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Integrated inventory models with two–level credit policy and a price negotiation scenario for price–sensitive Stock–dependent demand

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A B S T R A C T

In this research, the integrated inventory models are developed for price–sensitive stock–dependent demand and delay in payments are permissible. Two level trade credit police in the vendor–buyer and buyer–customer is considered. An easy–to–use solution algorithm is derived for the integrated models to determine the buyer's optimal pricing and ordering strategy. A negotiation scenario is incorporated to distribute the extra profit between the vendor and buyer. A numerical example and sensitivity analysis are given to validate the proposed models. It is observed that the total joint profit of the integrated system can increase even if the price discount is offered to the buyer in proposed models.

1. Introduction

Till early 70's, researchers were analyzing inventory models from the buyer's or vendor's end. However, Goyal (1976) argued that the supply chain comprising of the vendor and the buyer to determine optimal ordering strategy is mutually beneficial to achieve the minimum integrated total cost when the vendor's production is infinite. Banerjee (1986) assumed that the vendor's production rate is deterministic on the lot – for – lot basis for Goyal's (1976) model. Goyal (1988) relaxed lot – for – lot production and assumed that the vendor produces an integer multiple of the buyer's order per production run. There after, Ha and Kim (1997), Pan and Yang (2002), Chang et al. (2006), Hsiao (2008) presented the integrated vendor – buyer models with equal order sizes. These studies established that the vendor's cost as wall as the integrated total cost from the joint decision is smaller than the independent decision; however, the buyer's cost increases. Fascinated by the principal of mutual benefit, the vendor could make amends for the buyer's loss by using incentive strategies to entice the buyer to opt for joint decision. Chakravarty and Martin (1988), yang (2004), Wee and Yang (2007), suggested incentive of price discount to the buyer.

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Most of the inventory models assumed that the buyer settles the account with the vendor immediately on the arrival of the product in the inventory system. In business, the practice of offering trade credit by the vendor to the buyer is prevailing. The offer of trade credit encourages the buyer to increase the order quantity and boost up the demand. Goyal (1985) developed mathematical model for the buyer's optimal order quantity when permissible delay in payments is possible. Shah (1993) and Aggarwal and Jaggi (1995) analyzed inventory policies for deteriorating items under the condition of a permissible delay in payments. Chung (1998) derived analytic results for optimum procurement units when credit period is offered by the vendor to the buyer. Chu et al. (1998) established convexities of the objective function for Aggarwal and Jaggi's (1995) model. Teng (2002) calculated interest earned on the selling price instead of the purchase cost as taken by Goyal (1985). Teng et al. (2005) modeled inventory model for deteriorating when trade credit is offered and demand is price – sensitive. For details of articles on trade credit, one can refer to review article by Shah et al. (2010).

The passing of credit period from the customers to the buyer who is getting from the vendor is termed as the two – level credit policy. Huang (2003) applied the two – level credit policy into the classical EOQ model to determine the optimal order quantity. Teng and Goyal (2007) advised the customers to settle the account with the buyer at the end of the credit period. Jaggi et al. (2008) discussed inventory model for the two – level trade credit policy, when the demand is sensitive to the credit period offered by the buyer. Teng and Chang (2009) analyzed two – level trade credit policy when production rate is finite. The above-cited articles were discussed from the buyer's point of view. Due to globalization, the researchers have started analyzing integrated vendor – buyer inventory models with allowable trade credit; such as Abad and Jaggi (2003), Yang and Wee (2006), Chen and Kang (2007), Ho et al. (2008), Ouyang et al. (2008), Huang et al. (2009), Chen and Kang (2009) etc. Barratt (2004) stated that in supply chain management trust, mutuality, information exchange, openness etc. are important for establishing a long – term cooperative relationship among the trading players. The studies by Goyal (1976), Pan and Yang (2002), Yang (2004), Ho et al. (2008) were made to support the Barratt's argument. Chen and Kang (2007) developed integrated inventory model with two – level trade credit policy. The Chung et al. (2006), Yang et al. (2007) and Chung and Wee (2008) incorporated a negotiation scheme for agreeing to vendor's offering price to the buyer to achieve a win – win strategy. Chen and Kang (2009– a) formulated integrated inventory models under the two – level trade credit policy with price – sensitive demand and negotiation scheme. In this paper, the integrated inventory models are analyzed for price – sensitive stock – dependent demand with the two – level trade credit policy and a negotiation scheme. The concept of stock – dependent demand makes this contribution more practicable in the prevailing concept of super – malls.

The notations and assumptions are given in the following section. Section 3 deals with the development of three models considering the two – level credit policy with price – sensitive stock – dependent demand, viz. a non – integrated model, integrated model and integrated vendor – buyer model with a negotiation scheme to make compensation for the buyer's loss. A numerical example and sensitivity analysis are given to study the derived models in section 4. The conclusions are given in section 5.

2. Notations and assumptions

The proposed models are derived using the following notations and assumptions.

Notations

i = 1, 2, 3

A_b : buyer's ordering cost per order.

A_v : vendor's setup cost per production run.

F : fixed processing cost for vendor in dealing with each order

I_b : buyer's holding charge fraction per unit per unit time excluding interest charges

I_v : vendor's holding charge fraction per unit per unit time

C_v : vendor's production cost per unit item

C_{bi} : buyer's purchase cost per unit item

C_{bd} : buyer's purchase cost per unit item under the negotiation scheme offered by the vendor ($C_{bd} = C_{bi} - d$, $d > 0$).

P_{bi} : buyer's selling price per unit item in model i with $P_{bi} > C_{bi} > C_{bd} > C_v$

$R\left(P_{bi}, I_j(t)\right)$: annual stock – dependent rate which is a function of on – hand inventory at any instant of time t and decreasing function of buyer's selling price, i.e. $R\left(P_{bi}, I_j(t)\right) = \left(\alpha + \beta I_j(t)\right) P_{bi}^{-\eta}$

where $j = b \ or \ v$ (b for buyer's inventory and v for vendor's inventory)

α denotes fixed demand $(\alpha > 0)$

β denotes stock– dependent demand parameter (>0) and $\alpha >> \beta$

η mark – up , $\eta > 1$

δ : the ratio of demand rate to the production rate, where $0 < \delta < 1$

M : credit period offered by the vendor

N : credit period offered by the buyer

I_e : annual interest rate earned by the buyer

I_c : annual interest charges to be paid per \$ in stock to the vendor

I_o : annual interest rate for vendor's opportunity interest loss due to the delay payment

T_{bi} : cycle time in year unit of the buyer

$$\begin{cases} T_{bi1} , & if\ M \geq N\ and\ T_{bi} + N \geq M \\ T_{bi2} , & if\ M \geq N\ and\ T_{bi} + N < M \\ T_{bi3} , & if\ M < N \end{cases}$$

n_i : number of shipments of the buyer in model i

$TPB_i\left(T_{bi}, P_{bi}\right)$: total profit per unit time for the buyer (also denoted as TPB_i)

$$\begin{cases} TPB_{i1} , & if\ M \geq N\ and\ T_{bi} + N \geq M \\ TPB_{i2} , & if\ M \geq N\ and\ T_{bi} + N < M \\ TPB_{i3} , & if\ M < N \end{cases}$$

$TPV_i\left(n_i, T_{bi}, P_{bi}\right)$: total profit per unit time for the vendor (also denoted as TPV_i) in model i

$TP_i\left(n_i, T_{bi}, P_{bi}\right)$: total profit per unit time which is sum of TPB_i and TPV_i (also denoted as TP_i)

$$\begin{cases} TP_{i1} & , \quad if\ M \geq N\ and\ T_{bi} + N \geq M \\ TP_{i2} & , \quad if\ M \geq N\ and\ T_{bi} + N < M \\ TP_{i3} & , \quad if\ M < N \end{cases}$$

PTPG : percentage total profit gain
GMR : gross margin ratio

Assumptions

1. The supply chain under consideration consists of a single vendor and single buyer for a single product.
2. Demand rate is price – sensitive stock – dependent and production rate is greater than the demand rate.
3. Shortages are not allowed.
4. The lead – time is zero.
5. The two – level credit policy is implemented in which the vendor offers the buyer a credit period and the buyer also gives a credit period to the customers. The customers settle the account with the buyer when the credit period offered by the buyer is due.

3. Mathematical models

Here, the rate of change of inventory is due to price – sensitive stock – dependent demand. The differential equation governing the inventory status at any instant of time *t* is given by

$$\frac{dI(t)}{dt} = -\left(\alpha + \beta I(t)\right) P^{-\eta}, \qquad 0 \leq t \leq T$$

with $I(0)=Q$ and $I(T)=0$. Then the solution of the differential equation is

$$I(t) = \frac{\alpha}{\beta}\left[e^{\beta P^{-\eta}(T-t)} - 1 \right], \qquad 0 \leq t \leq T$$

and optimum procurement quantity is

$$Q = \frac{\alpha}{\beta}\left[e^{\beta P^{-\eta} T} - 1 \right] .$$

In this section, we develop three models. Firstly, we compute the vendor's profit model, which will be applied to all the three proposed models.

Vendor's profit model

The buyer's order $\left(n_i Q_i\right)$ is produced n_i times by the vendor. The different cost components of vendor's profit are as follows:

1. Set – up cost for the vendor $= \dfrac{A_v}{n_i T_{bi}}$
2. Process cost incurred by the vendor in dealing with each order $= \dfrac{n_i F}{n_i T_{bi}}$

3. Holding cost per unit time $= \dfrac{C_v\, I_v\, \alpha}{\beta^2 P_{bi}^{-\eta}\, T_{bi}}\Big[(n_i-1)(1-\rho)+\rho\Big]\left[e^{\beta P_{bi}^{-\eta} T_{bi}} - \beta P_{bi}^{-\eta}\, T_{bi} - 1\right]$

4. Opportunity interest loss per unit time $= \dfrac{C_b\, I_o\, M\alpha}{\beta\, T_{bi}}\left[e^{\beta P_{bi}^{-\eta} T_{bi}} - 1\right]$

5. Revenue per unit time for the vendor $= \dfrac{\left(C_b - C_v\right)\alpha}{\beta\, T_{bi}}\left[e^{\beta P_{bi}^{-\eta} T_{bi}} - 1\right]$

Hence, the total profit per unit time for the vendor is

$$TPV_i = \frac{\left(C_b - C_v\right)\alpha}{\beta\, T_{bi}}\left[e^{\beta P_{bi}^{-\eta} T_{bi}} - 1\right] - \frac{C_v\, I_v\, \alpha}{\beta^2 P_{bi}^{-\eta}\, T_{bi}}\Big[(n_i-1)(1-\rho)+\rho\Big]\left[e^{\beta P_{bi}^{-\eta} T_{bi}} - \beta P_{bi}^{-\eta}\, T_{bi} - 1\right] - \frac{C_b\, I_o\, M\alpha}{\beta\, T_{bi}}\left[e^{\beta P_{bi}^{-\eta} T_{bi}} - 1\right] - \frac{A_v + n_i\, F}{n_i\, T_{bi}}$$

3.1 Model 1: Non – integrated vendor – buyer model

The buyer's cost components are as follows:

6. Ordering cost per order $= \dfrac{A_b}{T_{b1}}$

7. Holding cost per unit time $= \dfrac{C_b\, I_b\, \alpha}{\beta^2 P_{b1}^{-\eta}\, T_{b1}}\left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta}\, T_{b1} - 1\right]$

Interest earned and interest charged are computed on the basis of the lengths of T_{b1}, M and N as follows. Various scenarios could be possible due to offer of two – level trade credit.

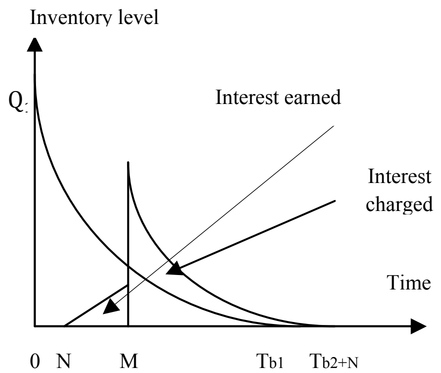

Fig. 1. Interest earned and charged under M ≥ N and $T_{b1+N} \geq M$

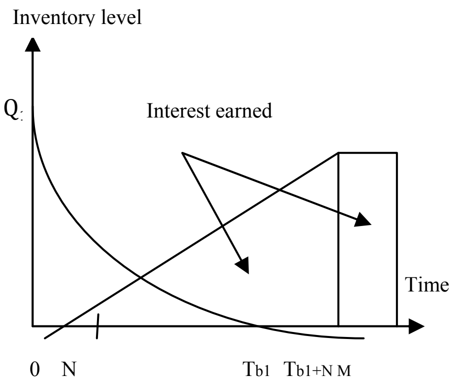

Fig. 2. Interest earned when M ≥ N and $T_{b1+N} < M$

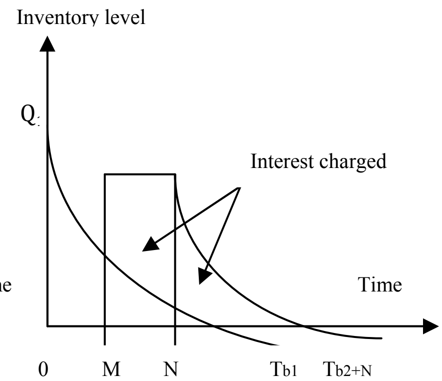

Fig. 3. Interest charged M ≤ N

Case 1. $M \geq N$

Scenario1: $T_{b1} + N \geq M$ [Fig. 1]

Under the two – level trade credit policy, the customer is allowed to settle for account when the credit period offered by the vendor is due. Therefore, the customer can pay off the buyer during $\left[N,\ T_{b1} + N\right]$. The buyer deposits the generated revenue into the bank to earn an interest before the delay period offered by the vendor is due. Hence,

8. Interest earned per unit time

$$= \frac{P_{b1} I_e \alpha}{\beta P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - e^{\beta P_{b1}^{-\eta} (T_{b1} - M + N)} - (M-N) \beta P_{b1}^{-\eta} e^{\beta P_{b1}^{-\eta} (T_{b1} - M + N)} \right]$$

9. Interest charged per unit time during $\left[M,\ T_{b1} + N \right]$

$$= \frac{C_b I_c \alpha}{\beta^2 P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - e^{\beta P_{b1}^{-\eta} (M-N)} - \beta P_{b1}^{-\eta} \left(T_{b1} + N - M \right) \right]$$

Scenario2: $T_{b1} + N < M$ [Fig. 2]

Here, the customers settle the accounts with the buyer when the credit period offered by the buyer is due. The buyer's interest charges are zero; and

10. Interest earned per unit time during [N, M]

$$= \frac{P_{b1} I_e}{\beta P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta} T_{b1} - 1 + \left(M - T_{b1} - N \right) P_{b1}^{-\eta} T_{b1} \left(e^{\beta P_{b1}^{-\eta} T_{b1}} - 1 \right) \right]$$

Case 2. $M \le N$ [Fig. 3]

In this situation, the buyer does not have purchase cost to be paid against the procured units, and hence, he borrows the total purchase cost from the bank. Hence, the interest earned is zero; and

11. Interest charged per unit time

$$= \frac{C_b I_c \alpha}{\beta^2 P_{b1}^{-\eta} T_{b1}} \left[\beta P_{b1}^{-\eta} T_{b1} (N-M) \left(e^{\beta P_{b1}^{-\eta} T_{b1}} - 1 \right) + e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta} T_{b1} - 1 \right]$$

12. Revenue per unit time for the buyer is

$$= \frac{(P_{b1} - C_b) \alpha}{\beta T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - 1 \right]$$

Based on the above scenarios, the buyer's total profit per unit time is as follows:

Case 1. $M \ge N$

Scenario1: $T_{b1} + N \ge M$

$$TPB_{11} = \frac{(P_{b1} - C_b) \alpha}{\beta T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - 1 \right] - \frac{A_b}{T_{b1}} - \frac{C_b I_b \alpha}{\beta^2 P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta} T_{b1} - 1 \right]$$
$$+ \frac{P_{b1} I_e \alpha}{\beta P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - e^{\beta P_{b1}^{-\eta} (T_{b1} - M + N)} - (M-N) \beta P_{b1}^{-\eta} e^{\beta P_{b1}^{-\eta} (T_{b1} - M + N)} \right]$$
$$- \frac{C_b I_c \alpha}{\beta^2 P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - e^{\beta P_{b1}^{-\eta} (M-N)} - \beta P_{b1}^{-\eta} \left(T_{b1} + N - M \right) \right] \qquad (1)$$

Scenario2: $T_{b1} + N < M$

$$TPB_{12} = \frac{(P_{b1} - C_b) \alpha}{\beta T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - 1 \right] - \frac{A_b}{T_{b1}} - \frac{C_b I_b \alpha}{\beta^2 P_{b1}^{-\eta} T_{b1}} \left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta} T_{b1} - 1 \right]$$

$$+\frac{P_{b1}\, I_e\, \alpha}{\beta P_{b1}^{-\eta}\, T_{b1}}\left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta}\, T_{b1} - 1 + P_{b1}^{-\eta}\, T_{b1}\left(M - T_{b1} - N\right)\left(e^{\beta P_{b1}^{-\eta} T_{b1}} - 1\right)\right] \qquad (2)$$

Case 2. $M \le N$

$$TPB_{13} = \frac{\left(P_{b1} - C_b\right)\alpha}{\beta\, T_{b1}}\left[e^{\beta P_{b1}^{-\eta} T_{b1}} - 1\right] - \frac{A_b}{T_{b1}} - \frac{C_b\, I_b\, \alpha}{\beta^2 P_{b1}^{-\eta}\, T_{b1}}\left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta}\, T_{b1} - 1\right]$$

$$-\frac{C_b\, I_c\, \alpha}{\beta^2 P_{b1}^{-\eta}\, T_{b1}}\left[e^{\beta P_{b1}^{-\eta} T_{b1}} - \beta P_{b1}^{-\eta}\, T_{b1} - 1 + \beta P_{b1}^{-\eta}\, T_{b1}\left(N - M\right)\left(e^{\beta P_{b1}^{-\eta} T_{b1}} - 1\right)\right] \qquad (3)$$

Solution procedure 1:

The optimal solution of P_{b1} and T_{b1} is obtained by solving $\frac{\partial\, TPB_{1j}}{\partial\, P_{b1}} = 0$ and $\frac{\partial\, TPB_{1j}}{\partial\, T_{b1}} = 0$, in case j = 1,2,3 for suitable model parametric values with the help of mathematical software. Then optimum value of n_1 (number of shipments) which maximizes the vendor's total profit per unit time can be computed, and hence total profit of the system (which is sum of the buyer's total profit per unit time and the vendor's total profit per unit time) can be obtained.

3.2 Model 2: Vendor – buyer integrated model

Here, the vendor and buyer determine optimal policy which maximizes the total profit of both the players, collectively. For the attainment of the goal when the end – user's demand is price – sensitive and stock – dependent, the total profit per unit time for the integrated vendor – buyer model is as follows. Here, it is assumed that the buyer's price is given.Similar to model 1, the total profit per unit time is formulated depending on lengths of T_{b2}, M and N as follows.

Case 1. $M \ge N$

Scenario1: $T_{b2} + N \ge M$

$$TP_{21} = TPB_{21} + TPV_2$$

$$= \frac{\left(P_{b2} - C_v\right)\alpha}{\beta\, T_{b2}}\left[e^{\beta P_{b2}^{-\eta} T_{b2}} - 1\right] - \frac{1}{T_{b2}}\left(A_b + \frac{A_v + n_2 F}{n_2}\right) - \frac{C_b I_b \alpha}{\beta^2 P_{b2}^{-\eta} T_{b2}}\left[e^{\beta P_{b2}^{-\eta} T_{b2}} - \beta P_{b2}^{-\eta}\, T_{b2} - 1\right]$$

$$+\frac{P_{b2}\, I_e\, \alpha}{\beta P_{b2}^{-\eta}\, T_{b2}}\left[e^{\beta P_{b2}^{-\eta} T_{b2}} - \left(M - N\right)\beta\, P_{b2}^{-\eta}\, e^{\beta P_{b2}^{-\eta}\left(T_{b2} - M + N\right)} - e^{\beta P_{b2}^{-\eta}\left(T_{b2} - M + N\right)}\right]$$

$$-\frac{C_b\, I_c\, \alpha}{\beta^2 P_{b2}^{-\eta}\, T_{b2}}\left[e^{\beta P_{b2}^{-\eta} T_{b2}} - e^{\beta P_{b2}^{-\eta}(M-N)} - \beta\, P_{b2}^{-\eta}\left(T_{b2} + N - M\right)\right]$$

$$-\frac{C_v\, I_v\, \alpha}{\beta^2 P_{b2}^{-\eta}\, T_{b2}}\left[\left(n_2 - 1\right)\left(1 - \rho\right) + \rho\right]\left[e^{\beta P_{b2}^{-\eta} T_{b2}} - \beta\, P_{b2}^{-\eta}\, T_{b2} - 1\right] \qquad (4)$$

Scenario 2: $T_{b2} + N < M$

$$TP_{22} = TPB_{22} + TPV_2$$

$$=\frac{\left(P_{b2}-C_v\right)\alpha}{\beta T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-1\right]-\frac{1}{T_{b2}}\left(A_b+\frac{A_v+n_2F}{n_2}\right)-\frac{C_bI_b\alpha}{\beta^2P_{b2}^{-\eta}T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-\beta P_{b2}^{-\eta}T_{b2}-1\right]$$

$$+\frac{P_{b2}I_e\alpha}{\beta P_{b2}^{-\eta}T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-\beta P_{b2}^{-\eta}T_{b2}-1+P_{b2}^{-\eta}T_{b2}\left(M-T_{b2}-N\right)\left(e^{\beta P_{b2}^{-\eta}T_{b2}}-1\right)\right]$$

$$-\frac{C_vI_v\alpha}{\beta^2P_{b2}^{-\eta}T_{b2}}\left[\left(n_2-1\right)\left(1-\rho\right)+\rho\right]\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-\beta P_{b2}^{-\eta}T_{b2}-1\right]-\frac{C_bI_oM\alpha}{\beta T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-1\right]\quad(5)$$

Case 2. $N\geq M$

$$TP_{23}=TPB_{23}+TPV_2$$

$$=\frac{\left(P_{b2}-C_v\right)\alpha}{\beta T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-1\right]-\frac{1}{T_{b2}}\left(A_b+\frac{A_v+n_2F}{n_2}\right)-\frac{C_bI_b\alpha}{\beta^2P_{b2}^{-\eta}T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-\beta P_{b2}^{-\eta}T_{b2}-1\right]$$

$$-\frac{C_bI_c\alpha}{\beta^2P_{b2}^{-\eta}T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-\beta P_{b2}^{-\eta}T_{b2}-1+\beta P_{b2}^{-\eta}T_{b2}\left(N-M\right)\left(e^{\beta P_{b2}^{-\eta}T_{b2}}-1\right)\right]$$

$$-\frac{C_vI_v\alpha}{\beta^2P_{b2}^{-\eta}T_{b2}}\left[\left(n_2-1\right)\left(1-\rho\right)+\rho\right]\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-\beta P_{b2}^{-\eta}T_{b2}-1\right]-\frac{C_bI_oM\alpha}{\beta T_{b2}}\left[e^{\beta P_{b2}^{-\eta}T_{b2}}-1\right]\quad(6)$$

Solution procedure 2 :

Step 1: Set $n_2=1$

Step 2: For j = 1, 2, 3 solve $\frac{\partial TP_{2j}}{\partial P_{b2}}=0$ and $\frac{\partial TP_{2j}}{\partial T_{b2}}=0$ simultaneously using mathematical software for given set of parameters.

Step 3: If $M\geq N$ then if $T_{b2}+N\geq M$ then compute TP_2 from Eq. (4), else compute TP_2 from Eq. (5), else compute TP_2 from Eq. (6).

Step 4. Increment n_2 by n_2+1.

Step 5. Repeat steps 2 – 4 till

$$TP_2\left(n_2^*-1,\ T_{b2}\left(n_2^*-1\right),\ P_{b2}\left(n_2^*-1\right)\right)\leq TP_2\left(n_2^*,\ T_{b2}\left(n_2^*\right),\ P_{b2}\left(n_2^*\right)\right)\geq TP_2\left(n_2^*+1,\ T_{b2}\left(n_2^*+1\right),P_{b2}\left(n_2^*+1\right)\right)$$

Step 6. Stop.

3.3 ***Model 3: Integrated vendor – buyer model with a negotiation scheme***

In negotiation scheme, the vendor offers a price discount to the buyer to compensate for the loss. The total profit per unit time is computed as follows:

Case 1. $M\geq N$

Scenario 1: $T_{b3}+N\geq M$

$$TP_{31}=\frac{\left(P_{b3}-C_v\right)\alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta}T_{b3}}-1\right]-\frac{1}{T_{b3}}\left(A_b+\frac{A_v+n_3F}{n_3}\right)-\frac{C_{bd}I_b\alpha}{\beta^2P_{b3}^{-\eta}T_{b3}}\left[e^{\beta P_{b3}^{-\eta}T_{b3}}-\beta P_{b3}^{-\eta}T_{b3}-1\right]$$

$$+\frac{P_{b3}I_e\alpha}{\beta P_{b3}^{-\eta}T_{b3}}\left[e^{\beta P_{b3}^{-\eta}T_{b3}}-\left(M-N\right)\beta P_{b3}^{-\eta}e^{\beta P_{b3}^{-\eta}\left(T_{b3}-M+N\right)}-e^{\beta P_{b3}^{-\eta}\left(T_{b3}-M+N\right)}\right]$$

$$-\frac{C_{bd}I_c\alpha}{\beta^2P_{b3}^{-\eta}T_{b3}}\left[e^{\beta P_{b3}^{-\eta}T_{b3}}-e^{\beta P_{b3}^{-\eta}\left(M-N\right)}-\beta P_{b3}^{-\eta}\left(T_{b3}+N-M\right)\right]$$

$$-\frac{C_v I_v \alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[(n_3-1)(1-\rho)+\rho\right]\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1\right]-\frac{C_{bd} I_o M \alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right] \quad (7)$$

Scenario 2: $T_{b3}+N < M$

$$TP_{32}=\frac{(P_{b3}-C_v)\alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right]-\frac{1}{T_{b3}}\left(A_b+\frac{A_v+n_3 F}{n_3}\right)-\frac{C_{bd} I_b \alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1\right]$$
$$+\frac{P_{b3} I_e \alpha}{\beta P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1+P_{b3}^{-\eta} T_{b3}\left(M-T_{b3}-N\right)\left(e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right)\right]$$
$$-\frac{C_v I_v \alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[(n_3-1)(1-\rho)+\rho\right]\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1\right]-\frac{C_{bd} I_o M \alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right] \quad (8)$$

Case 2. $M < N$

$$TP_{33}=\frac{(P_{b3}-C_v)\alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right]-\frac{1}{T_{b3}}\left(A_b+\frac{A_v+n_3 F}{n_3}\right)-\frac{C_{bd} I_b \alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1\right]$$
$$-\frac{C_{bd} I_c \alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1+\beta P_{b3}^{-\eta} T_{b3}\left(N-M\right)\left(e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right)\right]$$
$$-\frac{C_v I_v \alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[(n_3-1)(1-\rho)+\rho\right]\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-\beta P_{b3}^{-\eta} T_{b3}-1\right]-\frac{C_{bd} I_o M \alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}}-1\right]$$

where $C_{bd}=C_b-d$. (9)

For fixed n_3, T_{b3} and P_{b3}, the integrated profit functions TP_{31}, TP_{32} and TP_{33} are increasing function of discount d. (Appendix A). To make amends for the buyer's loss, the price scheme is carried out by computing the difference of total profits of models 1 and 3 for buyer and vendor both. Define, vendors extra profit as $EPV=TPV_3-TPV_1$ and that of buyer as $EPB=TPB_3-TPB_1$. Consider the relationship $EPV=\delta\, EPB$. For $\delta=0$, the total profit increment will be for the buyer. For $\delta=1$ the extra profit will be distributed equally between two players. The vendor will be beneficial for higher value of δ.

Solution procedure 3 :

- Step 1: Set δ, d.
- Step 2: Calculate the maximum profit of TPV_1 and TPB_1 from model 1.
- Step 3: Compute $EPV=TPV_3-TPV_1$ and $EPB=TPB_3-TPB_1$.
- Step 4. Solve $EPV=\delta\, EPB$ for C_{bd}.
- Step 5. Apply solution procedure 2 to calculate TPV_3^*, TPB_3^* and TP_3^*.
- Step 6. Knowing n_3^*, T_{b3}^*, P_{b3}^* compute optimum C_{bd}^*

4. Numerical example

In this section, we exhibit working of the proposed models by a numerical example. Consider, the parametric values as follows: α = 100000, β = 3.5, η = 1.5, A_b = 100, C_b = 5, I_b = 0.10, I_v = 0.16, A_v = 1200, F = 100, I_e = 0.09, I_o = 0.09, I_c = 0.12, C_v = 2.5, ρ = 0.8 and δ = 1. In Table 1, the solution is given for N = 0.0 and 0.05. Clearly, N = 0.0 is one – level trade credit policy which is the special case of the developed model. For N = 0.0, the total profit per unit time for the integrated vendor – buyer model increases by 11.03 % ($PTPG_2$) compared to non – integrated model (Model 1). The buyer looses $3629 and vendor gains $ 5651 in the integrated system. Hence, the buyer will be reluctant to opt for joint decision. To reduce the buyer's loss, the vendor offers the price discount to the buyer. Table 1 depicts that $PTPG_3$ increases to 11.99 % i.e. increases by 0.96 % when negotiation scenario is implemented. In Fig. 4, the changes in EPB and EPV are plotted with respect to discounted purchase cost C_{bd} per unit item. It indicates that both players will be benefited when C_{bd} = $ 3.85. For N = 0.05, i.e. offering trade credit to the customers lowers total profit and PTPG for the vendor and buyer. However, the buyer's optimal selling price P_b^* for each model increases very slightly compared to that when N = 0.0. This proves that the increase of the buyer's selling price cannot counter act the decrease of profit due to offer of credit period, N = 0.05 to the customers.

Table 1
Optimal Solution for the three models

N	Model	C_b or C_{bd}	n^*	T^*	P_b^*	TPV^*	TPB^*	TP^*	PTPG (%)
0.00	1	5.00	5	0.358	15.17	2662	16910	19572	_
	2	5.00	2	0.591	8.47	8313	13281	21594	11.03
	3	3.85	2	0.666	8.09	3836	18083	21919	11.99
0.05	1	5.00	4	0.458	15.32	2688	16825	19513	_
	2	5.00	2	0.614	8.54	8222	13285	21507	10.22
	3	3.86	2	0.688	8.14	3851	17988	21839	11.92

$PTPG_i = \left[\frac{TP_i}{TP_1} - 1\right] * 100$ %, i = 2, 3 (percentage of total profit gain)

In Table 2, the sensitivity analysis of the optimal solutions with respect to the negotiation factor; δ is carried out for N = 0.05 and it is observed the that increase in the negotiation factor δ increases the buyer's purchase cost per unit and selling price per unit slightly. It is observed that the gross margin ratio (GMR) has a negative change with increase in values of δ which is in favor of the buyer. On the other hand, for δ greater than 1, vendor is benefited the most.

Table 2
Sensitivity analysis with respect to δ for N = 0.05

δ	T_3^*	n_3^*	C_{bd}^*	P_{b3}^*	GMR	TPB_3^*	TPV_3^*	TP_3^*	$PTPG_3$
0	0.714	3	3.61	8.10	55.43	19781	2688	22469	11.96
0.10	0.696	3	3.65	8.12	55.05	19240	2905	22145	11.94
1	0.688	3	3.86	8.14	52.58	17988	3851	21839	11.92
10	0.669	2	4.05	8.15	50.31	17031	4743	21774	11.73
100	0.666	2	4.09	8.17	49.94	16848	4914	21762	11.58

$GMR = \left[\frac{P_{b3}^*}{C_{bd}^*} - 1\right] * 100$ %

In Table 3, sensitivity analysis of decision variables and targeted objective function is carried out by changing model parameters by – 40%, – 20%, + 20% and + 40%.

From Table 3, it is observed that gross margin ratio decreases for scale demand, stock – dependent parameter, mark – up, production utility ratio; δ, buyer’s ordering cost, vendor’s ordering cost, interest charged by the vendor to the buyer on the unsold stock whereas it is insensitive to interest earn by the buyer on the generated revenue. (See Fig. 5 and 6). The percentage gain in total profit of the integrated system increases significantly when there is an increase on stock – dependent parameter and interest charged by the vendor to the buyer.

Table 3
Sensitivity analysis with respect to model parameters when N = 0.05

Model parameters	% Change	n^*	T_{b3}^*	P_b^*	C_{bd}^*	GMR	TPB_3^*	TPV_3^*	TP_3^*	$PTPG_3$
α	– 40%	2	0.893	7.99	3.76	52.94	10942	2023	12965	16.58
	– 20%	2	0.687	7.83	3.75	52.11	10989	6355	17344	14.89
	+ 20%	2	0.623	7.78	3.75	51.80	21810	5152	26972	13.31
	+ 40%	2	0.575	7.74	3.74	51.68	25522	6171	31693	13.06
β	– 40%	2	0.585	8.06	3.78	53.10	17968	3827	21795	12.81
	– 20%	2	0.652	8.02	3.82	52.37	18032	3885	21917	12.93
	+ 20%	2	0.763	8.00	3.85	51.88	18119	3989	22108	13.18
	+ 40%	2	0.896	7.92	3.83	51.64	18507	4406	22913	15.85
η	– 40%	Not feasible								
	– 20%	Not feasible								
	+ 20%	2	0.629	6.33	3.77	40.44	8518	1591	5866	30.65
	+ 40%	2	0.674	5.53	3.70	33.09	4275	2643	11161	20.35
δ	– 40%	2	0.755	8.03	3.82	52.43	18071	3983	22054	12.73
	– 20%	2	0.719	8.09	3.84	52.53	18027	3917	21944	12.31
	+ 20%	2	0.636	8.21	3.87	52.86	17920	3716	21637	11.27
	+ 40%	Not feasible								
A_b	– 40%	2	0.672	8.12	3.85	52.59	18104	3799	21903	12.05
	– 20%	2	0.684	8.18	3.85	52.93	18000	3844	21844	11.99
	+ 20%	2	0.696	8.21	3.86	52.98	17946	3864	21810	11.98
	+ 40%	2	0.764	8.25	3.87	53.09	17911	3870	21781	11.97
A_v	– 40%	2	0.572	8.04	3.84	52.24	18037	4187	22224	12.25
	– 20%	2	0.631	8.09	3.86	52.29	17936	4082	22018	11.73
	+ 20%	2	0.739	8.18	3.87	52.69	17893	3777	21670	11.64
	+ 40%	2	0.788	8.22	3.88	52.80	17840	3671	21511	11.57
I_b	– 40%	2	0.761	8.07	3.87	52.04	18104	3978	22082	12.11
	– 20%	2	0.721	8.10	3.87	52.22	18047	3911	21958	12.06
	+ 20%	2	0.659	8.17	3.86	52.75	17936	3793	21729	11.82
	+ 40%	2	0.634	8.20	3.85	53.05	17881	3742	21623	11.63
I_v	– 40%	2	0.669	8.12	3.88	52.22	17961	3932	21893	11.57
	– 20%	2	0.658	8.19	3.88	52.63	17936	3928	21864	11.33
	+ 20%	2	0.632	8.24	3.89	52.79	17921	3686	21627	11.10
	+ 40%	2	0.590	8.33	3.92	52.94	17903	3532	21435	10.83
I_c	– 40%	2	0.779	8.05	3.88	51.80	18128	4007	22133	11.21
	– 20%	2	0.727	8.10	3.88	52.10	17980	3996	21976	11.29
	+ 20%	2	0.654	8.17	3.85	52.88	17929	3779	21708	11.38
	+ 40%	2	0.624	8.21	3.85	53.11	17861	3722	21584	11.56
I_e	– 40%	2	0.690	8.14	3.88	52.33	17978	3853	21831	11.89
	– 20%	2	0.688	8.14	3.88	52.33	17984	3856	21840	11.91
	+ 20%	2	0.687	8.13	3.86	52.52	17993	3851	21844	11.92
	+ 40%	2	0.684	8.13	3.86	52.52	17999	3850	21849	11.94

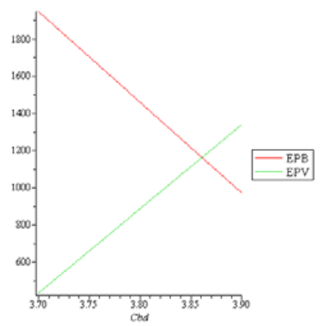

Fig. 4 EPV and EPB versus C_{bd}

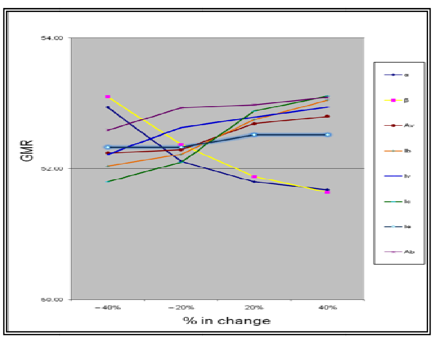

Fig. 5 Percentage change in GMR w.r.t. model parameters

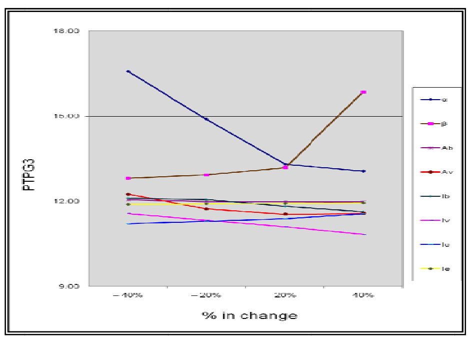

Fig. 6. Percentage change in $PTPG_3$ w.r.t. model parameters

5. Conclusions

This paper analyzes the impact of stock – dependent demand and the credit period offered by the buyer to the customer, called as the two – level trade credit policy, in the integrated models. The recursive solution procedure is established to determine the optimal solutions. It is suggested that the buyer can be encouraged for taking joint decision by incorporating offer of price discount in the unit price. The developed model is illustrated by a numerical example, and also sensitivity analysis is performed with respect to the negotiation factor to observe the changes in the buyer's gross margin ratio and percentage in total profit gain. For M = 0.1 and N = 0.05, for equal increase in δ, the buyer's total profit decreases and vendor's total profit increases and the gross margin reverses.

This study favors the mutual benefits and information sharing between the players of supply chain. In future research, the integrated models could be developed to study the other promotional scheme.

Appendix A.

For fixed n_3, T_{b3} and P_{b3}

The derivative of TP_{31} w.r.t. d is

$$\frac{dTP_{31}}{dd} = +\frac{I_b\,\alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - \beta P_{b3}^{-\eta} T_{b3} - 1\right] + \frac{I_c\,\alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - e^{\beta P_{b3}^{-\eta}(M-N)} - \beta P_{b3}^{-\eta}\left(T_{b3}+N-M\right)\right] + \frac{I_0 M\alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - 1\right] > 0$$

The derivative of TP_{32} w.r.t. d is

$$\frac{dTP_{32}}{dd} = \frac{I_b\,\alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - \beta P_{b3}^{-\eta} T_{b3} - 1\right] + \frac{I_0 M\alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - 1\right] > 0$$

The derivative of TP_{33} w.r.t. d is

$$\frac{dTP_{33}}{dd} = \frac{I_b\,\alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - \beta P_{b3}^{-\eta} T_{b3} - 1\right] + \frac{I_0 M\alpha}{\beta T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - 1\right]$$
$$\frac{I_c\,\alpha}{\beta^2 P_{b3}^{-\eta} T_{b3}}\left[e^{\beta P_{b3}^{-\eta} T_{b3}} - \beta P_{b3}^{-\eta} T_{b3} - 1 + \beta P_{b3}^{-\eta} T_{b3}\left(N-M\right)\left(e^{\beta P_{b3}^{-\eta} T_{b3}} - 1\right)\right] > 0$$

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