

**Periodic and continuous inventory models in the presence of fuzzy costs****Soheil Sadi-Nezhad<sup>a\*</sup>, Shima Memar Nahavandi<sup>a</sup> and Jamshid Nazemi<sup>a</sup>**<sup>a</sup>*Department of Engineering, Science & Research Branch, Islamic Azad University, Tehran Iran***ARTICLE INFO***Article history:*

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**ABSTRACT**

This paper presents two models, a periodic review model and a continuous review inventory model with fuzzy setup cost, holding cost and shortage cost. We use two methods in the name of signed distance and possibilistic mean value to defuzzify. Also we consider the lead time demand and the lead-time plus one period's demand as random variables. To validate the models and the solution procedures we apply them to a transformer manufacturing, 'Iran transfo', company. Furthermore we design a decision support system which can be used for efficient evaluation of the proposed models in fuzzy environment.

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**1. Introduction**

The increase in global competition especially among the manufacturing sectors motivates many contemporary organizations to spend more efforts for optimizing their global supply chain. Managing inventory as a primary function of supply chain, in an efficient and effective way, plays an important role on reducing the total cost of supply chain. The main purpose of the inventory management practices in all production companies is to have the required items ready in the predefined schedule with incurring minimum cost (Cakir & Canbolat, 2008). The proper control of their levels usually brings significant savings in costs. The development of inventory theory has evolved since the early 1920s, started with basic models with a few parameters to describe the deterministic components (Kao & Hsu, 2002). However, in the real world applications, estimating the precise value of parameters like demand and costs may not be possible. What a decision maker faces in this case is a fuzzy environment. In this regard, the preliminary models are changed to include more details using probabilistic models and fuzzy models, to overcome this limitation.

Fuzzy set theory introduced by Zadeh (1963), provides an alternate to handle the vague situations. It has been used in modeling of inventory systems with vague and imprecise parameters since 1980s. In recent years, various types of inventory problems have been developed in fuzzy environments. Yao and Lee (1999) introduced a backorder inventory model with fuzzy order quantity as triangular and trapezoidal fuzzy numbers and shortage cost as a crisp parameter. Yao and Su (2000) developed fuzzy inventory model without backorder by adopting an interval valued fuzzy number for fuzzifying the

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total demand quantity. Chang et al. (2004) proposed mixture inventory model involving variable lead time with backorders and lost sales by introducing the fuzziness of lead time demand, the average demand per year and the backorder rate of the demand during the stock-out period. Yao and Chiang (2003) considered the total cost of inventory without backorder. They fuzzified the total demand and cost of storing one unit per day into triangular fuzzy numbers and defuzzified by the centroid and the signed distance methods. Lin (2008) developed the inventory problem for a periodic review model with variable lead time and fuzzified the expected demand shortage and backorder rate using signed distance method to defuzzify. Wu and Yao (2003) fuzzified the order quantity and shortage quantity into triangular fuzzy numbers in an inventory model with backorder and they obtained the membership function of the fuzzy cost and its centroid. Handfield et al. (2009) developed a  $(Q, r)$  model based on fuzzy-set including fuzzy parameters like demand, lead time, supplier yield, and shortage cost. They also introduced a human risk attitude factor to quantify the decision maker's attitude toward the risk of stocking out during the replenishment period. The total cost was computed using centroid as defuzzification method. Tütüncü et al. (2009) used fuzzy set concepts to treat the uncertainty regarding costs in models of continuous review inventory control with or without backorder, with fuzzy costs and probabilistic demand then used Park's Median Rule to defuzzify. Vijayan and Kumaran (2008) introduced stochastic  $(Q, r)$  and  $(R, T)$  inventory models with a mixture of backorders and lost sales. They introduced fuzziness in the cost parameters to study the impact and sensitiveness of the impreciseness of cost components in the decision variables by trapezoidal fuzzy numbers and adopted the signed distance method to estimate the fuzzy cost function of the models. Dey and Chakraborty (2009) developed a periodic review model in a mixed imprecise and/or uncertain environment in which the customer demand, lead-time demand and the lead-time plus one period's demand are assumed to be fuzzy random variables. The fuzzy expected cost is defuzzified by its possibilistic mean value. Table 1 shows a summary of literature review.

**Table 1**  
The summary of literature review

Author	Year	Fuzzy Parameters	Defuzzification Method
Yao and Chiang	2003	total demand, holding cost	Centroid, Signed distance method
Wu and Yao	2003	order quantity, shortage quantity	centroid
Chang and Yao	2004	total demand, lead time demand, backorder rate	Signed distance method
Tutunchu and Akoz	2007	setup cost, holding cost, shortage cost, unit cost	Median rule
Vijayan and Kumaran	2008	setup cost, holding cost, shortage cost, lost sale cost	Signed distance method
Lin	2008	expected demand shortage, backorder rate	Signed distance method
Dey and Chakraborty	2009	total demand, lead-time demand and the lead-time plus one period's demand	Possibilistic mean value
Handfield, Warsing	2009	demand, lead time, supplier yield, and shortage cost	centroid
Sadinejad and Memar	2010	setup cost, holding cost, shortage cost	Signed distance method, Possibilistic mean value

Due to some fluctuations in inventory costs, these parameters are described as “approximately equal to”. Therefore, in this research we characterize these parameters as fuzzy variables in a periodic model and a continuous inventory model with backorder. Yao & Chiang (2003) introduced the vantage of signed distance method comparing to centroid method and we prefer to use this method.

Also for the ease of computation we use Possibilistic mean value as another method for defuzzification (Day & Chakraborty, 2009).

This article is organized as follows: Section 2 reviews some definitions and properties about fuzzy sets and defuzzification methods which would be used later. Section 3, deals with the methodology where the models and their optimal solutions are discussed. The subsection 3.1 explores the inventory problem for a periodic review and subsections 3.1.1 and 3.1.2 describes the defuzzified model by signed distance method and possibilistic mean value method. Section 3.2 introduces the continuous review inventory model in which section 3.2.1 and 3.2.2 are related to its defuzzification through signed distance and possibilistic mean value methods, respectively. Section 4 shows the results of the models for and Iranian case study called "Iran Transfo". In section 5, a decision support system with a user-friendly interface to obtain efficient solution of the proposed models is presented. Finally, conclusion remarks are given at the end to summarize the contribution of the paper.

## 2. Fuzzy preliminaries

Fuzzy set theory provides an appropriate framework to treat imprecision of modeling uncertainties and it offers more flexibility in describing these uncertainties.

Definition 1. A fuzzy set  $\tilde{a}$  on  $R = (-\infty, +\infty)$  is called a fuzzy point if its membership function is as follows,

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 & x = a \\ 0 & x \neq a \end{cases}$$

where  $a$  is called its support.

Definition 2. The fuzzy set  $\tilde{A} = (a, b, c)$  is called the triangular fuzzy number where  $a < b < c$ , if the membership function of  $A$  is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & \text{otherwise.} \end{cases}$$

Definition 3. The  $\alpha$ -cut of  $\tilde{A} = (a, b, c)$  where  $0 \leq \alpha \leq 1$  is:  $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$ .  $A_R(\alpha)$  and  $A_L(\alpha)$  are the left and right end points of  $A(\alpha)$  and are defined as follow,

$$A_{\alpha}^{-} = a + \alpha(b - a), \quad A_{\alpha}^{+} = c - \alpha(c - b) \quad (1)$$

Property1. For the fuzzy triangular numbers  $\tilde{A} = (a, b, c)$  and  $\tilde{B} = (p, q, r)$  we have,

$$\begin{aligned} \tilde{A}(+) \tilde{B} &= (a + p, b + q, c + r) \\ \tilde{A}(-) \tilde{B} &= (a - p, b - q, c - r) \end{aligned} \quad k(\cdot) \tilde{A} = \begin{cases} (ka, kb, kc), & k > 0 \\ (kc, kb, ka), & k < 0 \\ 0, & k = 0 \end{cases}$$

Definition 4. The defuzzification of  $\tilde{A}$  can be found by centroid or signed distance methods. The centroid of  $\tilde{A}$  is  $C(\tilde{A}) = \frac{a+b+c}{3}$  and the signed distance from  $\tilde{A}$  to 0 is defined as follows,

$$d(\tilde{A}, \tilde{0}) = \int_0^1 d([A_L(\alpha), A_R(\alpha)], \tilde{0}) d\alpha = \frac{a+2b+c}{4} \quad (2)$$

According to Kumaran (2007), it can be concluded that using  $d(\tilde{A}, \tilde{0})$  to defuzzify a fuzzy number, is better than using  $C(\tilde{A})$ .

### 2.1 Possibilistic mean value of a fuzzy number

For a given fuzzy number  $\tilde{A}$ , the interval-valued possibilistic mean is defined as,

$$M(\tilde{A}) = [M_L(\tilde{A}), M_R(\tilde{A})],$$

where  $M_L(\tilde{A})$  and  $M^U(\tilde{A})$  are the lower and the upper values of  $\tilde{A}$  and are, respectively, defined by Day and Chakraborty (2009).

$$M_L(\tilde{A}) = \frac{\int_0^1 \alpha A_\alpha^- d\alpha}{\int_0^1 \alpha d\alpha} \quad \text{and} \quad M_R(\tilde{A}) = \frac{\int_0^1 \alpha A_\alpha^+ d\alpha}{\int_0^1 \alpha d\alpha}$$

The possibilistic mean value of  $\tilde{A}$  is then defined as,

$$\bar{M}(\tilde{A}) = \int_0^1 \alpha (A_\alpha^- + A_\alpha^+) d\alpha \quad (3)$$

## 3. Methodology

We make use of the following notations through this paper:

- $T$  the time between replenishments;
- $R$  target inventory level;
- $Q$  lot size in units;
- $r$  reorder point;
- $A$  unit cost of placing an order for an item;
- $h$  inventory holding cost per unit per year;
- $\pi$  unit shortage cost;
- $D$  annual demand

### 3.1 periodic review inventory model

A periodic review system involves determining the amount of an item in stock at a specified, fixed time interval and placing an order that when added to the quantity on hand, will equal to predetermined maximum level. The target inventory level  $R$ , is the sum of expected demand during the lead time and replenishment period plus the safety stock. Due to fluctuations in demand or

variable lead time, stock out may occur, in which case a penalty per unit shortage is incurred. The purpose of the periodic inventory model is to find the optimal time between replenishment and maximum inventory level such that the combination of the order cost and the holding cost and shortage cost is minimized. The approximate mean total cost is expressed as follows,

$$C(R, T) = \frac{A}{T} + h \left[ R - DL - \frac{DT}{2} \right] + \frac{\pi}{T} B(R, T), \quad (4)$$

where  $B(R, T)$  is the expected shortage at the end of the cycle and is defined as follows,

$$B(R, T) = E(D_{L+T} - R)^+ = \int_R^{\infty} (x - R) f(x) dx, \quad (5)$$

where  $x$  denotes the lead-time plus one period's demand that follows a normal distribution.  $f(x)$  is normal probability distribution function with mean  $\mu_{L+T}$  and standard deviation  $\sigma \sqrt{T + L}$ .

To minimize the  $C(R, T)$  we solve  $\frac{\partial d(C(R, T), 0)}{\partial R} = 0$  which yields,

$$\int_R^{\infty} f(x, T) dx = \frac{hT}{\pi}. \quad (6)$$

The complementary cumulative distribution of  $x$  is as follows,

$$R = F^{-1} \left( 1 - \frac{hT}{\pi} \right). \quad (7)$$

The initial value of  $T$  concludes the value of  $R$  and consequently the value of  $C(R, T)$  can be calculated. This procedure is iterated until the condition  $C(R_{i-1}, T_{i-1}) - C(R_i, T_i) > 0$  is achieved in stage  $i$ , which means the cost needs to be decreased until it attains its minimum which yields  $T^*$  and  $R^*$  to be  $T^* = T_{i-1}$  and  $R^* = R_{i-1}$ .

Note that the values of  $\mu_{L+T}$  and  $\sigma_{L+T}$  must be recalculated in the cumulative distribution function for each iteration.

### 3.1.1 Fuzzy periodic review inventory model using signed distance method

Since evaluating actual costs in real world is very difficult, if not impossible, we consider all costs in the model in fuzzy sense. Consider the fuzzy costs as follows,

$$\begin{aligned} \pi &= (\pi - \lambda_1, \pi, \pi + \lambda_2), \\ h &= (h - \Delta_1, h, h + \Delta_2), \\ A &= (A - \delta_1, A, A + \delta_2). \end{aligned} \quad (8)$$

By substituting these fuzzy costs into (4) we get three expected annual costs as follows,

$$\begin{aligned} C_1(R, T) &= \frac{A - \delta_1}{T} + (h - \Delta_1) \left( R - DL - \frac{DT}{2} \right) + E(D_{L+T} - R)^+ \left( \frac{\pi - \lambda_1}{T} \right), \\ C_2(R, T) &= \frac{A}{T} + h \left[ R - DL - \frac{DT}{2} \right] + E(D_{L+T} - R)^+ \frac{\pi}{T}, \\ C_3(R, T) &= \frac{A + \delta_2}{T} + (h + \Delta_2) \left( R - DL - \frac{DT}{2} \right) + E(D_{L+T} - R)^+ \left( \frac{\pi + \lambda_2}{T} \right). \end{aligned} \quad (9)$$

Using the signed distance method given in (2) yields the following defuzzified value of  $C(R,T)$ ,

$$d(C(R,T),0) = C_2 + \frac{1}{4} \left[ \frac{\delta_2 - \delta_1}{T} + (\Delta_2 - \Delta_1) \left( R - DL - \frac{DT}{2} \right) + \left( \frac{\lambda_2 - \lambda_1}{T} \right) B(R,T) \right].$$

Similar to the crisp case, the optimal values of  $R$  and  $T$  are obtained by taking the derivative of  $C(R,T)$  with respect to  $R$ , which yields,

$$\int_R^\infty f(x,T) dx = \frac{\left[ h + \frac{\Delta_4 - \Delta_3}{4} \right] T}{\pi + \frac{\pi_6 - \pi_5}{4}},$$

or

$$R = F^{-1} \left( 1 - \frac{\left[ h + \frac{\Delta_4 - \Delta_3}{4} \right] T}{\pi + \frac{\pi_6 - \pi_5}{4}} \right). \quad (10)$$

The solution procedure proposed for the crisp case in the previous section is implemented here to find  $T^*$  and  $R^*$ .

### 3.1.2 Fuzzy periodic review inventory model using possibilistic mean value method

The aim of this section is to use the  $\alpha$ -cut of the fuzzy inventory costs to derive the  $\alpha$ -cut of the fuzzy total cost. The  $\alpha$ -cuts of the setup cost, holding cost and shortage cost are calculated using (1). Substituting the left endpoints of the costs in (4) results the left endpoint of the total cost and the right endpoints results the right endpoint of the total cost as follow,

$$\begin{aligned} C_\alpha^- &= \frac{A_\alpha^-}{T} + h_\alpha^- \left[ R - DL - \frac{DT}{2} \right] + B(R,T) \frac{\pi_\alpha^-}{T}, \\ C_\alpha^+ &= \frac{A_\alpha^+}{T} + h_\alpha^+ \left[ R - DL - \frac{DT}{2} \right] + B(R,T) \frac{\pi_\alpha^+}{T}. \end{aligned} \quad (11)$$

The possibilistic mean value of the above interval through (3) is determined as follows,

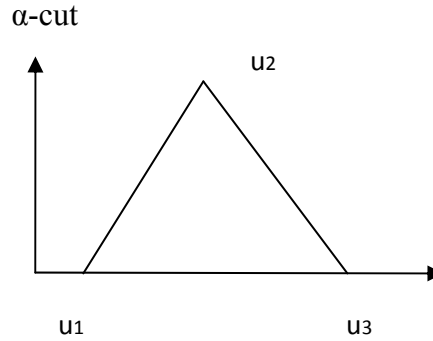
$$C(R,T) = \int_0^1 (C_\alpha^- + C_\alpha^+) d\alpha.$$

The procedure to find the optimal values is the same as section 3.1 but  $R$  is determined as follows where  $u$  is defined as,

$$u = 1 - \frac{hT}{\pi}, \quad (12)$$

with  $\alpha=0$ , the two lower edges and with  $\alpha=1$ , the upper edge of the triangle are made as follows,

$$u_1 = 1 - \frac{h_3 T}{\pi_1}, \quad u_2 = 1 - \frac{h_2 T}{\pi_2}, \quad u_3 = 1 - \frac{h_1 T}{\pi_3}. \quad (13)$$



The left and the right endpoints of this interval are as follows,

$$\begin{aligned} u_{\alpha}^{-} &= u_1 + \alpha(u_2 - u_1), \\ u_{\alpha}^{+} &= u_3 - \alpha(u_3 - u_2). \end{aligned} \quad (14)$$

Therefore, the defuzzified value using its possibilistic mean value given in (3) is calculated as follows,

$$u = \int_0^1 (u_{\alpha}^{-} + u_{\alpha}^{+}) d\alpha, \quad (15)$$

and  $R$  is also calculated as  $R = F^{-1}(u)$ .

### 3.2 Continuous review inventory system

In a continuous review inventory system, an order for a lot is placed whenever the quantity on hand is dropped to a predetermined level, known as the order point. The total cost expression of this model used in this paper is presented as follows,

$$C(Q, r) = \frac{AD}{Q} + h \left[ \frac{Q}{2} + r - \mu \right] + \left( \frac{\pi D}{Q} B(r) \right), \quad (16)$$

where  $B(r)$  is the expected demand shortage at the end of the cycle and is defined as follows,

$$B(r) = E(D_L - r)^+ = \int_R^{\infty} (x - r) f(x) dx, \quad (17)$$

where  $x$  is regarded as the value of lead-time demand with normal probability distribution function  $f(x)$ , mean  $\mu_L$  and standard deviation  $\sigma_L$ . Taking the first derivative of  $C(Q, r)$  with respect to  $Q$  and  $r$  yields the optimal values as follows,

$$Q^* = \sqrt{\frac{2D[A + \pi B(r)]}{h}}, \quad (18)$$

$$r^* = F^{-1}\left(1 - \frac{hQ}{\pi D}\right). \quad (19)$$

### 3.2.1 Fuzzy continuous review inventory model using signed distance method

By substituting fuzzy parameters in (16) and getting three total costs like the periodic model, the defuzzified cost using signed distance method is determined as follows,

$$d(C(Q, r), 0) = C(Q, r) + \frac{1}{4} \left[ \frac{(\delta_2 - \delta_1)D}{Q} + (\Delta_2 - \Delta_1) \left[ \frac{Q}{2} + r - \mu \right] + \left( \frac{(\lambda_2 - \lambda_1)D}{Q} B(r) \right) \right].$$

Taking the partial derivatives with respect to  $Q$  and  $r$  yields,

$$Q^* = \sqrt{\frac{2D \left[ A + \frac{(\delta_2 - \delta_1)}{4} + (\pi + \frac{(\lambda_2 - \lambda_1)}{4}) B(r) \right]}{h + \frac{(\Delta_2 - \Delta_1)}{4}}}, \quad (20)$$

and

$$r^* = F^{-1} \left( 1 - \frac{\left( h + \frac{(\Delta_2 - \Delta_1)}{4} \right) Q}{\left( \pi + \frac{(\lambda_2 - \lambda_1)}{4} \right) D} \right). \quad (21)$$

Since the optimum values of  $Q$  and  $r$  cannot be directly obtained by Eq. (20) and Eq. (21), the iterative procedure suggested by Hadley and Whitin (1963) is used to solve the equations. The initial value for  $Q$  is obtained by equating  $B(r)$  to zero and with the  $r$  resulted from substituting  $Q$  in (21) and  $B(r)$  is obtained by substituting  $r$  in (17) and the new  $Q$  is obtained by Eq. (20). Repeating this procedure until the condition  $Q_i = Q_{i-1}$  is met, will lead us to find the optimal values of  $Q$  and  $r$ .

### 3.2.2 Fuzzy continuous review inventory model using possibilistic mean value method

Similar to section 3.1.2, the  $\alpha$ -cut approach is used to treat the fuzziness and derive the total annual cost as follows,

$$C_\alpha^-(Q, r) = \frac{A_\alpha^- D}{Q} + h_\alpha^- \left[ \frac{Q}{2} + r - \mu \right] + \left( \frac{\pi_\alpha^- D}{Q} B(r) \right)$$

$$C_\alpha^+(Q, r) = \frac{A_\alpha^+ D}{Q} + h_\alpha^+ \left[ \frac{Q}{2} + r - \mu \right] + \left( \frac{\pi_\alpha^+ D}{Q} B(r) \right) \quad (22)$$

$$C(Q, r) = \int_0^1 (C_\alpha^- + C_\alpha^+) d\alpha.$$

The minimum, the mean and the maximum values of the  $Q$  represented by  $Q_1, Q_2$  and  $Q_3$ , respectively, are calculated as follows,

$$Q_1 = \sqrt{\frac{2D[A_1 + \pi_1 B(r)]}{h_3}} \quad Q_2 = \sqrt{\frac{2D[A_2 + \pi_2 B(r)]}{h_2}} \quad Q_3 = \sqrt{\frac{2D[A_3 + \pi_3 B(r)]}{h_1}}$$



The possibilistic mean using (3) is then presented as follows,

$$Q = \int_0^1 (Q_{\alpha}^{-} + Q_{\alpha}^{+}) d\alpha. \tag{23}$$

Repeating the same concept applied in section 3.1.2 we have,

$$u_1 = 1 - \frac{h_3 Q}{\pi_1 D}, \quad u_2 = 1 - \frac{h_2 Q}{\pi_2 D}, \quad u_3 = 1 - \frac{h_1 Q}{\pi_3 D}, \quad u = \int_0^1 (u_{\alpha}^{-} + u_{\alpha}^{+}) d\alpha, \quad r = F^{-1}(u). \tag{24}$$

The iterative process in section 3.2.1 is applied here to find optimal  $Q$  and  $r$ . Fig. 1. gives an overview of the inputs and outputs of the models.

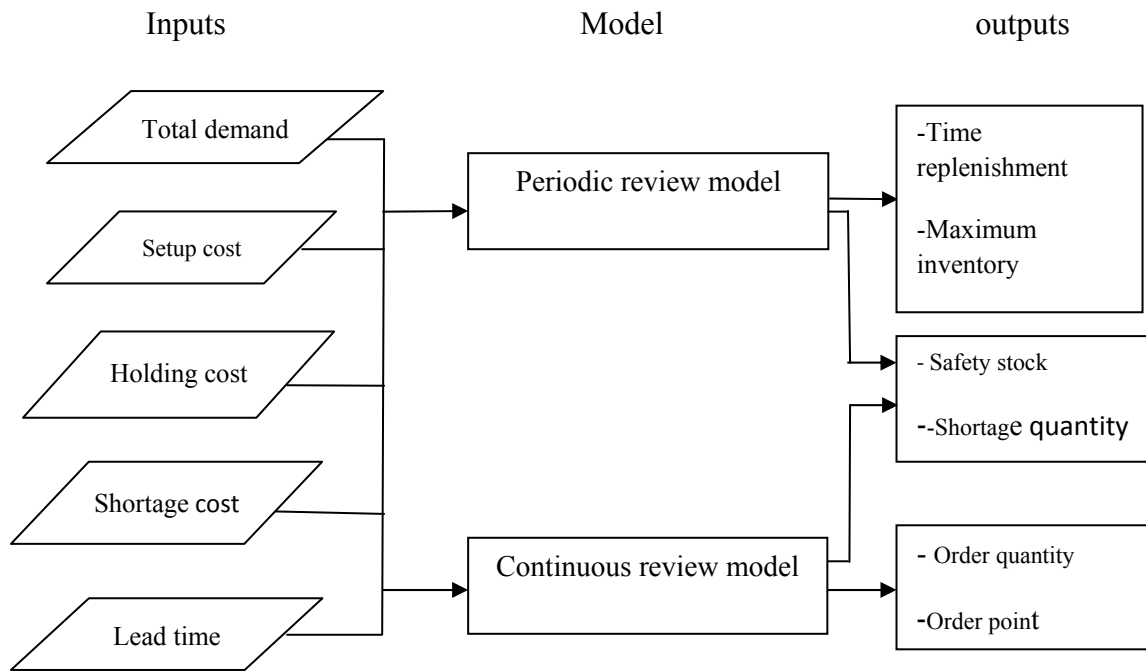


Fig.1. Inputs and outputs of the models

#### 4. Application in Iran Transfo

In the previous sections, we have shown how to modify the crisp models for continuous and periodic review inventory systems in order to consider the fuzziness associated with the costs. Recently, there has been an increase interest on using fuzzy theory for real-world applications (Modarres et al. (2010)). In order to validate and illustrate the results of the proposed models we have conducted an application in a private company called "Iran Transfo". We have also selected the most strategic items which constitute group products *A* and *B* in ABC inventory classification. The total demand, the lead time and the fuzzy costs, as inputs, are shown in Table 2. Deficiency of any of these items leads to an interruption on manufacturing which causes approximately 35,000,000 US dollar. The results for both models, with both defuzzification methods are determined in Table 3.

**Table 2**

Input parameters

name	Total demand	Lead time (month)	Fuzzy setup cost	Fuzzy holding cost
1	4.488	4	(95000, 100,000, 103,000)	(1000, 1500, 1800)
2	3240	1	(9700, 10000, 10200)	(115,120,122)
3	960	5	(118,000, 120,000, 121,000)	(580,600,620)
4	10800	6	(67,000, 70,000, 72,000)	(285,300,310)
5	10320	6	(56,000, 60,000, 62,000)	(200,225,250)

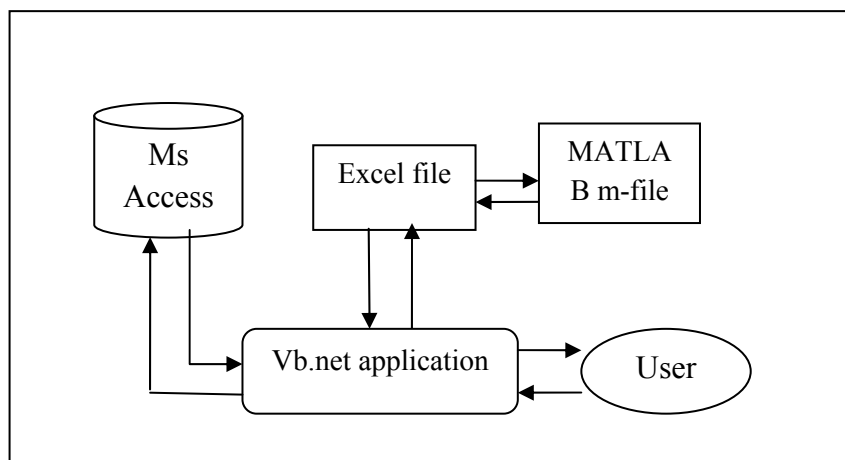
**Table 3**

The optimal solutions

	Fuzzy method	Periodic model results			Continuous model results		
		C(R,T)	R	T	C(Q,r)	Q	r
1	SDM	2,556,900	3078	54	2,288,800	784	2229
	$\alpha$ -cut	2,578,500	3077	54	2,308,100	790	2228
2	SDM	159,720	1503	72	125,660	736	576
	$\alpha$ -cut	159,990	1504	72	125,870	735	577
3	SDM	3,049,900	9621	54	2,634,900	1934	7622
	$\alpha$ -cut	2,975,400	1023	54	2,621,700	1832	7629
4	SDM	1,544,900	9867	72	1,399,100	2300	7336
	$\alpha$ -cut	1,547,200	9867	72	1,370,900	2195	7341
5	SDM	262,680	1266	216	230,280	644	577
	$\alpha$ -cut	272,630	1265	216	238,940	634	576

## 5. Decision support system

In today's timeliness production environment, it is extremely important for the decision makers to have access to the decision support tools in order to make rapid and accurate decisions. In this section a decision support system is developed to efficiently use the models and corresponding iterative procedures and to analyze the effectiveness of the proposed approaches when tackling real world problem instances. The technological background of the system is illustrated in Fig. 2.

**Fig. 2.** The structure of DSS

The proposed DSS model consists of three major components where the first one is a database component in which all inventory records are stored and can be modified for updates. The database is implemented with Microsoft access. The second component is a model based component which includes both continuous and periodic review models with user-defined parameters. The models and the procedures are written in MATLAB software. The third item is the user interface which is designed in Visual Basic. Transforming data between MATLAB and Visual Basic is performed by using an excel file as an intermediate to pass data between these two environments. The main interface window of DSS is given in Fig. 3. As we can observe, user can see the results for both models and choose the proper one according to their condition. Also user can choose the time period and see the optimal result for from the selected period. The point is that both models perform the calculation using both defuzzification methods and the results for the method which yields the minimum cost are shown in the form.

The screenshot shows the 'Main window' of the DSS application. It is divided into several sections:

- properties:** Includes 'product name' (copper), 'Total demand' (4488), and 'Lead time (month)' (4).
- Fuzzy costs:** A table with columns 'a1', 'a2', and 'a3' and rows 'Setup', 'Holding', and 'Shortage'.
 

	a1	a2	a3
Setup	95000	100000	103000
Holding	1000	1500	1800
Shortage	30000000	35000000	37000000
- Model type:** Radio buttons for 'Periodic review' and 'continues review' (selected). A 'Time period' dropdown menu is also present.
- Solve:** A button to execute the calculation.
- Results:** Two columns of results:
 

Continues model:		Periodic model:	
Optimal order quantity	784	Optimal replenishment time (month)	
Optimal reorder point	2229	Optimal maximum inventory	
Total cost	2288800	Total cost	
Safety stock	793	Safety stock	

Fig. 3. The main form of the DSS

## 6. Conclusion

Incorporating fuzzy set theory and probability theory into inventory models potentiates the model to tackle real world fuzziness and randomness. In this article, we have applied the fuzzy set theory to reformulate the two periodic and continuous inventory models where shortages are backordered with shortage cost are incurred. Also, we have used the probabilistic theory to express the demand in the replenishment time. In both models, we have used the triangular fuzzy numbers to represent the imprecise setup cost, holding cost and shortage cost and obtained the model with fuzzy total cost. The

periodic model gives the optimal values of review periods and maximum inventory levels and the continuous review model gives the optimal order quantity and order point. For these fuzzy models, we have employed two methods of defuzzification which are the signed distance and the possibilistic mean value. The implementation of the proposed method has also been illustrated using some numerical example. In addition, we have also proposed a decision support system with a user friendly interface, for efficient and effective use of the proposed models. Hence, the suggested inventory models are observed to be executable and useful for the decision maker in the real world.

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