A multi-objective possibilistic programming approach for locating distribution centers and allocating customers demands in supply chains

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ABSTRACT

In this paper, we present a multi-objective possibilistic programming model to locate distribution centers (DCs) and allocate customers' demands in a supply chain network design (SCND) problem. The SCND problem deals with determining locations of facilities (DCs and/or plants), and also shipment quantities between each two consecutive tier of the supply chain. The primary objective of this study is to consider different risk factors which are involved in both locating DCs and shipping products as an objective function. The risk consists of various components: the risks related to each potential DC location, the risk associated with each arc connecting a plant to a DC and the risk of shipment from a DC to a customer. The proposed method of this paper considers the risk phenomenon in fuzzy forms to handle the uncertainties inherent in these factors. A possibilistic programming approach is proposed to solve the resulted multi-objective problem and a numerical example for three levels of possibility is conducted to analyze the model.

1. Introduction

One of the primary issues in facility location problem is to locate a set of new facilities such that the transportation cost from various facilities to customers is minimized. Due to increasing importance of efficient design of supply chain networks, facility location problems in the context of supply chain management (SCM) have attracted attentions of many researchers. For comprehensive review on facility location models and solution approaches refer to Aikens (1985), Owen & Daskin (1998), and Klose & Drexel (2005). Especially, Mello et al. (2009) dedicated a review paper to facility location models in the supply chain management. In the SCM context we generally seek the best sites for locating distribution centers (DCs) or warehouses in a discrete solution space such that total fixed cost of locating DCs and variable transportation costs for distributing products (commodities) from manufacturing plants to customers through opened DCs are minimized. This type of problems is normally modeled as mixed integer programming (MIP) formulations. One function of DCs is consolidation such that they sort and combine products received from plants for shipment to customers. Moreover, providing a firm with flexibility in responding to changes in the marketplace in a quicker manner, and taking advantage of economies of scale in transportation costs are other benefits of using DCs (Amiri, 2006). Although the main concern is to select the best possible sites for DCs, but in the literature, plant location is also considered as decision variables (For example, Kaufman et al., 1977; Ro & Tcha, 1984, Pirkul & Jayaraman, 1998; Marin & Pledgerin, 1999; Amiri, 2006; Lu & Bostel, 2007). There are cases where no limit is assigned to plants and/or DCs (Kuehn & Hamburger, 1963; Kaufman et al., 1977; Ro & Tcha, 1984; Brimberg et al., 2000) and therefore the resulted models are formulated as uncapacitated facility location problem (UFLP), while there are also other cases where some realistic constraints such as production power of plants and storage space...
of DCs are taken into account and the resulted formulation is called capacitated facility location problem (CFLP) (Lee, 1991; Hindi & Basta, 1994, Pirkul & Jayaraman, 1996, 1998; Jayaraman & Pirkul, 2001; Amiri 2006; Keskin & Uster, 2007, among others). Tragantalsak et al. (2000) studied a two-echelon facility location problem where the facilities in the first echelon are considered as uncapacitated while the facilities in the second echelon are capacitated. There are many input parameters involved in configuration of a supply network which are either deterministic or stochastic such as distance or customers’ demands. Although some of the parameters, such as distance, could be deterministic but there are other important factors, like demand, which are subject to uncertainty and have been dealt with through stochastic programming models (Logendran & Terrell, 1988; Sherali & Rizzo, 1991; Hwang, 2002, Miranda & Garrido, 2008). A normal case to handle the uncertainty is to use historical data to find an estimate of the uncertain data. However, in many cases, when the historical data are unreliable or the period of the study is short, the probability distributions of customers’ demands cannot be obtained easily. Instead, we could use fuzzy decision making methods where we are able to use expert’s opinion in a form of linguistic terms such as little, moderate, large, etc to provide estimations for the uncertain parameters. Therefore, the fuzzy set theory (Zadeh, 1965) can be used to deal with this situation. There are many cases where facility location problems are analyzed using the concept of fuzzy programming (Darzentas, 1987; Rao & Saraswati, 1988). Bhattacharya et al. (1992) developed a fuzzy goal programming approach to deal with this problem. Zhou and Liu (2007) considered a capacitated facility location-allocation problem, and used a fuzzy programming method to solve it. In this paper we develop a fuzzy multi-objective mixed integer linear programming (FMOMILP) model for capacitated DC location and distribution decisions in supply chains where demands of customers and capacities of the DCs are assumed to have some possibility distribution, and risks associated with each potential DC location as well as each arc of the network are considered as fuzzy numbers. The possibility programming approach is used for transforming the resulting fuzzy model to its crisp equivalent, and the compromise programming method is adopted to solve this multi-objective model. Risk of each potential DC location is expressed using linguistic variables associated with natural disasters such as fire history, earthquake possibility, tornados, hurricanes, etc for evaluating each location in terms of disruption risks. Risks related to the arcs imply inherent risks in the transportation of products from each plant to each DC and from each DC to each customer. These risks may be viewed as risks related to different transportation modes. For instance, we assume the shipments of the products from DC A to customer C are less risky if the transportation facility is train and they are more risky if regular trucks are used, instead. We also consider other conditions affecting the quality of shipment between each two nodes in each two echelons of the network. Zhou and Liu (2007) studied locating of facilities in the continuous solution space and considered a single-stage distribution problem. The problem formulation of this study, however, deals with facility location decisions in discrete space in a two-stage distribution problem. Furthermore, we consider a bi-objective model whereas they dealt with the classical single-objective capacitated location-allocation problem.

The paper is organized as follows. In Section 2, we briefly review possibility programming for multi-objective linear programming models. Mathematical formulation of the proposed network design problem is developed in Section 3. In Section 4, we provide a numerical example for the problem under investigation and discuss the results. Finally, in Section 5, conclusions are given to summarize the contribution of the work.

2. Possibility programming for multi-objective linear programming models

Negi and Lee (1993) proposed a fuzzy multi-objective linear programming model as follows,

\[
\text{Maximize} \quad \sum_{j=1}^{n} \tilde{c}_{rj} x_j, \quad r = 1, 2, \ldots, p, \tag{1}
\]
subject to \[ \sum_{j=1}^{n} a_{ij}x_j \leq \tilde{b}_i, \quad i = 1, 2, \ldots, m, \] \[ x_j \geq 0, \quad j = 1, 2, \ldots, n, \] where \( x_j, \quad j = 1, \ldots, n \) are crisp decision variables. \( \tilde{c}_{ij} \) is the fuzzy coefficient of the \( j \)-th decision variable in the \( r \)-th, \( r = 1, \ldots, p \) objective function. \( \tilde{a}_{ij} \) is the fuzzy coefficient of the \( j \)-th decision variable in the \( i \)-th constraint \( (j = 1, \ldots, n, i = 1, \ldots, m) \), and \( \tilde{b}_i \) is the fuzzy right-hand side in the \( i \)-th, \( i = 1, \ldots, m \) constraint. \( \tilde{c}_{ij}, \tilde{a}_{ij} \) and \( \tilde{b}_i \) can be expressed as either trapezoidal or triangular fuzzy numbers. Here, we represent their trapezoidal form as \( \tilde{c}_{ij} = (\underline{c}_{ij}, \underline{c}_{ij}, \bar{c}_{ij}, \bar{c}_{ij}) \), \( \tilde{a}_{ij} = (\underline{a}_{ij}, \underline{a}_{ij}, \bar{a}_{ij}, \bar{a}_{ij}) \), \( \tilde{b}_i = (\underline{b}_i, \underline{b}_i, \bar{b}_i, \bar{b}_i) \), and their triangular form as \( \tilde{c}_{ij} = (\underline{c}_{ij}, \underline{c}_{ij}, \bar{c}_{ij}) \), \( \tilde{a}_{ij} = (\underline{a}_{ij}, \underline{a}_{ij}, \bar{a}_{ij}) \), \( \tilde{b}_i = (\underline{b}_i, \underline{b}_i, \bar{b}_i) \).

Applying the possibility programming approach to fuzzy multi-objective linear programming model (1)–(3) under exceedance as well as strict exceedance possibility in the case of trapezoidal fuzzy numbers is given below.

2.1. Case of exceedance possibility

Maximize \[ Z_r = \sum_{j=1}^{n} ((1 - \alpha)c_{rj} + \alpha \bar{c}_{rj})x_j, \quad r = 1, 2, \ldots, p, \] subject to: \[ \sum_{j=1}^{n} ((1 - \alpha)a_{ij} + \alpha \bar{a}_{ij})x_j \leq (1 - \alpha)\tilde{b}_i + \alpha \bar{b}_i, \quad i = 1, 2, \ldots, m, \] \[ x_j \geq 0, \quad j = 1, 2, \ldots, n, \] where \( \alpha \) is a pre-determined value which is the minimum required possibility, and falls in the interval of \((0,1]\).

2.2. Case of strict exceedance possibility

In this case, only constraint set (5) in the above Eq. (4)–(6) is replaced by the following constraint set,

\[ \sum_{j=1}^{n} ((1 - \alpha)a_{ij2} + \alpha \bar{a}_{ij})x_j \leq (1 - \alpha)\tilde{b}_i + \alpha \bar{b}_i, \quad i = 1, 2, \ldots, m. \]

In the case of triangular fuzzy numbers, \( c_{ij2} \) is replaced with \( c_{ij0} \), \( a_{ij1} \) and \( a_{ij2} \) are replaced with \( a_{ij0} \), and \( b_{i2} \) is replaced with \( b_{i0} \) in the models. Note that, in a given fuzzy model, it is possible to use trapezoidal numbers for some input parameters and triangular numbers for other parameters.

3. Model formulation

The network design problem considered in this section consists of three echelons of plants, DCs, and customers and the location decisions are made in the DC level. Furthermore, distribution decisions are made in two stages. In the first stage, products are shipped from capacitated plants to capacitated DCs, and in the second stage, shipments from capacitated DCs to customers (retailers) are considered in order to satisfy customers’ demands. Demand of each customer and capacity of each DC are assumed to have some possibility distributions which are expressed using trapezoidal fuzzy numbers. The goal is to find the best locations for DCs to be opened and to determine shipment quantity on each arc of the network such that the total cost and the total risk in the network are minimized. There
is also an upper bound ($p$) on the number of DCs to be opened. The SCM network under study is depicted in Fig. 1. We use the following notation for the formulation of the model.

$I$ set of customers, $i=1,\ldots,m$

$J$ set of potential DCs, $j=1,\ldots,n$

$K$ set of plants, $k=1,\ldots,K$

$\tilde{D}_i$ fuzzy demand of customer $i$

$\tilde{W}_j$ fuzzy capacity of DC $j$

$P_k$ capacity limit at plant $k$

$f_j$ fixed cost of opening a DC at site $j$

$c_{ij}$ unit transportation cost from DC $j$ to customer $i$

$e_{jk}$ unit transportation cost from plant $k$ to DC $j$

$\tilde{R}_{ij}$ fuzzy risk associated with the arc connecting DC $j$ to customer $i$

$\tilde{S}_{jk}$ fuzzy risk associated with the arc connecting plant $k$ to DC $j$

$\tilde{b}_j$ fuzzy risk associated with DC at site $j$

The decision variables are also as follows,

$z_j$ 1 if DC at site $j$ is opened, 0 otherwise, $\forall j$

$x_{ij}$ amount of product shipped from DC $j$ to customer $i$

$y_{jk}$ amount of product shipped from plant $k$ to DC $j$

The fuzzy supply chain network design (FSCND) problem can be formulated as follows,

Min $Z_1 = \sum_{j \in J} f_j z_j + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} \sum_{k \in K} e_{jk} y_{jk}$ \hspace{1cm} (1)$

Min $Z_2 = \sum_{i \in I} \sum_{j \in J} \tilde{b}_j x_{ij} + \sum_{i \in I} \sum_{j \in J} \tilde{R}_{ij} x_{ij} + \sum_{j \in J} \sum_{k \in K} \tilde{S}_{jk} y_{jk}$ \hspace{1cm} (2)$

s.t. $\sum_{j \in J} x_{ij} = \tilde{D}_i$, $\forall i \in I$, \hspace{1cm} (3)$

$\sum_{i \in I} x_{ij} \leq \tilde{W}_j z_j$, $\forall j \in J$, \hspace{1cm} (4)$
\[
\sum_{j \in J} z_j \leq p, \quad (5)
\]

\[
\sum_{i \in I} x_{ij} = \sum_{k \in K} y_{jk}, \quad \forall j \in J, \quad (6)
\]

\[
\sum_{j \in J} y_{jk} \leq P_k, \quad \forall k \in K, \quad (7)
\]

\[
x_{ij} \geq 0, \quad \forall i \in I, \forall j \in J, \quad (8)
\]

\[
y_{jk} \geq 0, \quad \forall j \in J, \forall k \in K, \quad (9)
\]

\[
z_j \in \{0,1\}, \quad \forall j \in J. \quad (10)
\]

The objective function (1) minimizes the total costs of the opening and the operating the DCs, and the total variable transportation costs in the network. The objective function (2) minimizes the total risk in the network which has three components: The risks associated with locating DCs, the risks of shipping products from plants to DCs, and the risks of distributing products from DCs to customers. Constraint set (3) ensures that the demand of each customer is satisfied. Constraint sets (4) and (7) ensure that the capacity restrictions at the DCs and the plants are not violated, respectively. Constraint (5) limits the number of DCs to be opened to the pre-specified value \(p\). Constraint set (6) is the flow conservation constraint at each DC. Finally, constraints (8) – (10) are non-negativity and integrality constraints, respectively.

To solve FSCND model, the compromise programming (CP) method is used. Compromise programming tries to find a solution that comes “as close as possible” to the ideal (optimal) values of each objective function (Zeleny, 1982). Here “Closeness” is defined by the \(L_p\) distance metric as follows:

\[
L_p = \left[ \sum_{i=1}^{k} \gamma_i \left( \frac{f_i - f_i^*}{f_i^*} \right)^p \right]^{1/p}
\]

for \(p = 1, 2, \ldots, \infty\),

in which \(f_1, f_2, \ldots, f_k\) are different and conflicting objective functions. \(f_i^* = \min(f_i)\), ignoring all other objectives, is called the ideal value for the \(i\)th objective and \(\gamma_i\) is the weight of objective \(i\). The \(x^*\) is called a compromise solution, if minimizes \(L_p\) by considering \(\gamma_i > 0, \sum \gamma_i = 1, \) and \(1 \leq p \leq \infty\).

Different efficient solutions can be obtained by considering different values for parameters \(p\) and \(\gamma_i\). As \(p\) increases, larger deviations get more weight, such that for \(p=\infty\), the largest deviation completely dominates the distance determination. However, the most common values are \(p = 1, 2, \text{ and } \infty\).

4. Numerical Example

In this section, a numerical example is studied to demonstrate the implementation of the proposed method and discuss the advantage of using the developed model. The example consists of two plants, six potential DC locations, and ten customers. The decision maker uses the linguistic variables shown in Table 1 to assess the risks associated with potential locations for DCs and risks associated with each arc of the network for transporting the product.
Table 1
Linguistic variables for assessing risk of each potential DC location and each arc of the network

<table>
<thead>
<tr>
<th>Variable</th>
<th>Linguistic Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low (VL)</td>
<td>(0,0,1,2)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(1,2,2,3)</td>
</tr>
<tr>
<td>Medium Low (ML)</td>
<td>(2,3,4,5)</td>
</tr>
<tr>
<td>Fair (F)</td>
<td>(4,5,5,6)</td>
</tr>
<tr>
<td>Medium High (MH)</td>
<td>(5,6,7,8)</td>
</tr>
<tr>
<td>High (H)</td>
<td>(7,8,8,9)</td>
</tr>
<tr>
<td>Very High (VH)</td>
<td>(8,9,10,10)</td>
</tr>
</tbody>
</table>

4.1. Model parameters

Model parameters are as follows:

\[
\tilde{D} = ((80,82,86,95),(75,80,83,86),(90,93,93,95),(87,90,92,97),(85,87,88,98),(78,82,83,85),
(76,81,82,89),(72,76,78,80),(80,86,89,90),(77,78,84,84)),
\]

\[
\tilde{b} = (L, F, VL, F, ML, H), \quad f = (15600, 17000, 15200, 16000, 17200, 18000), \quad P = (620, 570),
\]

\[
\tilde{W} = ((260,270,280,310),(270,280,290,300),(290,295,320,325),(300,310,320,340),(260,270,290,340),
(270,310,310,320)),
\]

Fig. 1. Supply chain network under study.
Since the coefficients of variables in the constraints are all crisp numbers, therefore, there is no difference between the case of exceedance possibility, and the case of strict exceedance possibility. The parametric crisp equivalent of vectors $\mathbf{D}^\sim$ and $\mathbf{b}^\sim$, and matrices $\mathbf{R}^\sim$ and $\mathbf{S}^\sim$ are as follows:

$\mathbf{D}^\sim = (95 - 9\alpha, 86 - 3\alpha, 95 - 2\alpha, 97 - 5\alpha, 98 - 10\alpha, 85 - 2\alpha, 89 - 7\alpha, 80 - 2\alpha, 90 - \alpha, 84 - 0\alpha),$

$\mathbf{W} = (310 - 30\alpha, 300 - 10\alpha, 325 - 5\alpha, 340 - 20\alpha, 340 - 50\alpha, 320 - 10\alpha),$

$\mathbf{R}^\sim = \begin{bmatrix} 9 - \alpha & 6 - \alpha & 6 - \alpha & 2 - \alpha & 8 - \alpha & 3 - \alpha & 6 - \alpha & 9 - \alpha & 3 - \alpha & 2 - \alpha \\ 6 - \alpha & 6 - \alpha & 8 - \alpha & 3 - \alpha & 9 - \alpha & 3 - \alpha & 10 & 6 - \alpha & 6 - \alpha & 8 - \alpha \\ 9 - \alpha & 3 - \alpha & 5 - \alpha & 9 - \alpha & 10 & 10 & 6 - \alpha & 6 - \alpha & 2 - \alpha & 6 - \alpha \\ 3 - \alpha & 5 - \alpha & 9 - \alpha & 3 - \alpha & 6 - \alpha & 5 - \alpha & 6 - \alpha & 6 - \alpha & 2 - \alpha & 6 - \alpha \\ 6 - \alpha & 6 - \alpha & 5 - \alpha & 6 - \alpha & 9 - \alpha & 6 - \alpha & 2 - \alpha & 3 - \alpha & 3 - \alpha & 6 - \alpha \\ 2 - \alpha & 3 - \alpha & 6 - \alpha & 3 - \alpha & 6 - \alpha & 9 - \alpha & 3 - \alpha & 8 - \alpha & 6 - \alpha & 9 - \alpha \end{bmatrix},$

$\mathbf{S}^\sim = \begin{bmatrix} 6 - \alpha & 5 - \alpha & 3 - \alpha & 9 - \alpha & 6 - \alpha & 2 - \alpha \\ 3 - \alpha & 9 - \alpha & 8 - \alpha & 6 - \alpha & 3 - \alpha & 3 - \alpha \end{bmatrix}.$

### 4.2. Solution results and analysis

The proposed model of this paper has been solved for $\alpha = 0, 0.5, 1$ which represent very low, moderate and very high possibilities. The optimal values of the objective functions for each value of $\alpha$, are given in Table 2. Results of the compromise programming (CP) model for $\gamma = 0.5$ ($i=1,2$), and $p=1$ are represented in Table 3. Table 4 summarizes the solution obtained for different values of $\alpha$ in terms of assigned DCs to plants, assigned customers to DCs, plant load and DC load ratios. Plant and DC load ratios are also depicted in Figures 2 and 3, respectively.

### Table 2

Optimal values of objective functions when solved individually

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>68459</td>
<td>67988.50</td>
<td>67618</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>9019</td>
<td>7477.50</td>
<td>6058</td>
</tr>
</tbody>
</table>
### Table 3
Results of the compromise programming (CP) model

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\alpha$</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_p$</td>
<td>0.063</td>
<td>0.062</td>
<td>0.060</td>
<td></td>
</tr>
<tr>
<td>$z_1$</td>
<td>77101</td>
<td>76415</td>
<td>75773</td>
<td></td>
</tr>
<tr>
<td>$z_2$</td>
<td>9019</td>
<td>7477.5</td>
<td>6058</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4
Summary of the solution obtained of the CP model for different values of $\alpha$.

<table>
<thead>
<tr>
<th>$p=1$</th>
<th>$\alpha$</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assigned customers</td>
<td>Load ratio</td>
<td>Assigned customers</td>
<td>Load ratio</td>
<td>Assigned customers</td>
</tr>
<tr>
<td>DC 1 load ratio</td>
<td>4,5,6,10</td>
<td>1.00</td>
<td>4,5,6,10</td>
<td>1.00</td>
</tr>
<tr>
<td>DC 2 load ratio</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>DC 3 load ratio</td>
<td>2,3,5,9</td>
<td>1.00</td>
<td>2,3,5,9</td>
<td>0.98</td>
</tr>
<tr>
<td>DC 4 load ratio</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>DC 5 load ratio</td>
<td>1,7,8</td>
<td>0.78</td>
<td>1,7,8</td>
<td>0.81</td>
</tr>
<tr>
<td>DC 6 load ratio</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assigned DCs</th>
<th>Load ratio</th>
<th>Assigned DCs</th>
<th>Load ratio</th>
<th>Assigned DCs</th>
<th>Load ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1 load ratio</td>
<td>3,5</td>
<td>0.53</td>
<td>3</td>
<td>0.51</td>
<td>3</td>
</tr>
<tr>
<td>Plant 2 load ratio</td>
<td>1,5</td>
<td>1.00</td>
<td>1,5</td>
<td>0.98</td>
<td>1,5</td>
</tr>
</tbody>
</table>

*Load ratio = amount of product shipped from DC(plant)/capacity of the DC(plant)*

From Figure 2 it is seen that while load ratio for plant 1 has little fluctuations, the load ratio for plant 2 is consistently dropping. This implies that under different possibilities the amount shipped from each plant to each DC will vary. Similarly, Figure 3 shows that while DC 1 is always fully loaded, the load ratio for DC 3 when possibility increases from $\alpha=0$ to $\alpha=0.5$ decreases, and simultaneously this value for DC 5 increases. Also, note that while for $\alpha=0,0.5$ all of the demand of customer 5 can be satisfied by DCs 1 and 3, for $\alpha=1$, DC 5 should also satisfy some portion of this customer’s demand.
5. Conclusion

In this paper, we have proposed a possibilistic programming approach for supply chain network design problem under fuzzy environment (FSCND). Specifically, we dealt with location and distribution decisions in a supply chain system. The proposed model of this paper has considered two different objectives in order to incorporate different risk factors associated with each location and each arc of the network such as opening a DC, connecting a plant to a DC, and a DC to a customer into the model. The fuzzy multi-objective mixed integer linear program (FMOMILP) also assumes possibility distributions for customers' demands and the capacities of DCs through fuzzy sets theory. The proposed model of this paper, in addition to minimizing the location and the transportation costs, also minimizes the total risk of location and distribution in the network through an integrated and comprehensive model. For the possibilistic programming model we have considered three levels of possibilities of very low, medium, and very high, and for each level of possibility we have analyzed the results through a numerical example.

![Fig.2. Plant load ratio](image1)

![Fig.3. DC load ratio](image2)

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References


