)\) is represented by Eq. (6). The above equations can be solved by using the boundary conditions 
\[I_1(0) = 0, \quad I_2(t_1) = 0, \quad I_3(t_2) = I_2(t_2), \quad I_3(t_3) = 0 \quad \text{and} \quad I_5(t_4) = 0,\]
respectively. The solution of the Eqs. (12-14) are given as follows,

\[
I_1(t) = (P - d_1)t, \quad 0 \leq t \leq t_1
\]  

(35)

\[
I_2(t) = (P - d_1)(t - t_1), \quad t_1 \leq t \leq t_2
\]  

(36)

\[
I_3(t) = -d_1(t - (P - d_1)t_1), \quad t_2 \leq t \leq \mu
\]  

(37)

\[
I_3(t) = -d_2(t - t_3), \quad \mu \leq t \leq t_3
\]  

(38)

\[
I_5(t) = -d_2(t - t_4), \quad t_3 \leq t \leq t_4
\]  

(39)

Also, \(I_1(t_1) = W \Rightarrow t_1 = \frac{W}{P - d_1}\) 
\[
I_3(\mu) = I_3(\mu) \Rightarrow \mu = \frac{(P - d_1)t_1 - d_2t_3}{d_2 - d_1}
\]  

(40)

\[
I_5(t_3) = W \Rightarrow t_3 = \frac{d_2t_4 - W}{d_2}
\]  

(41)

Total relevant cost of the system is given by

\[
TC_3 = C_s + C_p P t_2 + h_o \left\{ \frac{(P - d_1)}{2} t_1^2 + W(t_3 - t_1) + \frac{d_2}{2}(t_4 - t_3)^2 \right\}
\]  

\[+h_r \left\{ \frac{(P - d_1)}{2}(t_2 - t_1)^2 - \frac{d_1}{2}(\mu - t_2)^2 - (P - d_1)t_1(\mu - t_2) + \frac{d_2}{2}(t_3 - \mu)^2 \right\}
\]  

(43)

**Case 4** \(t_3 \leq \mu \leq t_4\)

![Graphical representation of the system in case 4](image)

Fig. 4. Graphical representation of the system in case 4

The system is governed by the following differential equations in case 4.

\[
\frac{dI_1(t)}{dt} = P - d_1, \quad 0 \leq t \leq t_1
\]  

(44)

\[
\frac{dI_2(t)}{dt} = P - d_1, \quad t_1 \leq t \leq t_2
\]  

(45)

\[
\frac{dI_3(t)}{dt} = -d_1, \quad t_2 \leq t \leq t_3
\]  

(46)
\[
\frac{dI_{s1}(t)}{dt} = -d_1, \quad t_3 \leq t \leq \mu \\
\frac{dI_{s2}(t)}{dt} = -d_2, \quad \mu \leq t \leq t_4
\]  

(47)  

(48)  

\( I_4(t) \) is represented by Eq. (6).  

The above equations can be solved by using the boundary conditions \( I_1(0) = 0, \) \( I_2(t_1) = 0, \) \( I_3(t_3) = 0, \) \( I_{s1}(t_3) = W \) and \( I_{s2}(t_4) = 0, \) respectively. The solution of the Eqs. (44-48) are given below:

\[
I_1(t) = (P - d_1)t, \quad 0 \leq t \leq t_1
\]  

(49)  

\[
I_2(t) = (P - d_1)(t - t_1), \quad t_1 \leq t \leq t_2
\]  

(50)  

\[
I_3(t) = -d_1(t - t_5), \quad t_2 \leq t \leq t_3
\]  

(51)  

\[
I_{s1}(t) = -d_1(t - t_5) + W, \quad t_3 \leq t \leq \mu
\]  

(52)  

\[
I_{s2}(t) = -d_2(t - t_4), \quad \mu \leq t \leq t_4
\]  

(53)  

Also, \( I_1(t_1) = W \Rightarrow t_1 = \frac{W}{P - d_1}, \)  

(54)  

\[
I_2(t_2) = I_3(t_2) \Rightarrow t_3 = \frac{1}{d_1} \left\{ Pt_2 - (P - d_1) t_1 \right\}
\]  

(55)  

\[
I_{s1}(\mu) = I_{s2}(\mu) \Rightarrow \mu = \frac{-d_1 t_3 + d_2 t_4 - W}{d_2 - d_1}
\]  

(56)  

Total relevant cost of the system is given by

\[
TC_4 = C_s + C_p Pt_2 + h_o \left\{ \frac{(P - d_1)}{2} t_1^2 + W (t_3 - t_1) - \frac{d_1}{2} (\mu - t_3)^2 + W (\mu - t_3) - \frac{d_2}{2} (t_4 - \mu)^2 \right\} \\
+ h_i \left\{ \frac{(P - d_1)}{2} (t_2 - t_1)^2 - \frac{d_1}{2} (t_3 - t_2)^2 \right\}
\]  

(57)  

The problem is to minimize \( TC_i \) where \( i = 1, 2, 3, 4 \) for each case.

4. Numerical example

The values of the parameters are considered in appropriate units as follows:

\[ C_s = 100, \quad R = 100, \quad G = 1500, \quad d_1 = 10, \quad a = 0.5, \quad b = 1, \quad h_o = 1, \quad h_i = 1.5, \quad W = 50, \quad t_4 = 12 \]

The above model is solved by using genetic algorithm approach. We have considered the following parameters of GA: population size = 50, probability of crossover = 0.6, probability of mutation = 0.2, and maximum generation=50. The results are mentioned below with the path of convergence in each case.
Case 1: \( 0 \leq \mu \leq t_1 \)
\( r=0.569, \ P=17.877, \ t_2=0.19 \) and TC= 99479.23

Case 2: \( t_1 \leq \mu \leq t_2 \)
\( r=0.484, \ P=16.000, \ t_2=0.5 \) and TC=838.65

Case 3: \( t_2 \leq \mu \leq t_3 \)
\( r=0.938, \ P=30, \ t_2=0.56 \) and TC=1105.22

Case 4: \( t_3 \leq \mu \leq t_4 \)
\( r=0.581, \ P=19.8, \ t_2=0.49 \) and TC=779.67

5. Conclusion

The objective of this paper was to model a producer’s cost minimization strategy while facing the special sale/reduction offer on the selling price of items. The impact of obsolete or out of season products trade on demand and particularly on the retailer’s reaction to speed-up sales, so as to alleviate the impact of the larger order’s loss results into the price cut of the item. Generally, the physical item has a life time (\( \mu \)) after which it undergoes decay or damage. At time \( \mu \), efficient administration decides to provide a special offer/reduction on the selling price to clear the stock and to preserve goodwill. In general, customers are encouraged to buy more at more reduced prices. As a result, the demand rate during a special offer/reduction can be taken as increasing function of the reduction rate (\( r \)).

This function is found to be suitable as per a prior survey of the market. In real life, demand fluctuates to a great extent with price and quality, with quality being uncertain with time, and many other variables and unexpected difficulties come across which may intimidate the continuous out flow of goods, leading to a large build stock. In several practical scenarios, the decision maker has the chance to lessen the selling price before the end of the cycle, in order to excite sales and circumvent ending the period with too much inventory. To deal with such situation, it is necessary to develop a system that can suck up the massive fluctuations at the least possible cost. Prices differ according to the demand and supply conditions in the market which in turn depends upon whether the market is subject to
competitive conditions, monopoly, duopoly or oligopoly. Almost all the manufacturers locate their prices on a cost-plus-profit or a market-price basis. However, irrespective of any approach, customers use to pay fair prices on the basis of the quality of the goods. For example, the textile and footwear industry sells products at cheap prices to maintain branded image in the market. In this context, this paper suggests such a demand function that is formulated so that all possible cases of the cost function are minimized by trading off; the inventory costs of the OW and the RW, the production cost, the setup cost and the different selling prices. The characteristics of the model are observed through formulation and the model is also solved using a GA approach. The formulation of this demand function \( d_2 \) with a limited capacity of OW and volume flexibility is novel and very much realistic in the field of inventory control.

References


