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Packing layout added value in sheet metal laser cutting operations considering raw material reuse

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A B S T R A C T

We approach an open dimension problem, in specific, a two-dimensional strip packing problem variation found in sheet metal laser cutting, where rectangular items must be cut from a metal sheet, aiming to increase the packing layout added value. Therefore, this research objective is to analyze the packing layout added value with raw material reuse and practical constraints found in real-life laser cutting operations. The Best Fit Decreasing Height heuristic was modified to reuse raw material and calculate the packing layout added value, being compared with three construction heuristics using a set of literature and generated instances. We show the modified best fit decreasing height heuristic obtained better results when compared to the selected heuristics, with a high sheet metal utilization by the original instance rectangles and efficient raw material reuse. Thus, for sheet metal laser cutting practical operations, the modified best fit decreasing height heuristic is suitable for generating good packing layouts, resulting in industrial benefits including cost savings, increased productivity, greater competitiveness, and sustainability. Approaching raw material reuse increased the packing layout added value in most solutions found, and should be considered in real-life laser cutting operations. However, prioritizing only raw material reuse is not ideal, since a high number of additional rectangles can cause manufacturing wastes including overproduction, stock, and extra processing.

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1. Introduction

Cutting and packing (C&P) problems are found in many managerial and industrial optimization issues, developing solutions to pack/cut a set of items packed/cut in one or more objects, minimizing an objective function respecting specific problem constraints (Côté & Iori, 2018). As optimization problems, most C&P problems are NP-Hard (Iori et al., 2021) and can be solved by exact approaches (Martello et al., 2003), hybrid methods (Hifi and Ouafi, 1998), and approximation methods (heuristics and metaheuristics) (Sweeney and Paternoster, 1992; Oliveira et al., 2016).

In manufacturing industries, cutting/packing layouts can enhance the industries' competitiveness, promoting leaner and more agile processes (Chen et al., 2014). Usually, the main C&P problem objective is to minimize the raw material waste, being of particular interest to mass-production industries due to financial impacts (Coffman & Shor, 1990). Developing an algorithm to solve an industrial C&P problem involves understanding the complexity of items' geometry and the constraints (Wang et al., 2022). Also, the algorithm should be adaptable to considering real-life practical scenarios found in different industries including, wood, glass, paper, steel, furniture, and textile (Hopper & Turton, 1999; Bennel and Oliveira, 2008; Chernov et al., 2010; Alvarez-Valdes et al., 2013; Júnior et al., 2022; Bertolini et al., 2024).

Considering steel industry practical applications, sheet metal laser cutting is highlighted, where several rectangles must be cut aiming to reduce the total area used and improve the packing layout added value, where the laser beam is focused and directed onto the steel material, allowing for controlled and accurate cutting, offering high speed and versatility to cut complex shapes

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(Adalarasan et al., 2015; Elsheikh et al., 2021; Madhukar et al., 2016). Improving packing layout added value in sheet metal laser cutting decreases manufacturing costs by reducing wastes including overproduction, inventory, and extra-processing. Efficient sheet metal cutting pattern arrangement ensures only necessary raw material is used (Eswaramoorthi et al., 2011). Consequently, better inventory management is obtained, with less need for excessive raw material stockpiling (Vinodh et al., 2012). Also, due to the precise cutting patterns found, the necessity for extra processing steps is lowered, resulting in reduced labor and material costs associated with reworking errors and defects (Buer et al., 2018). Figure 1 shows the packing layout added value concept.

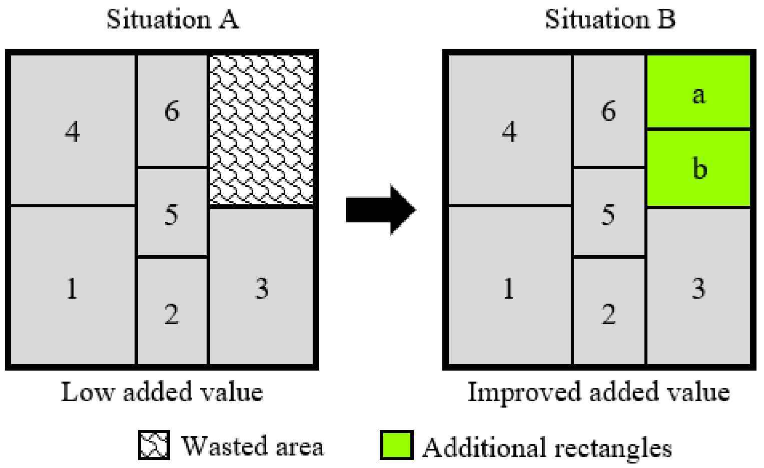

Fig. 1. Packing layout added value concept

A packing layout has a higher added value when rectangles are effectively packed in the sheet metal, adhering to cutting process constraints and increasing raw material utilization. For Scenario A, raw material reuse was not considered. Thus, the empty spaces found are wasted areas, decreasing raw material utilization and returning a low packing layout added value. In Situation B, an initial packing layout was found and, with the raw material reusing possibility, justified by packing layout empty spaces and market demand. Two additional rectangles (rectangles "*a*" and "*b*") were included, resulting in a packing layout without empty spaces, improving added value. Generally, metal unused during the cutting process is sold as scrap to foundry companies for recycling. However, within the circular economy framework, metal can not only be recycled but also directly reused, extending its useful life (Gaustad et al., 2018). Incorporating additional rectangles into empty spaces that would otherwise be discarded as scrap can be a solution to reuse raw material. Also, in mass production settings, additional rectangles increase raw material utilization by minimizing waste, decreasing the number of sheet metals required, lowering the demand for new material production, and subsequently reducing the environmental impact associated with excessive resource consumption (Nascimento et al., 2018). The objective of this article is to analyze packing layout added value in sheet metal laser cutting considering raw material reuse, classified according to Wäscher et al. (2007) as a rectangular two-dimensional strip packing problem (2D-SPP). Therefore, a modified Best-Fit Decreasing Height heuristic (M-BFDH) algorithm was proposed to approach packing layouts' added value and determine patterns and characteristics found in higher-added value packing layouts.

In recent years, the 2D-SPP has been analyzed approaching different practical problems. Wei et al. (2019) address the 2D-SPP with unloading constraints, where rectangles belong to different customers, aiming to minimize strip length while ensuring feasible unloading. An open space-based first-fit heuristic and a randomized local search for optimization are proposed. Liu et al. (2023) address the 2D-SPP in industrial settings, considering demand uncertainty and variable-sized strips, accounting for the bullwhip effect and the need for diverse strip dimensions for varying customers. A robust optimization model using a box uncertainty set and column generation to enhance solution accuracy is presented. Vasilyev et al. (2023) introduce a 2D-SPP extension, where rectangles' dimensions depend on the strip the rectangles are packed, proposing integer programming formulations and heuristic-based algorithms (skyline best-fit and randomized local search) to solve the problem efficiently. For 2D-SPP applications related to the steel industry, Chen et al. (2014) presented a rectangular layer-packing algorithm combined with a modified genetic algorithm and particle swarm optimization to solve the constrained 2D-SPP, with constraints specifically related to rectangles' dimensions derived from a steel plant. Xu and Yang (2022) focused on optimizing steel plate cutting using a roll-fed disc shearing process, developing a multi-objective mixed-integer nonlinear programming model and a genetic algorithm. Neuenfeldt et al. (2023) explored 2D-SPP heuristics and linear programming methods to reduce waste in sheet metal cutting, including cutting time, quality, and movement efficiency. Similarly, Francescatto et al. (2023) adapted a 2D-SPP mathematical model to analyze how plasma cutting constraints affect sheet metal waste in the steel industry. Na and Yang (2023) propose a deep learning-based pairing method to reduce nesting complexity in shipbuilding, improving sheet metal utilization, reducing pairing time, arrangement time, and scrap rate by using pairwise clustering and a neural network classifier. Yao et al. (2025) address the 2D-SPP to minimize strip height while accounting for defects generated from oxidized skin and cracks found in metal sheets, proposing an integer programming formulation, an exact two-stage approach using Benders' decomposition, and a skyline-based heuristic to enhance solution quality.

A gap was identified regarding research approaching packing layout added value in sheet metal laser cutting operations. Therefore, the article's contributions are:

- Presents a rectangular 2D-SPP variation approaching packing layouts added value in sheet metal laser cutting process, discussing the relation between added value, rectangles positioning, and raw material reuse, to understand the impact from different packing layouts with practical constraints, approximating industry practitioners and academia;
- Describes patterns and characteristics found in higher added value packing layouts, generating insights used by industry practitioners to improve laser cutting operations, promoting cost reduction and financial gains from effective rectangles' packing;
- Proposes a modification to a classic constructive heuristic (M-BFDH) to find added-value packing layouts focused on a different objective than the classic 2D-SPP (strip's height minimization), returning better results related to packing layout added value, since the M-BFDH was developed considering intrinsic characteristics of the addressed problem, mainly raw material reuse and variable sheet metal dimensions.

The remainder of this paper is structured as follows. Section 2 presents a literature review related to the development and use of level heuristics, specifically the Best-Fit Decreasing Height heuristic. Section 3 details the constraints found in laser cutting operations and the problem approached. Section 4 describes the M-BFDH developed to analyze the packing layout added value. Section 5 analyzes the results obtained with the M-BFDH. Finally, the conclusion is presented in Section 6. Additionally, Appendix A introduces the basic notation used, and Appendix B shows packing layouts obtained with the M-BFDH.

2. Literature review

Over the years, heuristic approaches have been developed to address C&P problems (Oliveira et al., 2016). For instance, the bottom-left (BL) heuristic proposed by Baker et al. (1980) has found application in solving strip packing, bin packing, knapsack, and cutting stock problems. Widely recognized as a well-known and fundamental approach, the BL heuristic aims to position each rectangle as low and to the left as possible. Despite the BL heuristic limitations, mainly generating empty spaces between rectangles in the packing layout, researchers have proposed modifications to enhance the positioning logic and reduce empty spaces. The Bottom-Left-Fill heuristic, introduced by Chazelle (1983), considers empty spaces as valid positions during the packing process. Also, the Improved Bottom-Left heuristic by Liu and Teng (1999), prioritizes downward movement during the packing process to achieve better solutions. With the improvements proposed for the BL heuristic, new constructive heuristics were developed, such as the best-fit (BF) heuristic, introduced by Burke et al. (2004), which dynamically selects rectangles based on available space in the packing layout and prioritizes the rectangles that fit perfectly into the empty spaces. Imahori and Yagiura (2010) enhanced the BF heuristic by implementing balanced binary search trees to improve its performance and achieve a better time complexity. Asik and Özcan (2009) extended the BF heuristic with a bidirectional approach, considering not only the usual empty spaces but also vertical spaces for rectangles' packing. Özcan et al. (2013) proposed the Modified Bidirectional BF heuristic, introducing the rectangles packing in groups as a key difference. Additionally, to address the empty spaces resulting from imperfect fits, Leung and Zhang (2011) developed the Fast Layer-Based Heuristic, aiming to maintain a flatter skyline during the rectangle packing.

Concomitant with the BL heuristic development, Coffman et al. (1980) introduced a new heuristic logic for cutting and packing problems, denominated level constructive heuristics. This approach involved packing rectangles in the layout using different levels, where each level was determined by the height of the tallest rectangle placed within it. In the Next Fit Decreasing Height heuristic (Coffman et al., 1980), rectangles were sorted in non-increasing order of height and placed in the leftmost position possible within each level. If a level did not have enough space for the next rectangle, a new level was opened. However, the Next Fit Decreasing Height heuristic had limitations related to empty spaces, as smaller rectangles could not be packed in closed levels (Oliveira et al., 2016). To address this issue, Coffman et al. (1980) proposed the First-Fit Decreasing Height heuristic, where levels were not closed, allowing smaller rectangles to be packed in the lowest possible level. To enhance the level heuristics results, Berkey and Wang (1987) introduced the Best-Fit Decreasing Height (BFDH) heuristic. In this approach, rectangles are not placed at the lowest possible level where the rectangles can fit, but rather at the level where the unused horizontal space is reduced among all potential fitting levels. To improve the level heuristics results, Ntene (2007) developed the Size Alternating Stack heuristic, which divides rectangles into two lists, one with narrow and the other with wider rectangles. The two lists are compared, the tallest rectangle is selected to determine the level height, and rectangles from the alternating list are placed on top of each other in the level created. Ortmann et al. (2010) used ideas from Ntene and van Vuuren (2009) to develop the Modified Size-Alternating Stack Algorithm and concepts from Lodi et al. (1999) to develop the Stack Ceiling with Re-sorting algorithm. Similarly, Cui et al. (2008) proposed a concept of dividing the strip into sections, each with its level determined by the rectangle placed in the BL corner. This approach allows for the flexible placement of additional rectangles within each level, and a recursive function is proposed to optimize the level space utilization. Furthermore, in recent years, several articles have explored level heuristics. Bortfeldt and Jungmann (2012) introduced the Strip Packing by Tree Search algorithm, adapting the approach developed by Fanslau and Bortfeldt (2010), which involves dividing the strip into layers. Cui et al. (2013) built upon the work presented in Cui et al. (2008) by guiding the recursive function to utilize arrangements that have been successful in previous levels. Buchwald and Scheithauer (2016) presented an improved version of the first-fit decreasing height heuristic described in Coffman et al. (1980). Chen et al. (2015) integrated a rectangular layer-packing algorithm with either a modified genetic algorithm or particle swarm optimization algorithm to address the two-dimensional orthogonal packing problem. For the generalized bin packing problem, Baldi and Bruglieri (2017) investigated the effectiveness of using level heuristics. Wei et al. (2014) proposed a block-based layer algorithm specifically designed for the 2D-SPP. Kokten and Sel (2022) combined first-fit increasing and first-fit decreasing height techniques with simulated annealing for the cutting stock problem. Brandão and Pedroso (2014) achieved effective solutions for the one-dimensional cutting stock problem using a pattern-based heuristic incorporating best-fit decreasing and first-fit decreasing height strategies.

Compared to classic cutting and packing heuristics such as BL and BF, level heuristics have received relatively less attention, with the most recent approach being, as far as the authors know, in 2024, due to poor performance in achieving objectives such as minimizing height, reducing bin and stock number, as well as improving area utilization in classic C&P problems. However, for maximizing the packing layout added value in sheet metal cutting considering raw material reuse, level heuristics, specifically the BFDH approach, can generate good solutions. Therefore, the proposed M-BFDH addresses the packing layout added value in sheet metal laser cutting considering the possibility of reusing raw material and varying both sheet metal's dimensions, differentiating from level heuristics literature, where the focus is on only minimizing object height or the number of bins/stocks.

3. Problem description

We approached the industrial packing problem found primarily in the steel industry, where rectangles are obtained from sheet metal through a laser cutting operation. The laser cutting operation uses thermal separation, in which the material, when in contact with the laser beam, suffers a local increase in temperature until completely melting or vaporizing, generating a heat affected zone (HAZ) (Rajaram et al., 2003; Çaidas & Hasçalik, 2008; Adalarasan et al., 2015). In laser cutting operations, a minimum distance between rectangles in the packing layout, represented by α, is necessary during the cutting process due to the laser beam width. For example, consider the cut of two $100 \times 100\,mm$ adjacent rectangles, and a laser beam with a $40.0\,mm$ width, if $\alpha = 0\,mm$, $2.0\,mm$ will be removed from each rectangle, resulting in two $98 \times 100\,mm$ rectangles. During the cutting process, α must be considered to maintain the rectangles' dimensional quality, being equal to or higher than the laser beam width. Also, α prevents deformation related to the thermal propagation from the HAZ generated during the cut. Thus, if α is respected, the adjacent rectangles from the rectangle being cut are not impacted by thermal propagation and do not require a subsequent machining operation, reducing process costs. Compared with different sheet metal cutting processes (plasma, oxy-fuel, and waterjet cutting), the α required for laser cutting is lower, varying from $0.10\,mm$ to $1.0\,mm$ on average, being affected by material type, gas used, material thickness, laser power, cutting speed, gas volume, gas pressure, nozzle diameter, and nozzle standoff (Ghany & Newishy, 2005; Çaidas & Hasçalik, 2008; Adalarasan et al., 2015; Madhukar et al., 2016; Elsheikh et al., 2021). Consequently, α has an impact on the packing layout and must be considered when approaching laser cutting practical operations. Similarly, due to manufacturing, transport, and storage, damage and deformation can occur at the sheet's edges, making the area unavailable for cutting (Francescatto et al., 2023). Therefore, a minimum (safety) distance between the rectangles and the sheet's edges is considered, represented as β, being subtracted from the sheet metal's dimensions. Fig. 2 shows a packing layout respecting the constraints found in sheet metal laser cutting operations, in specific, the minimum distance between rectangles in the packing layout (α) and the minimum distance between rectangles and the sheets' edges (β).

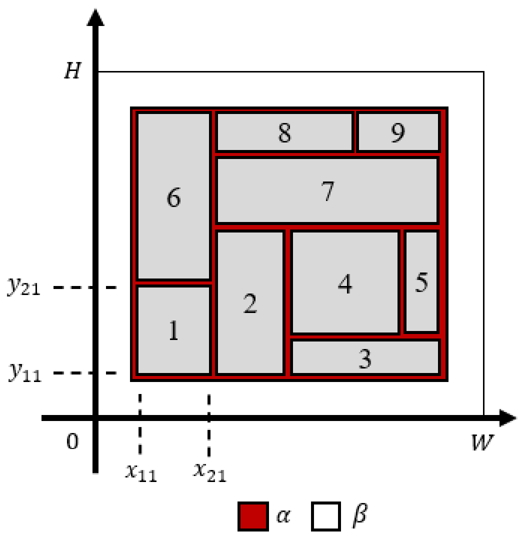

Fig. 2. Packing layout found in sheet metal laser cutting operations

Let n be the number of rectangles that must be packed in a sheet metal. A set of rectangles $I = \{1, 2, ..., n\}$ are packed without overlap to find the packing layout height (H). The sheet width (W) and the rectangles' characteristics are previously known, representing an offline packing. Each rectangle is defined by two dimensions, height (h_i) and width (w_i), where $i \in I$. The rectangles' position inside the sheet metal is defined by the coordinates x_{1i}, y_{1i}, x_{2i}, and y_{2i}. Since the objective is to maximize the packing layout added value, both sheet metal dimensions, W and H, can vary. Also, rectangles can be rotated, and no constraint is related to guillotine cutting. The possibility of reusing raw material by inserting additional rectangles in the initial packing layout empty spaces, as shown in Fig. 3, is considered. Furthermore, as a production planning constraint, the additional rectangles' dimensions must be equal to the dimensions of at least one rectangle from the instance that originated the packing layout, meaning the additional rectangles' dimensions are not randomly determined. For an empty space to be considered for reuse, at least the smallest area rectangle within the instance that originated the packing layout must fit the empty space. However, depending on the empty space dimensions, different additional rectangles' combinations can be allocated in the empty spaces.

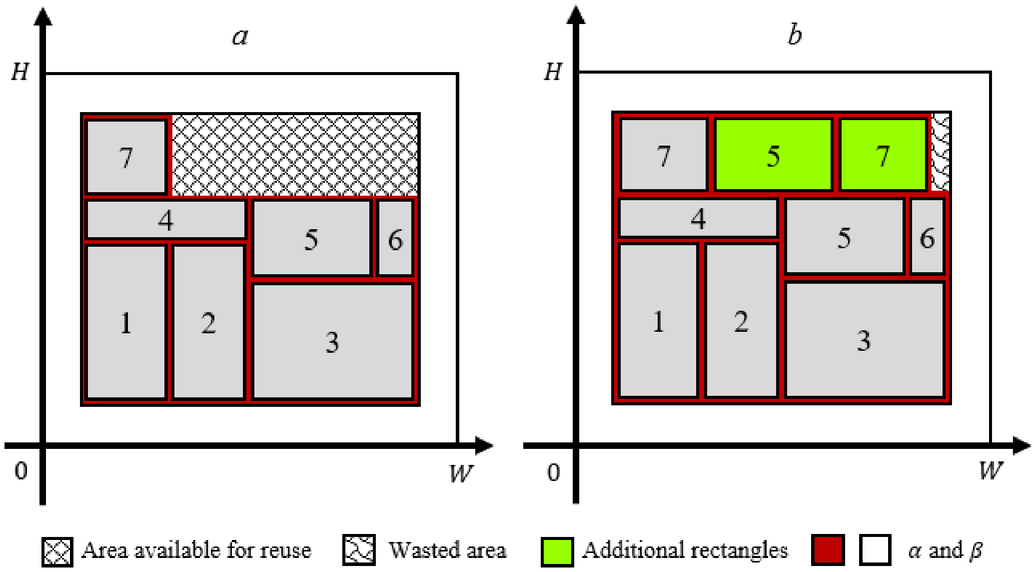

Fig. 3. Raw material reuse procedure

Fig. 3a shows a packing layout with only the instance's original rectangles, returning an area available for reuse. In Fig. 3b, rectangles 5 and 7 are considered additional rectangles utilized for raw material reuse, resulting in a new layout with increased added value. However, due to the empty space dimension, different instance rectangles could be used as additional rectangles, including a combination between rectangles 4 and 6 or rectangles 5 and 6. Also, a rectangle can be reused in the available area as many times as necessary. The added value index (ξ), representing the packing layout added value, obtained from the cutting area classification (Eq. (1)), is used as a reference to find the packing layout with the highest added value. A_1 refers to the metal sheet area used to obtain the packing layout, found by the multiplication between the calculated H and the considered W ($H \times W$). Furthermore, A_1 includes the area occupied by α and β. A_2 relates to the instance original rectangles area and A_3 represents the valid empty spaces area for reuse.

$$\xi = \frac{A_2F_1 + A_3F_2}{A_1} \tag{1}$$

The respective cutting areas are multiplied by factors to quantify the added value. Factor 1 (F_1) quantifies the instance original rectangles added value. Similarly, Factor 2 (F_2) quantifies the additional rectangles added value. F_1 and F_2 significantly depends on how each industry determines the added value. Due to the variability and intrinsic difficulty in assigning specific values for F_1 and F_2, value ranges can be considered to approach different practical scenarios, providing a sheet metal laser cutting complete analysis. F_1 and F_2 are dimensionless variables following a proportional distribution, where $F_1 + F_2 = 1$ and $F_2 < F_1$. Due to market and industry demands, the added value for the original (F_1) and additional (F_2) rectangles are different (Cherri et al., 2013; Gracia et al., 2013; Kos and Duhovnik, 2002). Additional rectangles are placed after the packing layout formation, not considered in the initial production planning and control, being used as a possible solution to reduce raw material waste. However, additional rectangles can originate overproduction, indirectly leading to operational wastes including inventory, extra-processing, and transportation (Buer et al., 2018; Eswaramoorthi et al., 2011; Jasti and Kodali, 2014). To reduce overproduction wastes, finding companies with a demand for the additional rectangles is an option, profiting from the raw material that would be discarded. Also, stocking the raw material in which additional rectangles could be cut to wait for an appropriate demand is impractical and inefficient, since smaller raw materials cannot be utilized in cutting operations due to restrictions imposed by laser machines, which are designed to handle larger pieces of material and struggle with positioning smaller ones. Furthermore, the original rectangles are already considered in production planning and control, being essential to reach market demands, resulting in less operational waste. Consequently, F_2 must be lower than F_1. To better understand the relation between packing layout added value and original and additional rectangles packing, two variables were created. The original rectangles sheet metal utilization percentage (ϑ) represents the sheet metal percentage used by the instance's original rectangles. Higher ϑ values relate to fewer empty spaces in the packing layout. Furthermore, the additional rectangles sheet metal utilization percentage (μ) represents the sheet metal percentage used by the additional rectangles. An increase in μ indicates more packing layouts empty spaces filled with additional rectangles. From the added value inherent complexity, it is not possible to assign a specific numerical value. Thus, ξ is a percentage-based value, depending on F_1 and F_2. A high ξ percentage represents a high added value packing layout. For example, for $F_1 = 0.9$, resulting in $F_2 = 0.1$, the maximum added value possible is $\xi = 90.0\%$, representing a packing layout with only original rectangles ($A_1 = A_2$ and $\vartheta = 100.0\%$) and without empty spaces ($A_3 = 0$ and $\mu = 0.0\%$). Hypothetically, if possible, the worst added value possible is $\xi = 10.00\%$, returning a packing layout with only additional rectangles ($A_1 = A_3$ and $\mu = 100.0\%$) and without original rectangles ($A_2 = 0$ and $\vartheta = 0.0\%$). However, a packing layout with $A_2 = 0$ is unfeasible, since the additional rectangles are dependent on empty spaces formed from the original rectangles packing. Finally, additional constraints are considered during the packing and raw material reuse: (i) all rectangles in an instance must be packed before the additional rectangles; (ii) the original rectangles and the additional rectangles must be packed inside the sheet metal dimensions; (iii) a packing layout does not need additional rectangles to be valid; and (iv) additional rectangles overlap is not allowed.

4. M-BFDH heuristic

This section describes the modified BFDH proposed to maximize the packing layout added value (ξ) considering sheet metal laser cutting operations. Section 4.1 explains the M-BFDH constructive heuristic procedure related to obtaining the initial packing layout and placing the additional rectangles. Section 4.2 details the two improvement rules used to find better added-value packing layouts.

4.1 M-BFDH constructive heuristic

From the BFDH potential to generate good solutions related to the packing layout added value, a modified version is proposed. The M-BFDH addresses the original BFDH limitations, particularly reducing empty spaces and the overall area used. Also, the M-BFDH constructive heuristic objective, separated into two phases (Section 4.1 and Section 4.2), is to find a ξ that represents the packing layout with the highest added value. Therefore, the sheet metal is divided in layers, and the rectangles are packed following a decreasing dimension rule, due to rectangle rotation, in an unlimited number of levels (Algorithm 1a).

Algorithm 1a. M-BFDH heuristic constructive phase – Initial layout

Input: A list of n rectangles to be packed, the rectangles' dimensions, and W.
Output: A feasible packing of rectangles.

```
Sort the rectangles in descending perimeter order;
if the first rectangle (n = 0) then
  position the rectangle in the sheet metal bottom-left corner;
if n > 0 then
  create an envelope based on the first rectangle position and W;
  for each remaining rectangle do
    reorganize the rectangles list following the scoring system considering the envelope dimensions;
      if the rectangle fits the envelope created then
          pack the rectangle;
        create a new envelope based on the rectangle position and W;
      create a level based on the height difference between the placed rectangle and the previous rectangle;
          store the created level dimensions;
      if no rectangle fits the envelope created then
        reorganize the levels created from last to first;
          for every level created do
          reorganize the rectangles list following a scoring system considering the level dimensions;
            if the rectangle fits the level then
              pack the rectangle;
                create a new envelope based on the rectangle position and the remaining level dimensions;
                exclude the level used from storage;
                break loop;
            if no rectangle fits the level then
              create an envelope based on the next-level dimensions;
                exclude the level used from storage;
              break loop;
            if there are no more levels stored and there are still rectangles to be packed then
              reorganize the remaining rectangles list by decreasing height;
              pack the rectangle in the leftmost corner possible;
              create an envelope based on the rectangles' position and W;
                break loop;
if all rectangles are packed then
  report rectangles' coordinates;
  report H;
```

To initiate the constructive process, the list of rectangles is organized following a decreasing perimeter order. At each iteration, only one rectangle is packed. The first rectangle packed has the largest perimeter. After the rectangle selection, the remaining rectangles are packed following a bottom-left logic, being positioned in the lowest and leftmost positions available, respecting the minimum distance constraints α and β. With each rectangle packing, a new "envelope" is created, considering the rectangle dimensions, the remaining W, α, and β. The envelope is a rectangular empty area starting at the last packed rectangle coordinates and ending at W. After creating the envelope, a score system is used to define the next rectangle packed. The scoring system considers the envelope height, compared with the remaining rectangles' height or width (rotation possibility), added α. The best possible score is equal to 0, where the compared rectangle has the same height or width as the envelope height. If the compared rectangle dimension is greater than the envelope height, a negative score is obtained, and the rectangle cannot be packed. However, if the score obtained compared with the remaining rectangle dimension is positive, the rectangle can be packed. If the compared rectangle dimension is smaller than the envelope's height, the score obtained is the difference between the envelope's height and the compared rectangle's dimension. The rectangles list is organized following an increasing score order (excluding negative scores), and, in case of a tie, the rectangle with the largest perimeter is selected. Both rectangle dimensions are compared with the envelope height, allowing rotation based on the dimension with the best score. Fig. 4 shows the envelope and level creation process, as well as the score system. When the rectangle score is higher than zero, a level is created considering the height difference between the rectangle and the envelope. The levels are used when no rectangle fits the envelope created, being organized from last to first. Consequently, the last level created is the first level on the level list.

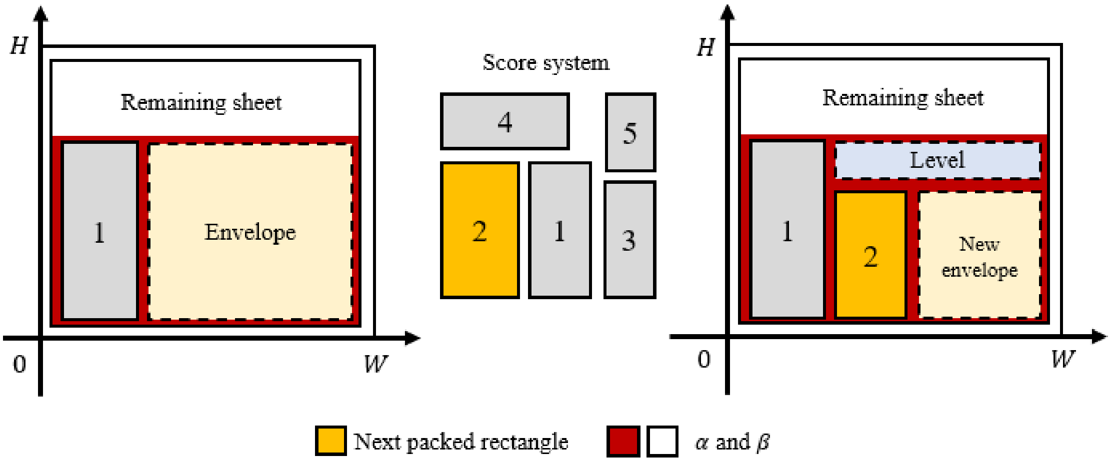

Fig. 4. Envelope and level creation process

If no rectangle fits the envelope created, three situations are verified: (i) the rectangle does not fit the envelope created but can be packed in the first level stored; (ii) the rectangle does not fit the envelope or the first level stored but can be packed in any of the remaining levels; and (iii) the rectangle does not fit either the envelope or any of the levels stored. In Fig. 5, for the packing layout shown in Situation A, the rectangles cannot be packed in the envelope created. Thus, the rectangles are compared with the last level stored, using the score system. If no rectangle can be packed in the last level stored (Situation B), the rectangles are compared with the remaining stored levels, following the scoring system. When a rectangle is packed in any level, a new envelope is created, starting the subsequent rectangle packing process. In Situation C, no rectangle can be packed in the envelope created or any level stored. Therefore, the rectangles list is reorganized based on the highest rectangle dimension, and the first rectangle from the list is packed in the leftmost location possible, creating a new layer and restarting the packing process for the remaining rectangles.

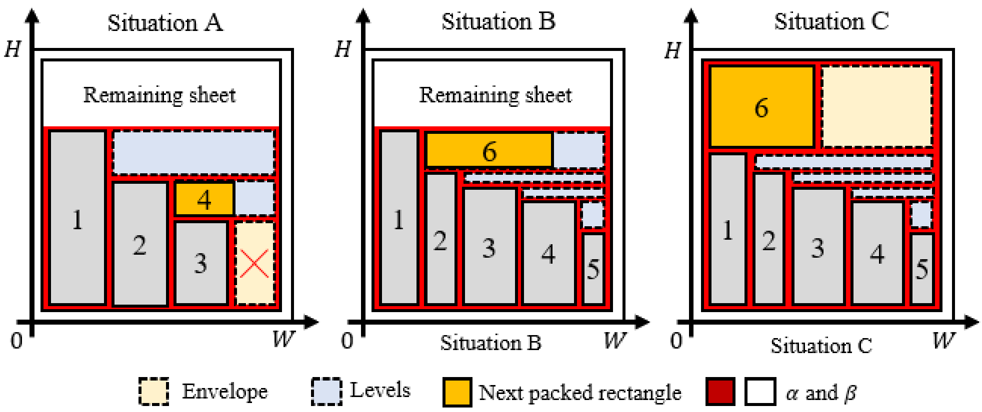

Fig. 5. Rectangle packing process

The process repeats until all rectangles are packed. After, the H and rectangles coordinates are reported, finishing the original rectangles packing process, resulting in the instance initial packing layout. Also, to improve the packing layout added value, raw materials can be reused. Therefore, the M-BFDH heuristic second phase begins (Algorithm 1b), placing additional rectangles in the packing layout empty spaces.

Algorithm 1b. M-BFDH heuristic constructive phase – Additional rectangles' packing

Input: Rectangles' coordinates and H.
Output: A feasible packing layout, with raw material reuse.

generate a feasible packing layout;
for all rectangles **do**
  compare the rectangles' position with the right adjacent rectangles;
  if the adjacent rectangles have the same height as the rectangle being compared **then**
    empty space is invalid;
  else
    create and store an empty space;
for each empty space **do**
  determine if the empty space is valid;
  determine the empty space area;
  determine the empty space additional rectangles characteristics;
  determine the number of additional rectangles that fit the empty space;
  determine the coordinates for each rectangle;
place the additional rectangles in the packing layout;
return resulting ξ;

Using the rectangles' coordinates from the packing layout obtained, the empty spaces are determined. Each rectangle is compared with their right adjacent rectangles, due to the packing logic used, which prevents empty spaces on the sheet metal left side. In addition, when comparing the rectangles to find empty spaces, two situations can occur, as shown in Fig. 6.

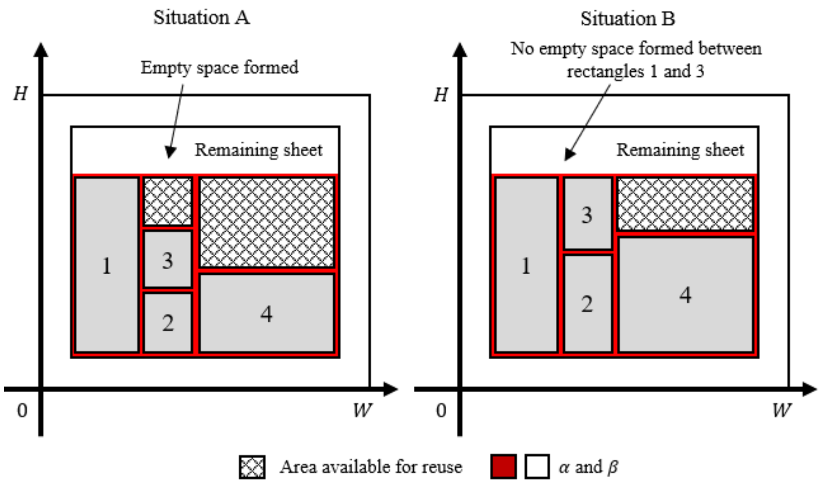

Fig. 6. Example of empty spaces

In Situation A, the adjacent rectangles (2 and 3) combined do not have the same height as rectangle 1, generating an empty space. However, for Situation B, the adjacent rectangles combined have the same height as rectangle 1, not originating an empty space. The empty spaces obtained are characterized as shown in Situation A, prioritizing rectangular empty spaces, since a better raw material reuse was found. Each empty space validity is assessed by determining if the empty space can accommodate the instances' smallest area rectangle. The valid empty spaces area is stored to be used in the ξ calculation. Subsequently, the additional rectangles' characteristics to be placed within the empty space are defined. The characteristics may involve simply placing the smallest area rectangle in the empty space multiple times, or combining rectangles of varying dimensions. From the characteristics defined, the number of rectangles that can fit in the empty space, along with their respective coordinates, is determined. Furthermore, the additional rectangles can be rotated to maximize the empty space utilization. Finally, the added value index (ξ) is calculated following Eq. (1).

4.2 Solution improvements rules: First rectangle variation and W variation

Improvement rules are applied to find packing layouts with increased added value, modifying the M-BFDH constructive phase. Therefore, for each improvement rule modification, a new constructive solution is generated, following Algorithm 1a (from line 2) and Algorithm 1b. The first improvement rule, called "first rectangle variation", is related to changing the packing order for the first rectangle. Initially, the first packed rectangle has the highest perimeter. However, beginning the packing with different rectangles returns new packing layouts. Consequently, packing layouts are generated considering each rectangle in the first constructive iteration, with and without rotation. For the second improvement rule, called "W variation", variations in W are considered, separated into 5% intervals, with the maximum variation being equivalent to 50% of W, preventing the metal sheet from becoming excessively narrow, leading to impractical packing layouts (Francescatto et al., 2023). The first packing layout is obtained using the original W. The subsequent packing uses $W = 0.95W$, decreasing until the last packing, where $W = 0.50W$. Both improvement rules are applied together. For each change in the initial rectangle packing order, the W variations are applied. Thus, considering an instance with 100 rectangles, 2,000 packing layouts will be obtained, equivalent to the first and second improvement rules combined. Finally, ξ is calculated for each packing layout and used as a reference to find the highest added value.

5. Computational experiments

Section 5 presents the results found with the M-BFDH application, divided into seven subsections. Section 5.1 describes the laser cutting and sheet metal parameters' characterization. Section 5.2 details the instances considered. Section 5.3 introduces the heuristics used for comparison purposes. Section 5.4 presents the results found for the added value index (ξ). Section 5.5 explores the raw material reuse impact on packing layout added value. Section 5.6 analyzes the improvement rules proposed. Finally, in Section 5.7 practical implications from the results are described.

5.1 Parameters' characterization

Laser-cutting machine parameters directly impact the cutting quality, influencing thermal effect, dimensional accuracy, efficiency, energy consumption, and equipment lifespan (Elsheikh et al., 2021; Madhukar et al., 2016; Adalarasan et al., 2015). The parameters considered for the laser cutting operation analyzed were: 4.0 mm thick stainless steel sheet metal, 3000 mm/min cutting speed, 1,500 $watts$ of cutting power, oxygen gas, 4.0 bar gas pressure, 7.0 m^3/h gas volume, 0.45 mm cutting gun nozzle diameter, and 0.55 mm nozzle standoff (Berkmanns & Faerber, 2008). Thus, from the defined parameters, an approximate minimum distance between the rectangles equivalent to $\alpha = 0.5\ mm$ is required (Berkmanns & Faerber, 2008). The minimum distance from the sheet metal edges (β) relates to the particularity of each industry which uses laser cutting, varying according to manufacturing process characteristics, equipment used, operators' qualification, raw material quality, as well as sheet metal transportation and storage inside the factory (Francescatto et al., 2023). Thus, a value of $\beta = 1.0\ mm$ was proposed, based on the considered laser cutting machines working area, $3000\ x\ 1500\ mm$ (Trumpf, 2023; Bystronic, 2023; Amada, 2023).

5.2 Instances selection

Literature instances were selected to represent practical characteristics found in sheet metal laser cutting. However, instances with $W < 150\ mm$ returned unfeasible packing layouts due to the parameters' characterization, as well as α and β values. For instances with $n < 15$, there were not enough rectangles to verify the positioning logic impact proposed in Section 4. Also, since the dimensions in each instance were considered in cm, for instances with the smallest rectangle area equal to $1\ cm^2$, the packing layout empty spaces found were completely filled, resulting in similar values for packing layout added value (ξ). Only instances with $W \geq 150\ mm$, $n \geq 15$, with an area greater than $1\ cm^2$ for the smallest rectangle, and H and W dimensions respecting the $3000\ \times 1500\ mm$ proportion were considered. From the literature, 50 instances were selected, classified as *beng* (Bengtsson, 1982), *bwmv* (Berkey and Wang, 1987), *C* (Hopper & Turton, 2001), *cgcut* (Christofides and Whitlock, 1977), *N* (Hopper, 2000), *ngcut* (Beasley, 1985), *nice*, and *path* (Wang and Valenzela, 2001) classes. Considering only the literature, a limitation regarding the instances' basic characteristics variation, including W, n, and smallest rectangle area minimum value was verified. Fifty new instances were generated using the 2DCPackGen (Silva et al., 2014), varying from $n = 22$ to $n = 121$, $W = 1100\ mm$ to $W = 1500\ mm$, and $15\ cm^2$ to $66\ cm^2$ (smallest area rectangle), organized in eight classes, resulting in a uniform distribution of the characteristics not approached by the literature instances. The 50 new instances classes are differentiated based on the values of n, W, and the smallest area rectangle. The classification "*small and square, short and tall, long and narrow, or big and square*" was used to obtain the rectangles' dimension, with the distribution curve characterized by the function $f(x, 2, 5)$. Therefore, the rectangles' size is randomized, and the distribution of the remaining parameters trends to the smallest values to respect the maximum sheet metal dimension. Table 1 shows the selected instances. Also, the 50 new instances are available at https://github.com/MatheusFrancescatto/ODP-instances.git, and the 50 literature instances are available at https://www.euro-online.org/websites/esicup/data-sets.

Table 1
Instances selected

Literature instances		New instances	
Class	Number of instances	Class	Instances number
beng	1	*ODP1*	6
bwmv	13	*ODP2*	6
C	13	*ODP3*	6
cgcut	1	*ODP4*	6
N	9	*ODP5*	6
ngcut	3	*ODP6*	6
nice	5	*ODP7*	7
path	5	*ODP8*	7
Total	50		50

5.3 Heuristics for comparison

Three constructive heuristics from the literature were used to compare the results found, BL (Baker et al. 1980), BF (Burke et al. 2004), and BFDH (Berkey and Wang, 1987). The BL and BF heuristics were selected for being widely used approaches for solving C&P problems, focusing on reducing the packing layout H. In addition, the BFDH was selected for being the level heuristic modified. However, unlike the M-BFDH heuristic, the BFDH does not focus on improving packing layout added value. Also, BL and BF are combined with the Tabu Search (TS) described in Neuenfeldt et al. (2023), since the combination of BL and BF with TS returned better solutions. The heuristics used for comparison are justified since the BL, BF, and BFDH constructive heuristics are widely utilized in industrial settings, showing simplicity, computational efficiency, and ability to produce satisfactory results in practical timeframes. In addition, through the years, the heuristics serve as standard benchmarks for evaluating new approaches related to C&P. By comparing novel methods with the BL, BF, and BFDH, new or modified heuristic approaches can be validated, contributing to C&P research advancement. Furthermore, the W variation was also considered for BL and BF. Consequently, the W value which returned the best added value solution in the constructive phase was fixed for the TS iterations. For the BFDH, the same improvements employed in the M-BFDH heuristic were considered. Lastly, the original BL, BF, and BFDH constructive heuristics, considering the minimum distance constraints α and β, varying W, and rectangle rotation, focusing on reducing sheet metal H without raw material reuse, were also used for comparison. Table 2 summarizes the heuristics used for comparison. To simplify the results representation, the original heuristics BL, BF, and BFDH are described as O-BL, O-BF, and O-BFDH, respectively. Similarly, the BL, BF, and BFDH heuristics focusing on maximizing packing layout added value (ξ), are represented by A-BL, A-BF, and A-BFDH.

Table 2
Heuristic adopted for comparison.

Heuristic	Improvements	Objective	Raw material reuse	Representation
BL	W variation and TS	Maximize ξ	Allowed	A-BL
		Minimize H	Not allowed	O-BL
BF	W variation and TS	Maximize ξ	Allowed	A-BF
		Minimize H	Not allowed	O-BF
BFDH	First rectangle variation and W variation (Section 4.2)	Maximize ξ	Allowed	A-BFDH
		Minimize H	Not allowed	O-BFDH
M-BFDH (Section 4.1)	First rectangle variation and W variation (Section 4.2)	Maximize ξ	Allowed	M-BFDH

5.4 Results analysis for the added value index (ξ)

As seen in Section 3, ξ is a reference value to compare packing layouts to find which has the highest added value, being defined with F_1 and F_2, which quantifies the cutting areas added-value. Solutions with a high ξ are related to a higher packing layout added value. Therefore, two variations in F_1 and F_2 were considered to approach different practical scenarios in sheet metal laser cutting: "Variation A" ($F_1 = 0.9$ and $F_2 = 0.1$) and "Variation B" ($F_1 = 0.6$ and $F_2 = 0.4$). "Variation A" represents an industrial scenario where F_1 is much higher compared to F_2, since the production planning was developed to cut the original rectangles. Also, for "Variation A", the additional rectangles are not easily commercialized, increasing operational process wastes, consequently lowering F_2 value. "Variation B" represents an uncommon industrial scenario where the additional and original rectangles' added-value are similar, since both rectangle types can be efficiently commercialized, represented by close F_1 and F_2 values.

Table 3
Average ξ for each instance class considering "Variation A".

Class	Average added value index ξ (in %)						
	A-BL	**O-BL**	**A-BF**	**O-BF**	**A-BFDH**	**O-BFDH**	**M-BFDH**
beng	71.3	72.2	73.2	72.2	73.1	70.3	**74.6**
bwmv	78.0	76.7	75.2	73.4	79.5	77.8	**80.6**
C	77.6	76.9	76.3	74.9	75.4	70.2	**79.4**
cgcut	72.5	71.7	73.5	74.5	75.6	70.8	**78.9**
N	80.0	78.8	75.8	74.2	73.6	68.7	**81.3**
ngcut	**72.7**	67.9	69.9	65.6	70.1	58.9	72.0
nice	77.3	77.1	74.5	68.6	80.2	75.7	**82.0**
path	81.7	81.3	80.7	77.5	73.2	65.0	**84.1**
ODP1	83.5	82.6	80.6	77.0	83.8	82.8	**84.3**
ODP2	82.8	81.9	79.6	76.5	83.9	82.7	**84.3**
ODP3	82.4	80.5	80.5	78.4	82.9	81.7	**83.7**
ODP4	81.1	80.8	80.4	78.1	82.4	80.7	**83.6**
ODP5	82.4	81.1	80.8	79.3	82.1	80.4	**83.7**
ODP6	81.1	79.5	79.7	78.0	81.9	80.1	**83.3**
ODP7	81.5	80.3	79.5	77.6	81.9	80.0	**83.3**
ODP8	80.1	78.2	78.4	74.9	80.7	77.9	**82.9**
Average ξ	79.7	78.7	77.8	75.4	79.2	76.0	**81.9**
t	600.0	600.0	600.0	600.0	104.4	103.9	97.1

The best solutions for each instance class were highlighted.
High ξ solutions have a packing layout with higher added value.
Maximum $\xi = 90.0\%$.
t: average time to find the solution (in seconds).

Table 3 and Table 4 show the average added value index (ξ) for all instance classes considering "Variation A" and "Variation B", respectively. Considering F_1 and F_2 for both variations, the maximum possible ξ value is 90.0% for "Variation A" and 60.0% for "Variation B", representing a packing layout without empty spaces ($A_1 = A_2$), apart from α and β, excluding the possibility of packing additional rectangles ($A_3 = 0$). For the TS, 10 tests of 60 seconds each were conducted. Furthermore, for O-BL and O-BF, the result for ξ was from the TS iteration which returned the smallest H. For the A-BFDH, O-BFDH, and M-BFDH heuristics, the time limit for finding a solution was 600s. The algorithms were implemented in C++ and tested on an Intel Core i3-10510U with 1.8 gigahertz CPU and 8 gigabytes of RAM with Windows 11 operating system.

Table 4
Average ξ for each instance class considering "Variation B".

Class	Average added value index ξ (in %)						
	A-BL	O-BL	A-BF	O-BF	A-BFDH	O-BFDH	M-BFDH
beng	49.4	48.2	50.9	48.2	50.5	46.9	**51.3**
bwmv	53.6	50.6	53.6	48.9	54.6	51.9	**54.9**
C	52.9	51.4	53.0	49.4	52.8	46.8	**53.8**
cgcut	45.5	48.4	52.0	49.0	52.2	47.2	**52.6**
N	53.7	52.6	52.7	49.4	52.5	48.9	**54.4**
ngcut	49.8	45.3	**50.2**	43.7	49.9	39.3	49.7
nice	52.1	51.1	52.4	46.8	54.3	50.5	**54.4**
path	55.2	54.0	55.6	51.5	53.6	43.3	**56.5**
ODP1	55.9	54.6	53.0	48.8	**56.5**	55.2	**56.5**
ODP2	55.6	54.3	52.7	49.5	**56.6**	55.1	56.5
ODP3	55.5	54.4	54.3	51.9	**56.3**	54.4	**56.3**
ODP4	55.3	53.7	53.6	51.7	55.9	53.8	**56.3**
ODP5	55.7	54.3	53.7	51.9	56.1	53.6	**56.4**
ODP6	54.9	52.9	52.9	50.9	55.6	53.4	**56.1**
ODP7	55.2	53.7	54.3	52.2	55.8	53.3	**56.3**
ODP8	54.1	51.6	54.2	48.5	55.7	52.2	**56.1**
Average ξ	54.1	52.4	53.4	49.8	54.6	50.7	**55.3**
t	600.0	600.0	600.0	600.0	104.5	104.1	97.3

The best solutions for each instance class were highlighted.
High ξ solutions have a packing layout with higher added value.
Maximum $\xi = 60.0\%$.
t: average time to find the solution (in seconds).

For both variations, the best results were found with the M-BFDH heuristic. Considering the individual instances results, for "Variation A", 90 instances showed a better packing layout added value with the M-BFDH heuristic, and, for "Variation B", M-BFDH returned the best solutions for 69 instances. Regarding common ("Variation A") and uncommon ("Variation B") industry scenarios, M-BFDH proved to be effective in maximizing the packing layout added value, especially considering "Variation A". The main factors impacting ξ are the original rectangles sheet metal utilization percentage (ϑ) and the additional rectangles sheet metal utilization percentage (μ), described in Section 3. Fig. 7 shows the relation between the average ϑ and ξ for each instance class, considering all heuristics adopted. The individual instances results for ξ, ϑ, μ, and H are presented at https://github.com/MatheusFrancescatto/ODP-instances.git. Also, ϑ has a larger impact on the solution when compared to μ, due to F_1 being always greater than F_2. In addition, the empty spaces for sheet metal reuse are obtained from the original rectangles' packing layout. Therefore, packing layouts with increased added value are found in instances with high ϑ, characterizing fewer empty spaces and high sheet metal utilization by the original rectangles.

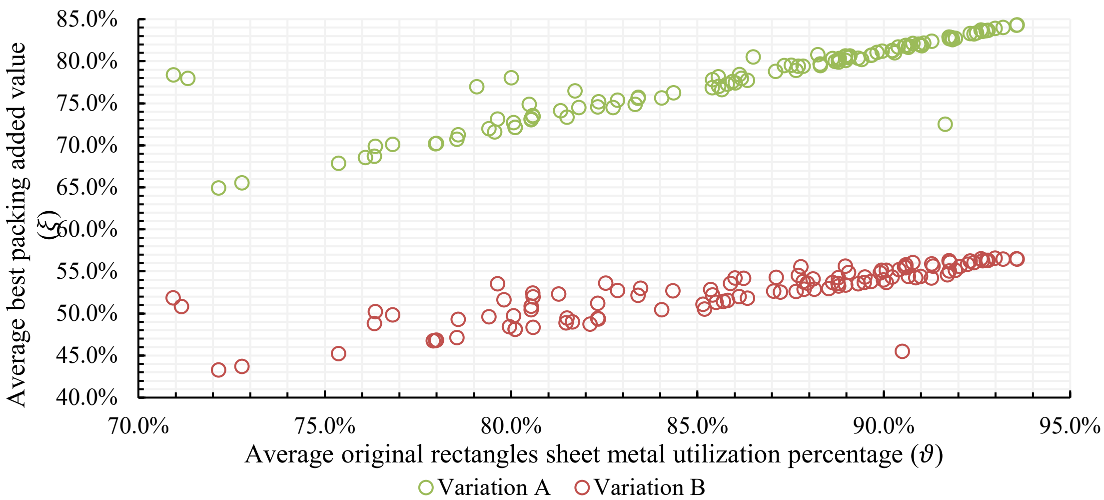

Fig. 7. Relation between the average original rectangles sheet metal utilization percentage (ϑ) and average best packing layout added value (ξ) for each instance class from Table 1, considering all heuristics

The highest ϑ values were found with the M-BFDH. M-BFDH constructive logic effectively places the instance's original rectangles in the sheet metal due to the scoring system, maintaining the level or layer "skyline" as straight as possible by selecting adequate rectangles, consequently reducing empty spaces. Furthermore, the empty spaces found are uninterrupted and allocated in the packing layout right side, facilitating additional rectangles' packing. Fig. 8 shows the *C23* packing layout for the M-BFDH, A-BF, A-BL, and A-BFDH. Also, in Appendix B, examples of packing layouts found with the M-BFDH are shown (Fig. B.1).

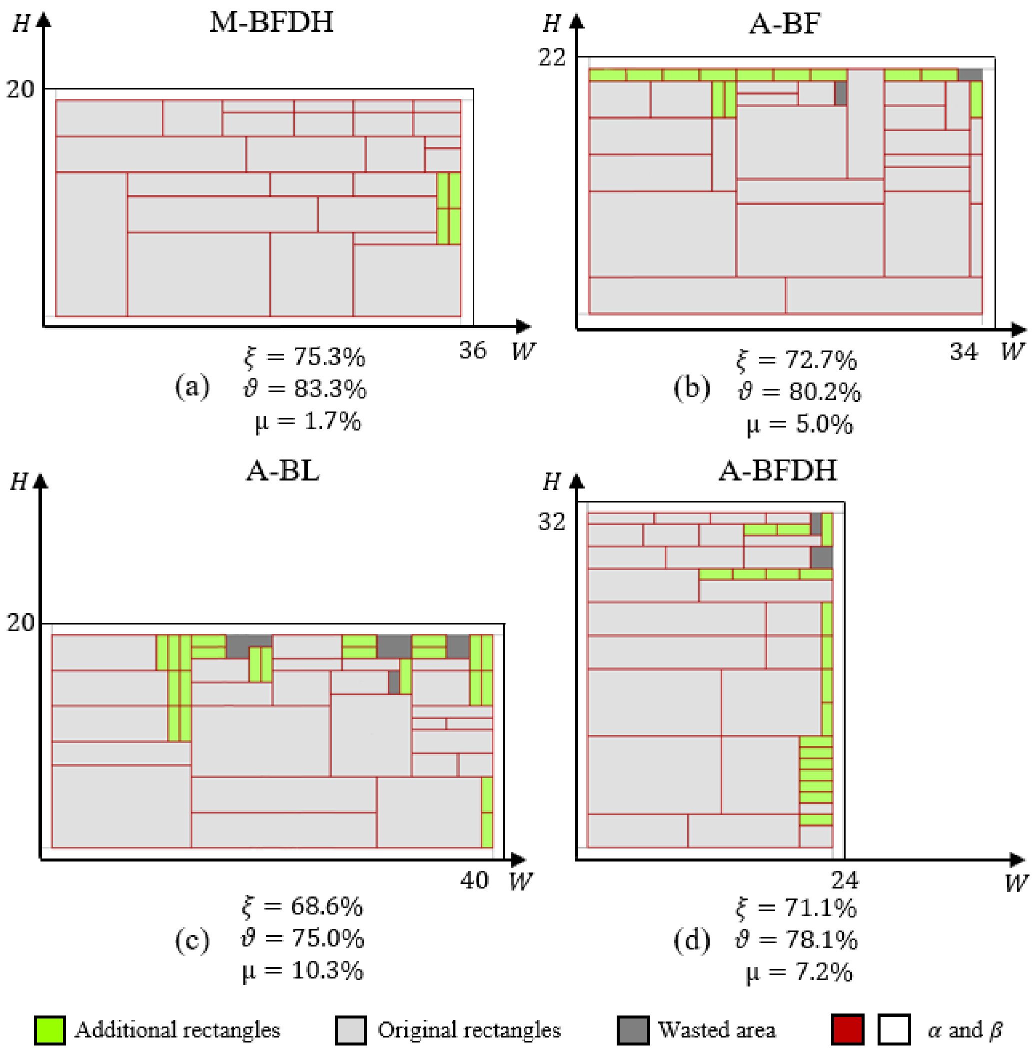

Fig. 8. *C23* packing layout for "Variation A" with the M-BFDH, A-BF, A-BL, and A-BFDH (dimensions in *cm*)

A better sheet metal utilization by the original rectangles is verified for the M-BFDH, resulting in fewer empty spaces (μ = 1.7%). Furthermore, the empty spaces are completely packed with additional rectangles. Also, although the compared heuristics have a larger area available for reuse, when compared to M-BFDH, the ξ value found is lower, indicating that the added values from the additional rectangles packed are not enough to overcome the ϑ impact. The combination of packing layouts with a high ϑ and the effective additional rectangles packing resulted in greater added value solutions for the M-BFDH.

5.5 Raw material reuse impact

The raw material available for reuse is derived from empty spaces found in the original rectangles' packing layout. Compared to the original rectangles sheet metal utilization percentage (ϑ), the additional rectangles sheet metal utilization percentage (μ) has a smaller packing layout added value impact. However, filling the empty spaces found with additional rectangles, instead of material disposal, always increases added value, being beneficial for the scenarios approached in "Variation A" and "Variation B". Figure 9 shows the relation between the average ξ and μ for each instance class. Only the results found with heuristics where raw material reuse was allowed are shown.

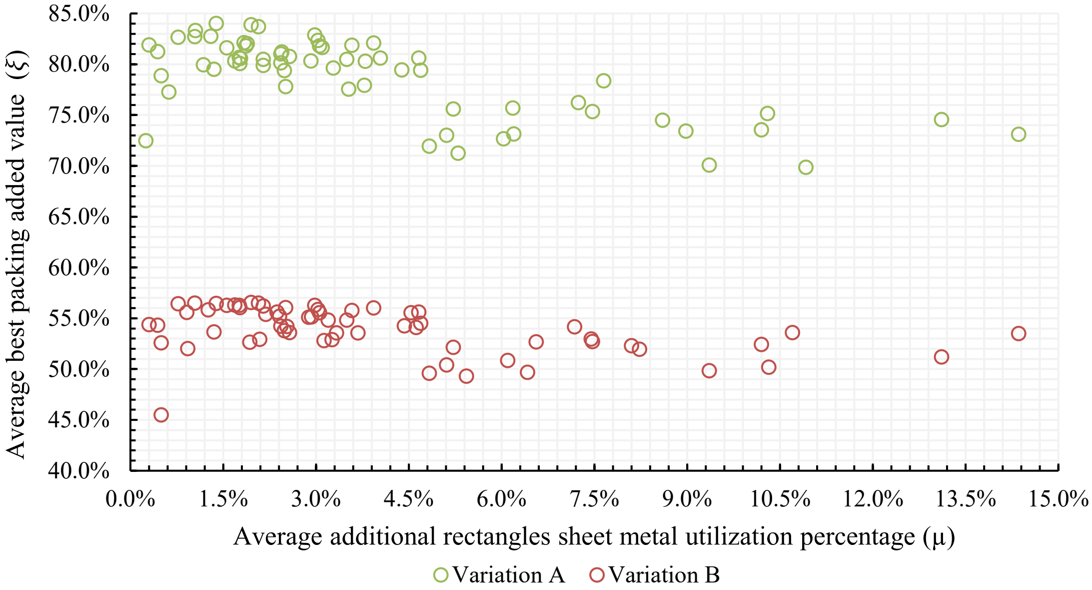

Fig. 9. Relation between the average additional rectangles sheet metal utilization percentage (μ) and average best packing layout added value (ξ) for each instance class from Table 1 for the M-BFDH, A-BL, A-BF, and A-BFDH.

For "Variation A", as raw material reuse decreases, the packing layout added value increases, consequently increasing ξ. Low μ values indicate less packing layout empty spaces and a higher sheet metal utilization by the original rectangles. Also, the increase in μ impacts negatively the solutions' quality, reflecting a packing layout with lower ϑ and decreasing added value. Due to the high F_1 value and low F_2 value, in "Variation A", differences between the packing layout added value with smaller and higher μ are more evident. Considering "Variation B", the raw material reuse impact on packing layout added value increases, due to F_1 and F_2 close values. Low μ instances still presented the best solutions. However, the difference in solution quality between instances with high and low μ values decreased, represented by a more homogeneous distribution. Fig. 10 shows the *C22* instance packing layout with the M-BFDH and O-BF.

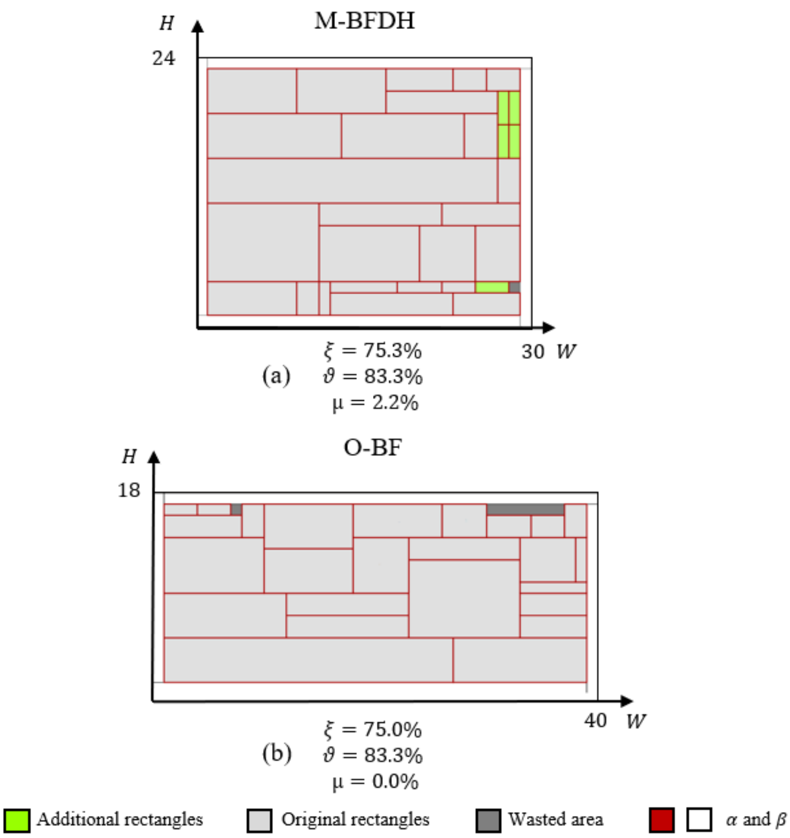

Fig. 10. *C22* packing layout for "Variation A" with the M-BFDH and O-BF (dimensions in cm).

For instances with close or equal ϑ values, heuristic methods that effectively place additional rectangles in the layout empty spaces showed the highest added value. For *C22,* both O-BF and M-BFDH returned the same ϑ value. However, since O-BF does not allow raw material reuse, the packing layout with the M-BFDH resulted in a higher ξ value. Due to the smaller area rectangle dimension ($h = 7\ cm$ and $w = 13\ cm$), even with the additional rectangles' rotation possibility, the empty spaces found in the M-BFDH packing layout are not effectively reused. For A-BFDH, the empty spaces generated are larger, being reused by the smallest area rectangle. However, to find larger empty spaces, the total sheet metal area increases, reducing the original rectangles sheet metal utilization percentage (ϑ) value and decreasing packing layout added value (ξ). Thus, for instances with the smallest area rectangle with higher dimensions, e.g., *cgcut2*, *N5*, *nice1*, *nice2*, *nice3*, *nice4*, and *path2*, even with a low μ, M-BFDH has the highest ϑ values, resulting in increased added value packing layouts.

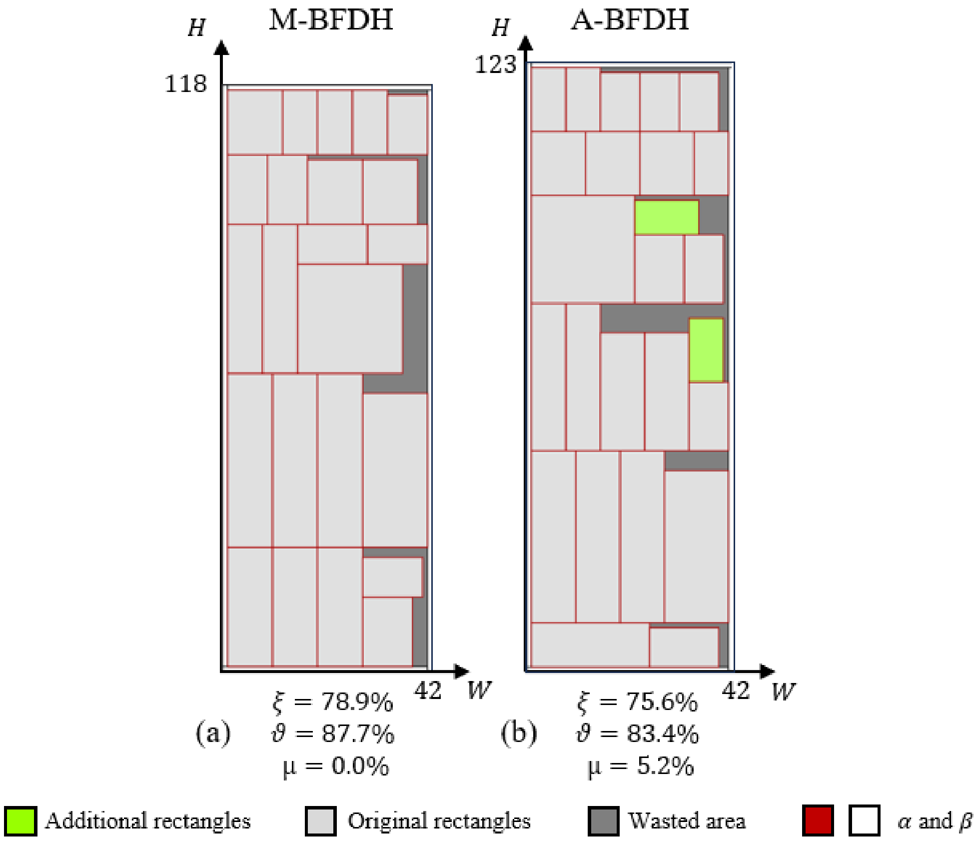

Fig. 11. Packing layout for *cgcut2* considering "Variation A" with the M-BFDH and A-BFDH (dimensions in cm).

Heuristics focused on reducing sheet metal H showed the worst solutions when compared to heuristics focusing on maximizing packing layout added value. Also, despite a packing layout with a reduced H resulting in a high sheet metal usage, not considering raw material reuse lowered the added value, as seen for O-BL, O-BF, and O-BFDH. Therefore, for laser cutting practical scenarios involving metal sheets, raw material reuse should be considered to increase the manufacturing added value by improving sheet metal use.

5.6 Improvements rules impact

Two improvement rules were proposed to increase the packing layout added value in the M-BFDH. As verified in Section 4.2, for the first rectangle variation rule, since the first rectangle packed is not submitted to the score order, at each iteration, a different rectangle is placed as the first rectangle. For the W variation rule, the original instance's W is reduced, in 5% intervals, until the maximum value of 50%. The improvement rules are applied together, for each variation in W, a packing layout considering all instance's rectangles (with and without rotation) as the first rectangle packed is constructed, and the packing layout with the best-added value (ξ) is selected.

Fig. 12 shows the improvement rules impact for Variation A, presenting the average ξ by instance class for the M-BFDH to compare packing layouts found without improvement rules, with each improvement rule applied individually, and with both improvement rules applied together. Only the results found for Variation A are analyzed in this section, as the packing layouts found with M-BFDH for Variation A and Variation B are equal. Therefore, the difference in packing layout added value, when comparing improvement rules, is the same.

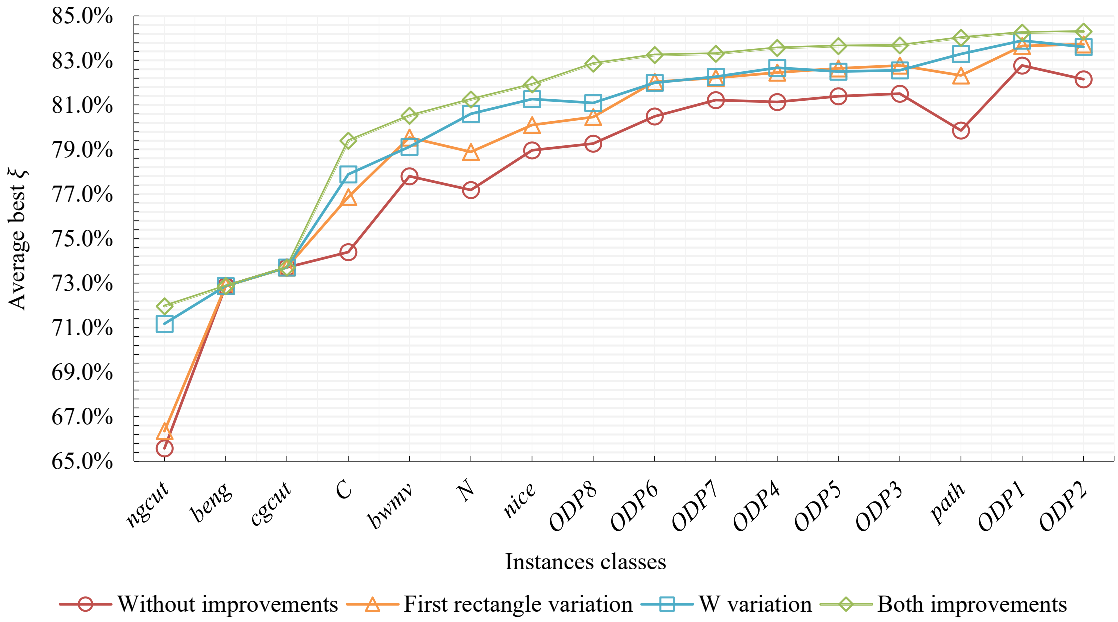

Fig. 12. M-BFDH improvement rules impact.

The improvement rules were effective to find higher added value packing layouts. Compared to the solutions found without improvements, all instances (excluding *beng1* and *cgcut2*) presented an increase in ξ. Furthermore, on average, the increase in ξ was 2.76%, with some instance classes, including *C*, *ngcut*, and *path*, returning an increase higher than 4.00%. To find higher added value packing layouts, the combination between varying the sheet metal W and changing the first packed rectangle must be considered in laser cutting operations with the M-BFDH. Similarly, for the first rectangle variation and W variation applied individually, both were able to enhance packing layout added value. The W variation presented better results when compared to changing the first packed rectangle. Consequently, eleven instance classes returned superior ξ values with only the W variation, and the average packing layout added value improvement provided with varying W was higher (1.89%) when compared to changing the first packed rectangle (1.27%). The disparity can be attributed to the significant changes in the packing layout from varying W, creating higher rectangles' combinations inside the sheet metal. Consequently, varying W has a greater potential for rearranging rectangles, resulting in packing layouts with distinct characteristics from the original layout, returning more possibilities to achieve increased added value. For the first rectangle variation, although it enhances packing layout added value, only variating the first rectangle does not provide as many opportunities for rectangle repositioning, consequently resulting in a comparatively smaller added value increase. However, with both improvement rules, more packing layouts with distinct characteristics are found due to W variation, and, on a small scale, due to changing the first rectangle. Therefore, additional opportunities to discover higher added value packing layouts were created, increasing the ξ by an average of 2.76%. In Fig. 13, considering the instances approached, by examining both improvement rules together, trends can be determined related to the W values used.

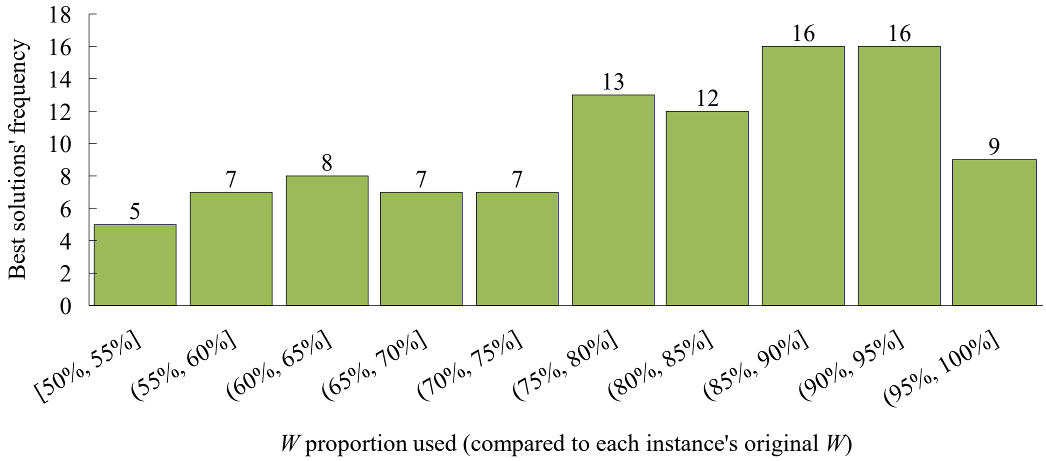

Fig. 13. Frequency distribution chart for the M-BFDH best solutions (highest ξ) considering the two improvement rules related to the W proportion used for the best solutions

For the M-BFDH results found considering the two improvement rules applied together, 66.0% of instances presented the highest added value packing layouts using a W between 75% to 100% of the original instance's W. Packing layouts with a higher reduction (50% to 70% of W) are less common since by limiting the sheet metal's W, the packed rectangles tend to form "stacks", where rectangles are packed above each other due to limited horizontal space, which can increase the number of empty spaces. However, usually, the empty spaces created from the rectangles "stacks" are not suited for reuse, due to smaller dimensions, reducing original rectangles sheet metal utilization percentage (ϑ) and decreasing packing layout added value (ξ). Also, not changing the instance's W is not ideal for packing layout added value, since only 9 instances maintained the original W dimension. For the M-BFDH applied in laser cutting operations, it is better to work between 75% to 95% of the original instance's W. Although every W reduction develops rectangles "stacks", a smaller reduction in W can be beneficial, leading to a more efficient rectangle allocation in the M-BFDH layers and levels, reducing empty space, consequently increasing packing layout added value.

5.7 Practical implications

Packing layouts with high added value are essential for the sheet metal laser cutting industry, impacting the product's monetary value. Benefits including higher prices, competitive differential, reduced price sensitivity, operational efficiency, and continuous innovation are consequences of the search for high added value packing layouts. Considering only the packing layout, the added value relates to the sheet metal filling by the instances' original rectangles combined with the additional rectangles from raw material reuse. To obtain high added-value packing layouts, heuristics that increase sheet metal utilization by the original instance rectangles, resulting in fewer empty spaces, are necessary. Furthermore, heuristics adapted to reuse raw material returned packing layouts with greater added value when compared to classic heuristics for C&P problems, focusing only on reducing sheet metal H. Therefore, reusing raw material by placing additional rectangles in the packing layout empty spaces should be considered in sheet metal laser cutting practical applications, positively impacting the added value found. Furthermore, since a significant portion of steel production worldwide is coming from recycling practices, with the smelting process during steel recycling operations being expensive and consuming a tremendous amount of energy, direct sheet metal reuse without smelting can be environmentally and economically advantageous over recycling (Ali et al., 2019).

Essentially, prioritizing only raw material reuse is not ideal. The additional rectangles packed derive from the original instance rectangles packing layout. Thus, for high material reuse, more empty spaces are required, characterizing a low sheet metal usage by the original rectangles, decreasing the added-value. Furthermore, a high number of additional rectangles number causes manufacturing wastes including overproduction, stock, extra processing, and transportation (Buer et al., 2018; Eswaramoorthi et al., 2011; Jasti and Kodali, 2014). The additional rectangles' impact on production waste relates to each industry's particular scenario. Industries where additional rectangles can be easily commercialized cause lower manufacturing wastes, resulting in a larger increase in added value from raw material reuse (Cherri et al., 2013). The possibility of changing W must be considered to increase packing layout added value in sheet metal laser cutting. For "Variation A" and "Variation B", using M-BFDH, A-BL, A-BF, and A-BFDH, in 69.0% of instances, the best solutions were found with modified W. Also, smaller variations in W, between 75% to 95% of the original instance's W, returned better added-value packing layouts when compared to variations between 50% to 70%. Furthermore, despite the reduction in W consequently increasing the H found, the variation enables the possibility of better rectangle placement in the sheet metal, improving raw material usage. For the first packed rectangle, due to the problem's geometric characteristics, is not possible to find a tendency related to the first packed rectangle dimensions and the packing layout added value.

For sheet metal laser cutting practical applications, M-BFDH is suitable for generating good packing layouts, resulting in industrial benefits including cost savings, increased productivity, greater competitiveness, and sustainability. M-BFDH showed the best results for the added value index (ξ), with a high metal sheet usage by the instance original rectangles, combined with effective raw material reuse, resulting in packing layouts with greater added value. Heuristics focused on reducing H were not effective in increasing packing layout added value, revealing the need for specific approaches considering practical scenarios with different objectives related to the manufacturing process, increasing operational research impact in the industry. The ξ values found relate to the production of only one packing layout for each tested instance. For production planning considering more than one packing layout produced, even small increases in added value return exponentially positive impacts for practical scenarios. Also, is harder to find packing layouts with an added value close to the packing layouts represented by the maximum ξ value (90.0% for "Variation A" and 60.0% for "Variation B"), due to the difficulty in reducing empty spaces, apart from α and β.

Finally, comparing results, the generated instances presented better packing layout added value for all heuristics considering both variations. Therefore, it is important to include instances with different practical characteristics when evaluating algorithms. The generated instances reflect complexities, variations, and characteristics found in real-world sheet metal plasma cutting scenarios. By evaluating algorithms with practical instances, insights related to the heuristics' robustness, scalability, and efficiency, are obtained to aid manufacturing process decision-making. Additionally, the generated instances can help identify potential issues including bias, error propagation, or unexpected behavior that might not be found in literature instances. Thus, instances based on practical sheet metal cutting scenarios are essential for ensuring algorithms can perform effectively in real-world applications.

6. Conclusion

This research approached the practical problem related to sheet metal laser cutting with practical constraints, aiming to maximize the packing layout added value by modifying the classic BFDH constructive heuristic. The M-BFDH divides the sheet metal into layers and levels, effectively placing rectangles, reducing packing layout empty spaces. In addition, the possibility of reusing raw material by packing additional rectangles in the packing layout empty spaces was considered. Two improvement rules were proposed to increase added value: First rectangle variation and W variation. From the heuristics considered, M-BFDH presented the best solutions, with a higher packing layout added value in 90% of instances for "Variation A" and 69% of instances for "Variation B". The high packing layout added value from the M-BFDH relates to better sheet metal usage by the instance original rectangles, having the most impact on solution quality. Also, reusing raw material by packing additional rectangles combined with the two improvement rules proposed, increased the packing layout added value, and should be considered in practical sheet metal laser cutting applications. Furthermore, since raw material reuse depends on the packing layout empty spaces, the additional rectangles have a lower impact on the packing layout added value when compared to the original instance rectangles. A gap was found in the C&P literature related to laser cutting practical applications, considering different manufacturing parameters. This research fills the gap being the first article, to the authors' knowledge, specifically addressing maximizing packing layout added value considering a sheet metal laser cutting practical scenario. Furthermore, from the results' robustness, supported by an extensive set of instances with varying characteristics, the M-BFDH potential to effectively address the proposed problem is highlighted.

Considering practical scenarios, several factors are directly and indirectly impacted by a packing layout with higher added value, resulting in industrial gains including lower manufacturing process wastes related to overproduction, stock, extra processing, and transportation; cost savings and increased productivity through manufacturing process waste reduction; reduction of raw material scrap and consequently sustainability increase; and competitiveness growth from the cited factors' combination. In summary, the examination of a rectangular 2D-SPP variation in the sheet metal laser cutting process analyzes the relationship between packing layouts, added value, and raw material reuse. This study emphasizes the necessity of considering diverse objectives in C&P problem formulation to bridge the gap between industry needs and academic research. Moreover, the proposed modification, M-BFDH, improves packing layout added value, being specifically developed to better approach raw material reuse and W variations. Also, by analyzing patterns in high added-value packing layouts, industry practitioners can gain insights in laser-cutting operations to improve financial gains and reduce process costs. Furthermore, this research highlights raw material reuse as a viable alternative to just recycling steel scrap from the packing layouts, since the empty spaces found in the packing layouts can be used to produce new rectangles. The M-BFDH heuristic can be used for different sheet metal cutting processes, such as plasma, waterjet, and oxyfuel cutting. However, an adjustment in α and β values is necessary, and, for complex scenarios, additional constraints related to the cutting process approach must be considered. Lastly, future research can address different sheet metal practical aspects, including the economic impact of a greater added value packing layout, the environmental consequences of considering raw material reuse, and an analysis of the manufacturing process when considering different production scales.

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Appendix A: Basic notation

α: minimum distance between rectangles;
β: minimum distance between rectangles and the sheet metal edges;
n: number of rectangles;
I: set of rectangles;
i: rectangle belonging to I;
H: sheet metal height solution;
W: sheet metal width;
h_i: rectangle height;
w_i: rectangle width;
x_{1i}: rectangle coordinate in the x-axis;
x_{2i}: rectangle coordinate in the x-axis;
y_{1i}: rectangle coordinate in the y-axis;
y_{2i}: rectangle coordinate in the y-axis;
ξ: added value index;
A_1: total sheet metal area;
A_2: original instance rectangles area;
A_3: valid empty spaces area for reuse;
F_1: instances original rectangles added value quantifier;
F_2: additional rectangles added value quantifier;
ϑ: sheet metal utilization percentage by the original instance rectangles;
μ: sheet metal utilization percentage by additional rectangles.

Appendix B: Examples of M-BFDH packing layout

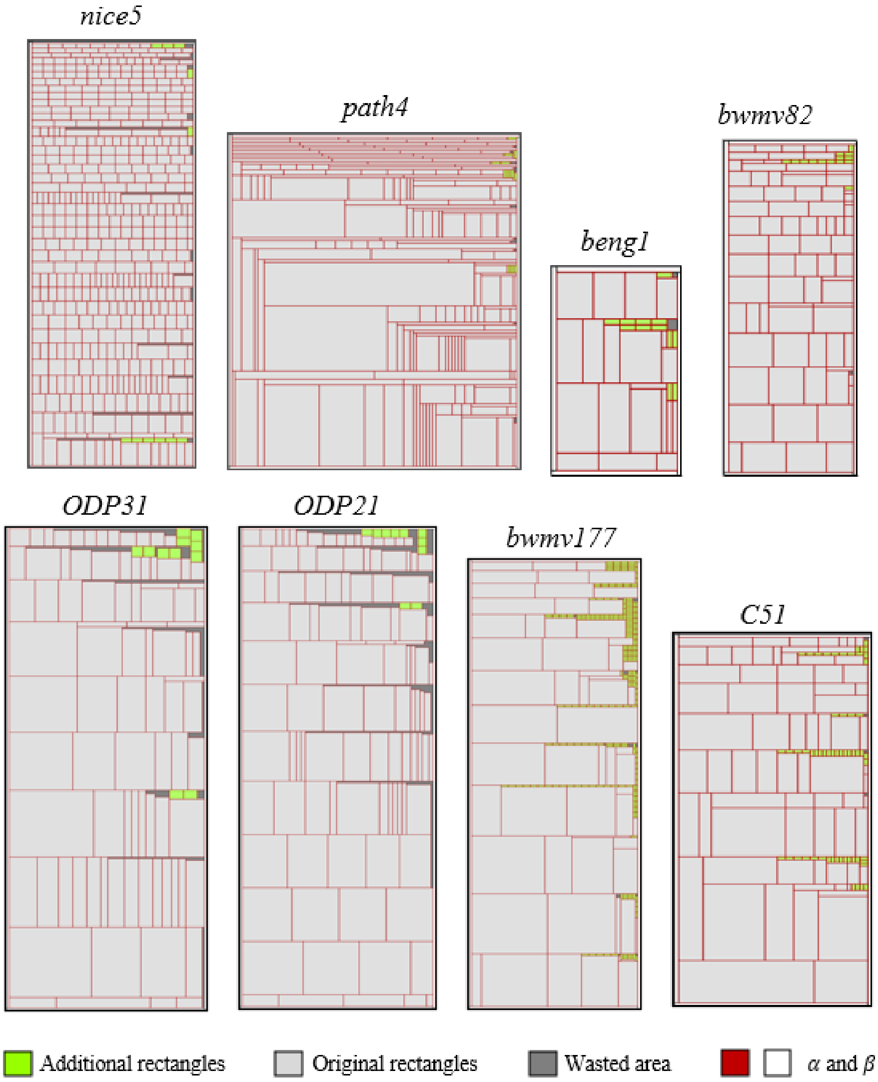

Fig. B.1. M-BFDH packing layout for *nice5*, *path4*, *beng1*, *bwmv82*, *ODP31*, *ODP32*, *bwmv177*, and *C51*.

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