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Technology licensing contracts in supply chains with carbon cap-and-trade and vertical shareholding

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ABSTRACT

This study explores technology licensing in a low-carbon supply chain under cap-and-trade regulations, with an upstream firm holding partial shareholding in a downstream firm. We established a Stackelberg game to analyze four licensing strategies: free, fixed fee, royalty, and revenue-sharing. We investigate the effects of vertical shareholding and cap-and-trade regulation, as well as whether technology licensing yields a more favorable outcome compared to non-licensing and which licensing strategy proves superior. The findings reveal that when the upstream firm holds a higher share in the downstream firm, it results in increased profits for the upstream firm, the supply chain system, and consumer surplus, but decreased profit for the downstream firm. Furthermore, when carbon emission quotas are sufficiently high (low), a higher carbon trading price leads to increased (decreased) supply chain profitability, while inevitably decreasing consumer surplus. Increased carbon emission quotas consistently contribute to increased supply chain profitability, but have no impact on consumer surplus. All licensing contracts enhance the profitability of the upstream firm, the supply chain system, as well as consumer surplus, with revenue-sharing emerging as the most effective strategy. However, whether technology licensing promotes social welfare depends on factors such as the carbon emissions per unit of product and the environmental impact of each unit of carbon emission.

1. Introduction

In recent years, a growing number of nations across the globe has made their schedules for achieving carbon neutrality targets, such as China's carbon neutrality target in 2060 (Wang et al., 2021; Lee & Park, 2020) and Europe's carbon-neutral continent goal in 2050 (Guo et al., 2022). To realize the goal of carbon neutrality, cap-and-trade regulations, as a most effective measure to reduce carbon emission, have been embraced by many countries worldwide. (Tang and Yang, 2020; Liu et al., 2021). Under cap-and-trade regulations, governments allocate carbon emission caps to firms and allow them to engage in carbon trading within the market. Should firms' carbon emissions surpass the assigned cap, they can choose to procure extra carbon quotas from the carbon trading market, thereby elevating their operating costs. Conversely, if firms' carbon emissions fall below the cap, they can sell surplus quotas, generating augmented revenue. Undoubtedly, the potential for heightened operating costs and increased revenues serves as an incentive for firms to diligently endeavor towards reducing carbon emissions. Empirical evidence highlights that global corporations have made substantial investments in carbon neutrality technologies to reduce carbon emissions. For example, Apple has invested up to $200 million towards expanding its Restore Fund in order to attain carbon neutrality across its business (Cai & Jiang, 2023). However, such substantial investment costs for carbon neutrality technologies may pose challenges for firms, especially small and medium-sized enterprises that have limited capital and lack technical R&D capabilities, hindering their ability to actively pursue carbon emission reduction initiatives. Under these

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realistic circumstances, firms without independent innovation capabilities usually choose to adopt Technology licensing approach to acquire new carbon neutrality technologies and overcome technical barriers (Yan and Yang, 2018) Technology licensing not only facilitates licensees in enhancing operational efficiency, reducing carbon emissions, and lowering operating costs (Qiao & Su, 2021; Chen et al., 2022), but also enables innovative firms (licensors) to maximize the value of their new technologies and generate substantial licensing revenues (Yan & Yang, 2018). This has prompted numerous firms, including AT&T, IBM, Apple, Huawei, Qualcomm, and others, to actively engage in technology licensing as a pivotal business strategy (Huang & Wang, 2017). Nowadays, carbon neutrality technology licensing has become a common practice within supply chains to reduce carbon emission. For example, Carbios licenses its PET biorecycling technology to industry leaders like Nestlé Waters, PepsiCo, and Suntory Beverage & Food Europe, empowering them to produce 100% recycled plastic bottles and significantly reduce carbon emissions by approximately 30% (Mohan, 2021). Similarly, Qualcomm’s licensing of its 3G/4G technologies to smartphone manufacturers such as MIUI and Apple (Jordan, 2016). These observations reflect a fact that carbon neutrality technology licensing significantly affects supply chain firms’ operational decisions and profitability. However, regarding how to license technologies, supply chain firms have adopted different approaches. For instance, HP, Facebook, and Microsoft adopt free licenses for their technologies to low-carbon innovators (Bauer et al., 2021). Carbios require upfront fixed fees from licensees authorized to produce PET plastic bottles. Huawei applies Royalty licensing in which licensers charge fees based on product quantity (Huawei, 2023). Furthermore, Firms also employ revenue-sharing contracts, as seen with Qualcomm’s requirement for Apple to pay licensing fees based on the total price of each iPhone sold (Tibken, 2017). Undoubtedly, different licensing strategies carry varying impacts on supply chain firms’ operational decisions and profitability.

In practice, some empirical evidence indicates that vertical shareholding is the main factor contributing to differences in technology licensing strategies. Vertical shareholding is a common and important business strategy and enables firms within the supply chain to share their partners’ profits according to the shares held. Prominent examples include Gillette’s 22.9% stake in Wilkinson Sword (Shelegia and Spiegel, 2015), as well as Bosideng and Red Dragon holding shares in Dashang Group (Chen et al., 2017; Fan et al., 2023). Xia et al. (2021) and Sun et al. (2023) demonstrated the significant impact of vertical shareholding on supply chain firms’ contractual relationships, green R&D decisions, and corresponding profitability. Further, Din and Wu (2015) reveals that a retailer's vertical shareholding has a significant effect on the manufacturer's royalty and fixed-fee licensing decisions. These observations highlight that vertical shareholding also influences firms’ motivation for technology licensing and their choices regarding licensing contracts. Drawing from observed the interaction between technology licensing and vertical shareholding, this study aims to explore the following research questions.

(1) How does vertical shareholding affect supply chain members’ decisions on licensing, pricing, and profitability conditional on cap-and-trade regulation?
(2) What are the optimal wholesale price and market price decisions under different carbon neutrality technology licensing models in a low-carbon supply chain?
(3) Does technology licensing contribute to improving supply chain profitability, consumer surplus, and social welfare? Which technology licensing contract is superior from the perspectives of the supply chain, consumers, and society?

To answer the above questions, we consider a low-carbon supply chain consisting of an upstream firm and a downstream firm, where the upstream firm owns partial passive shareholding in the downstream. Both firms operate under cap-and-trade regulation. We assume that the upstream firm acts as the supply chain leader and the technology innovator. The upstream firm can choose to license its technology to the downstream firm through various licensing contracts, including free licensing, fixed fee licensing, royalty licensing, and revenue-sharing licensing. Technology licensing assists the downstream firm to reduce carbon emission and production cost. This study makes the following significantly contributions summarized as follows.

(1) While existing literature on the low-carbon supply chain primarily focuses on the impact of cap-and-trade regulations on operational decisions of the supply chain. We enrich the existing literature by introducing technology licensing and investigating its impact on the supply chain members’ price decisions and profitability.
(2) In contrast to existing vertical shareholding literature, few of which have considered its impact in a green and low-carbon supply chain, we apply vertical shareholding to a low-carbon supply chain subject to cap-and-trade regulations and investigate the impact of vertical shareholding on supply chain operational decisions. More importantly, we investigate the impact of vertical shareholding under various technology licensing contracts, taking into consideration both vertical shareholding and technology licensing.
(3) Existing literature on technology licensing focuses primarily on competing firms and investigates fixed fee licensing and royalty licensing contracts, while we complement it by considering technology licensing between supply chain firms and expanding contract types for technology licensing, such as free licensing and revenue-sharing licensing. Additionally, we analyze the influence of vertical shareholding on licensing decisions and licensing contract choice, which is rarely studied in the current vertical shareholding research.

The rest of this research is organized as follows. Section 2 summarizes the relevant literature. Section 3 outlines the assumptions for low-carbon supply chain game models that take into account cap-and-trade regulations, vertical shareholding, and technology licensing. In Section 4, we formulate game models for non-licensing and licensing scenarios, characterizing optimal decisions of the supply chain members and comparing non-licensing and licensing models. Section 5 investigates the impact of cap-and-trade regulation and vertical shareholding. Section 6 investigates which technology licensing contract is superior from the supply chain, consumer, and societal viewpoints. The conclusion of this work is summarized in Section 7.

2 Literature review

This study is closely related to three streams of literature: (1) low-carbon supply chain; (2) vertical shareholding; and (3) technology licensing. The following is a summary of the literature from these three streams.

2.1 Low-carbon supply chain

The first stream of literature aims to explore how incentive strategies and cap-and-trade regulations affect carbon emission reduction in the low-carbon supply chain. As for incentive strategies, Yang and Chen (2018) explore how revenue-sharing and cost-sharing incentive strategies affect the carbon emission reduction efforts between a manufacturer and a retailer conditional on an assumption of consumers' environmental preferences. Li et al. (2019) reveal that the bargaining on revenue-sharing and cost-sharing contracts in supply chain incentivizes upstream to increase emissions abatements efforts. Yu et al. (2020) investigate the distinct impacts of cost-sharing and revenue-sharing strategies on carbon emissions within the supply chain, taking into account reference emissions and cost-learning effects. He et al. (2020) establish a low-carbon service supply chain model to explore the impact of the service integrator's cost-sharing contract on the service provider's optimal carbon reduction decision. The aforementioned literature shows that incentive strategies including revenue sharing and cost sharing contracts indeed affect the carbon emission reductions in supply chain systems, but does not explore whether technology licensing as an important strategy affects carbon emission reduction in the supply chain. Additionally, the impact of cap-and-trade regulations on carbon emission strategies and outcomes is not addressed in these works. We aim to fill this void by exploring the potential of technology licensing as a means to reduce carbon emissions within the framework of cap-and-trade regulations, providing a more comprehensive understanding of low-carbon supply chain management.

As for cap-and-trade regulations, Wang et al. (2021) employ differential game models to investigate the impacts of cooperation and non-cooperation among supply chain members on optimal carbon emission reduction decisions. Cui and Jiang (2023) also use differential game models to investigate the effects of cap-and-trade regulations, consumers' low-carbon preferences, and channel power structures on optimal pricing and carbon emission reduction decisions in supply chains. Bai et al. (2023) optimize the manufacturer's energy conservation investment decision and the supplier's quality decision in centralized, decentralized, and partially integrated supply chains under cap-and-trade regulation. Yang (2023) considers a dual-channel supply chain with a manufacturer and an e-commerce platform firm, and investigates the manufacturer's optimal carbon emission reduction efforts and the e-commerce platform firm's optimal green investment strategies under cap-and-trade regulations. Other studies in this stream of literature include Yang and Chen (2024), Fu and Song (2023), Tang and Yang (2020), Qu et al. (2021), Liu et al. (2021), Mondal and Giri (2022), Guo et al. (2022), Xu et al. (2023a), and Xu et al. (2023b), among others. The literature mentioned above has discussed carbon emission reduction in different supply chain environments and investigated optimal decisions regarding carbon emission reduction efforts under cap-and-trade regulations. However, they all fail to consider the carbon emission reduction induced by technology licensing. In contrast to the existing studies, our research focuses specifically on carbon emission reduction through technology licensing for carbon neutrality. We aim to explore the effects of various technology licensing contracts on carbon emission reduction within the supply chain.

2.2 Vertical shareholding

The second body of research pertinent to our study focuses on vertical shareholding and its influence on supply chain operational decisions. For example, Chen et al. (2017) consider a supply chain with vertical shareholding and show that increasing the leader's shares held by the follower does not affect supply chain profitability, while increasing the follower's shares held by the leader promotes the profits of both the chain and the leader. Fu and Ma (2019) investigate the impacts of vertical shareholding on wholesale price and production quantity decisions in pull and push supply chains and propose coordination schemes for win-win outcomes. Li et al. (2021) consider different channel structures in competing supply chains and show that partial vertical centralization, where the manufacturer owns shares in its exclusive retailer, can be an equilibrium structure under certain conditions. Zhang and Meng (2021) develop a value-creation model for a closed-loop supply chain with vertical shareholding, highlighting the benefits of increased vertical shareholding on the value of each party. Xiao et al. (2021) explore quality investment and vertical shareholding in a supply chain with an assembler and multiple suppliers, concluding that vertical shareholding stimulates the assembler to invest more to enhance product quality in the pull system rather than the push system. Xia et al. (2021) demonstrate that vertical shareholding leads to greater investment in green quality improvement in a supply chain, under both manufacturer Stackelberg and retailer Stackelberg supply chain game models. Sun et al. (2023) explore the impact of vertical shareholding on recovery rate and carbon emission reduction effort in a low-carbon closed-loop supply chain, demonstrating that regardless of the channel power structures, both the recovery rate and carbon emission reduction effort increase with vertical shareholding.

While the aforementioned studies shed light on the effects of vertical shareholding on supply chain operational decisions and profitability under various channel structures, they rarely consider its influence on carbon emission reduction. Despite the fact that Xia et al. (2021) and Sun et al. (2023) have conducted investigations into green quality investment strategies within the context of a green and low-carbon supply chain, they both neglected to explore it under cap-and-trade legislation. Our study differs from these previous works in three key aspects. Firstly, we investigate a low-carbon supply chain with vertical shareholding within the framework of cap-and-trade regulations. Secondly, we propose that carbon emission reduction can be achieved through carbon neutrality technology licensing rather than direct investment. Finally, we analyze different technology licensing contracts within the supply chain and examine the impact of vertical shareholding on the choice of these contracts.

2.3 Technology licensing

This study also adds to the stream of literature on technology licensing. Some recent studies have explored this topic within the context of competing firms. For example, Zhao et al. (2014) compare licensing strategies with fixed fees, royalty fees, and two-part tariffs between two firms with quality differences, while accounting for network effects. Zhang et al. (2016) propose a three-stage duopoly game model in which the outcome of the innovator's R&D is uncertain, and study how product differentiation and technology spillover affect the innovator's optimal licensing strategy choice between fixed-fee and royalty licensing contracts. Chen et al. (2022) studied the profitability of rival manufacturers and social welfare under cap-and-trade regulations, focusing on fixed-fee licensing and two-part tariff licensing. Chen et al. (2023) examined the decision of a technology innovation manufacturer to license cost-reducing technology to the mid-cost manufacturer, the high-cost manufacturer, or both. Hong et al. (2024) investigated how technology licensing affects firms' capacity under competition. Additionally, studies have investigated technology licensing in remanufacturing settings within closed-loop supply chains, where manufacturers generally license remanufacturers. Huang and Wang (2017) investigated the impact of information sharing on remanufacturing licensing options in a closed-loop supply chain involving a manufacturer, a distributor, and a third party. Hong et al. (2017) study the optimal recycling decisions of a manufacturer and a remanufacturer in a Cournot duopoly model, considering fixed-fee licensing and royalty licensing agreements. Huang and Wang (2019) show that the manufacturer prefers licensing the third party for remanufacturing, and fixed-fee licensing can benefit both parties when the remanufacturing cost is sufficiently low. Chai et al. (2020) explore licensing strategies between an OEM and an independent remanufacturer (IR) under cap-and-trade regulations, concluding that royalty (fixed-fee) licensing is preferable if the fixed-fee is low (high). Qiao and Su (2021) and Yang et al. (2022) study the impact of patent licensing fees on the distribution channel choice of remanufacturers. Liu et al. (2022) study OEM technology licensing to improve remanufactured product quality and consumer preferences, finding that two-part tariff licensing is consistently superior to fixed-fee and royalty licensing.

Our study differs from the previously listed research in the following aspects. Firstly, this study explores technology licensing within the supply chain, an area that has received limited attention, with the exception of Huang and Wang (2017). While some studies have examined technology licensing in closed-loop supply chains, they have primarily focused on horizontal competition between manufacturers and remanufacturers, neglecting the vertical competition between manufacturers and their upstream or downstream partners. Second, this study considers technology licensing under cap-and-trade regulations, while in the above studies, only Chai et al. (2020) and Chen et al. (2022) have taken this into account. Thirdly, in contrast to previous research that predominantly focused on fixed-fee and royalty licensing contracts, or two-part tariff licensing contracts combining fixed fee and royalty components, this study expands the scope by exploring free licensing and revenue-sharing licensing. Finally, and most importantly, this study complements prior studies by incorporating the concept of vertical shareholding within the supply chain and examining its impact on technology licensing strategies.

3. Model descriptions

We consider a supply chain consisting of an upstream firm and a downstream firm. Acting as the leader of the Stackelberg game, the upstream firm has a passive partial ownership holding in the downstream firm, which functions as the follower. Both firms operate under carbon cap-and-trade regulations, and the upstream firm holds a technological advantage over the downstream firm in terms of carbon neutrality, enabling it to effectively reduce carbon emissions and lower production costs. This advantageous position allows the upstream firm to license its carbon neutrality technology to the downstream firm, thereby enhancing supply chain efficiency and reducing carbon emissions. Within the framework of the supply chain game model, we make the following key assumptions.

1. We assume that the upstream and the downstream firms conduct their product transactions via the wholesale price contract. The downstream firm first purchases products from the upstream firm at a wholesale price w, and then sells products in the final market to consumers at a market price p. Assuming that the inverse demand faced by the downstream firm is $p = 1 - q$, where q denotes the amount of products demanded by consumers as well as the quantity produced by supply chain firms. This form of inverse linear demand function is aligned with prior operation management studies (Huang and Wang, 2019; Chai et al., 2020; Qiao and Su, 2021; Liu et al., 2022).

2. Assuming that the upstream firm possesses a significant technological advantage as a carbon neutrality innovator, it follows

that the upstream firm outperforms the downstream firm in terms of emitting less carbon emissions per unit of product during its production processes. We denote the carbon emissions per unit of product for the upstream and downstream firms as e_u and e_d, respectively. Without loss of generality, we normalize $e_u = e$ and $e_d = e(1+r)$, where $r(>0)$ represents the degree of difference in carbon emissions between the upstream and downstream firms. A higher value of r signifies a greater gap in carbon emissions resulting from the production processes of the two firms. It becomes evident that this difference in carbon emissions serves as a pivotal driver for the licensing of carbon neutrality technology between firms within the supply chain.

3. Assuming that the government has allocated carbon emission quotas for both the upstream and downstream firms, denoted as k_u and k_d, respectively. For the sake of simplicity, we assume that $k_u = k_d = K$, implying that both supply chain firms have been assigned equal carbon emission quotas. However, this assumption does not compromise the analysis presented in this study (Chen et al., 2022). The difference between the firms' actual carbon emissions (eq and $e(1+r)q$) and their respective carbon emission quotas determines whether they need to acquire extra carbon credits or sell excess carbon quotas in the carbon trading market. We introduce the carbon trading price as λ. Therefore, the carbon trading costs for the supply chain firms are measured as $\lambda(eq-K)$ and $\lambda[e(1+r)q-K]$, respectively. However, if the upstream firm licenses its carbon neutrality technology to the downstream firm, it is expected that the downstream firm's product will exhibit a reduced unit carbon emission from $e(1+r)$ to e. As a result, the downstream firm will incur the same carbon trading cost as the upstream firm, namely $\lambda(eq-K)$.

4. It is assumed that the carbon neutrality technology also contributes to cost reduction in production. For instance, Pepsi Cola's investment in green research and development, specifically in reusable plastic shipping containers, resulted in savings of $196 million. Hence, we posit that the upstream firm holds a cost advantage over the downstream firm, with production costs denoted as $c_u = c$ and $c_d = c(1+\theta)$ for the upstream and downstream firms, respectively. Here, $\theta(>0)$ denotes the cost differential between the supply chain firms, and a larger θ implies a greater differential in production costs, indicating inefficiency in the downstream firm's operations. However, if the downstream firm is granted a license to adopt the carbon neutrality technology of the upstream firm, its operational efficiency would improve, leading to a reduction in production costs from $c(1+\theta)$ to c.

5. In this study, we explore several licensing options available to the upstream firm, including the free licensing contract, fixed fee licensing contract, royalty licensing contract, and revenue-sharing contract. Under the free licensing contract, no transfer fee is imposed on the downstream firm for acquiring the carbon neutrality technology. Conversely, the fixed fee licensing contract requires the downstream firm to pay a fixed fee, denoted as $F(>0)$, to the upstream firm. Alternatively, under the royalty licensing contract, the downstream firm is obliged to pay a per-unit royalty fee, represented by $\delta(>0)$, to the upstream firm. Lastly, the revenue-sharing contract involves the downstream firm sharing a portion, denoted as $\xi(>0)$, of its revenue with the upstream firm.

6. Both firms operating within the supply chain are assumed to exhibit rational economic behavior, making strategic decisions guided by the principle of profit maximization. Additionally, suppose the upstream firm has passive partial ownership holding in the downstream firm, i.e., it owns a share $s(0 < s \leq 1/2)$ of the downstream firm. As a result, the upstream firm is entitled to receive dividends from the downstream firm in proportion to its ownership share. Thus, we assume that in determining the optimal decisions, the objective function of the upstream firm incorporates a weight of s to account for the downstream firm's profitability.

The supply chain Stackelberg game, which comprises upstream and downstream firms, contains four stages. In stage 1, the upstream firm, which serves as an innovator of carbon neutrality technology and a leader within the supply chain, makes a take-it-or-leave-it licensing decision on carbon neutrality technology in determining the licensing fees by free, fixed fee, royalty rate, or revenue-sharing rate. In stage 2, the downstream firm, acting as the upstream firm's follower, decides whether or not to accept the license offer. In stage 3, the upstream firm determines its optimal wholesale price to maximize profit. Finally, the downstream firm serves as the goods to customers by determining the optimal price in the market to maximize profit.

4 Equilibriums

4.1 The benchmark model

In this subsection, we consider a benchmark in which the upstream firm does not license its carbon neutrality technology to the downstream firm. The upstream firm chooses a wholesale price to maximize its profit, while the downstream firm decides on the optimal market price. We denote this benchmark model as "N". The supply chain decisions made by firms can be described below:

$$\max_{w} U_u^N = \pi_u^N + s\pi_d^N \tag{1}$$

$$\max_{p} U_d^N = (1-s)\pi_d^N \tag{2}$$

In the above Eq. (1) and Eq. (2), U_u^N and U_d^N denote the upstream and downstream firms' respective profits after accounting for ownership weight. π_u and π_d denote the initial profits of the upstream and downstream firms, which can be expressed as:

$$\pi_u^N = (w-c)q - \lambda(eq-K),$$
$$\pi_d^N = [p-w-c(1+\theta)]q - \lambda[e(1+r)q-K].$$

The Stackelberg game can be solved by backward induction. First, using Eq. (2), we can solve for the downstream firm's optimal market price decision from $\frac{\partial U_d^N}{\partial p} = 0$ and obtain:

$$p = \frac{1}{2}[1+c+w+c\theta+\lambda e(1+r)] \tag{3}$$

Next, by substituting Eq. (3) into Eq. (4), we can solve for the upstream firm's optimal wholesale price decision from $\frac{\partial U_u^N}{\partial w} = 0$. To ensure the equilibrium results of the benchmark model are positive, we require $1-c(2+\theta)-\lambda e(2+r) > 0$. Therefore, we have Corollary 1 as follows.

Corollary 1: *In the benchmark model, the equilibrium results are as follows:*

$$w^{*N} = \frac{1-c\theta-\lambda er+s[1-c(1+\theta)-\lambda e(1+r)]}{2-s},\ p^{*N} = \frac{3-2s+c(2+\theta)+\lambda e(2+r)}{2(2-s)},\ q^{*N} = \frac{1-c(2+\theta)-\lambda e(2+r)}{2(2-s)},$$
$$U_u^{*N} = \frac{[1-c(2+\theta)-\lambda e(2+r)]^2+4K\lambda(2-s)(1+s)}{4(2-s)},\ U_d^{*N} = \frac{(1-s)\{[1-c(2+\theta)-\lambda e(2+r)]^2+4K\lambda(2-s)^2\}}{4(2-s)^2},$$
$$U_c^{*N} = U_u^{*N} + U_d^{*N} = \frac{3[1-c(2+\theta)-\lambda e(2+r)]^2+8K\lambda(2-s)^2}{4(2-s)^2},\ CS^{*N} = \frac{1}{2}(q^{*N})^2 = \frac{[1-c(2+\theta)-\lambda e(2+r)]^2}{8(2-s)^2}.$$

4.2 The NL model

In the NL model, the upstream firm freely licenses its carbon neutrality technology to the downstream firm, thereby enabling the downstream firm to achieve parity with the upstream firm in terms of both carbon emissions and production costs per unit of product. Since no transfer payment is involved in this licensing arrangement, we can describe the initial profit functions of the upstream and downstream firms as follows:

$$\pi_u^{NL} = (w-c)q - \lambda(eq-K) \tag{4}$$
$$\pi_d^{NL} = (p-w-c)q - \lambda(eq-K) \tag{5}$$

where the superscript "NL" represents the NL model. With Eq. (4) and Eq. (5), the decision problems of the upstream and downstream firms are as follows:

$$\max_{w} U_u^{NL} = \pi_u^{NL} + s\pi_d^{NL} \tag{6}$$
$$\max_{p} U_d^{NL} = (1-s)\pi_d^{NL} \tag{7}$$

We employ Backward induction to solve this Stackelberg game. First, we solve for the downstream firm's optimal market price decision. According to the FOC $\frac{\partial U_d^{NL}}{\partial p} = 0$, we obtain:

$$p = \frac{1}{2}(1+c+w+e\lambda) \tag{9}$$

Then, Substituting Eq. (8) into Eq. (6), by solving $\frac{\partial U_u^{NL}}{\partial w} = 0$, we can obtain the upstream firm's optimal wholesale price. Therefore, Corollary 2 is derived as follows.

Corollary 2: *In the NL model, the equilibrium results are as follows:*

$$w^{*NL} = \frac{1-s(1-c-\lambda e)}{2-s},\ p^{*NL} = \frac{3-2s+2(c+\lambda e)}{2-s},\ q^{*NL} = \frac{1-2(c+\lambda e)}{2(2-s)},$$
$$U_u^{*NL} = \frac{[1-2(c+\lambda e)]^2+4K\lambda(2-s)(1+s)}{4(2-s)},\ U_d^{*NL} = \frac{(1-s)\{[1-2(c+\lambda e)]^2+4K\lambda(2-s)^2\}}{4(2-s)^2},$$

$U_c^{*NL} = U_u^{*NL} + U_d^{*NL} = \frac{3[1-2(c+\lambda e)]^2+8K\lambda(2-s)^2}{4(2-s)^2}$, $CS^{*NL} = \frac{1}{2}(q^{*NL})^2 = \frac{[1-2(c+\lambda e)]^2}{8(2-s)^2}$.

Proposition 1: $w^{*NL} > w^{*N}$, $p^{*NL} < p^{*N}$, $q^{*NL} > q^{*N}$, $U_u^{*NL} > U_u^{*N}$, $U_d^{*NL} > U_d^{*N}$, $U_c^{*NL} > U_c^{*N}$, *and* $CS^{*NL} > CS^{*N}$.

Proposition 1 reveals a key finding in the NL model: both upstream and downstream firms experience increased profits compared to the benchmark model. This signifies the feasibility of freely licensing carbon neutrality technology. Interestingly, the upstream firm, despite not charging a licensing fee, chooses to set a higher wholesale price. However, this does not necessarily result in a higher market price. In contrast, the downstream firm strategically reduces the market price due to the incorporation of carbon neutrality technology, leading to lower carbon trading and production costs. As a result, customer demand increases, improving profitability for both supply chain firms. Without a doubt, the supply chain system profit and consumer surplus are significantly increased.

4.3 The FL model

In this subsection, we shall investigate the FL model, which involves the upstream firm offering a fixed fee contract for the licensing of its carbon neutrality technology. In this situation, the downstream firm pays a lump amount $F(> 0)$ to the upstream firm for the utilization of the carbon neutrality technology. Thus, under fixed-fee licensing, the initial profit functions of supply chain entities are provided by:

$$\pi_u^{FL} = (w-c)q - \lambda(eq-K) + F \tag{9}$$
$$\pi_d^{FL} = (p-w-c)q - \lambda(eq-K) - F \tag{10}$$

where the superscript "FL" represents the FL model. By Eq. (9) and Eq. (10), the decisions problems of the upstream and downstream firms are characterized as:

$$\max_{F,w} U_u^{FL} = \pi_u^{FL} + s\pi_d^{FL} \tag{11}$$
$$\max_{p} U_d^{FL} = (1-s)\pi_d^{FL} \tag{12}$$

We solve the Stackelberg game by backward induction. Similar to the NL model, the downstream firm first chooses its optimal market price decision to maximize profit. Thus, solving $\frac{\partial U_d^{FL}}{\partial p} = 0$ yields $p = \frac{1}{2}(1+c+w+e\lambda)$. Subsequently, we solve the upstream firm's optimal wholesale price from $\frac{\partial U_u^{FL}}{\partial w} = 0$, and obtain $w = \frac{1-s(1-c-e\lambda)}{2-s}$. Therefore, the corresponding profits of the upstream and the downstream firms are as follows:

$$U_u^{FL} = \frac{[1-2(c+\lambda e)]^2 + 4K\lambda(2-s)(1+s) + 4F(2-s)(1-s)}{4(2-s)} \tag{13}$$
$$U_d^{FL} = \frac{(1-s)\{[1-2(c+\lambda e)]^2 + 4K\lambda(2-s)^2 - 4F(2-s)^2\}}{4(2-s)^2} \tag{14}$$

In fixed fee licensing stage, by Eq. (13) and Eq. (14), the upstream firm's decision problem on the fixed fee can be rewritten as:

$$\begin{aligned} &\max_{F} U_u^{FL} = \frac{[1-2(c+\lambda e)]^2 + 4K\lambda(2-s)(1+s) + 4F(2-s)(1-s)}{4(2-s)} \\ &s.t.\, U_d^{FL} \geq U_d^{*N} \end{aligned} \tag{15}$$

The above Eq. (15) implies that for the upstream firm to maximize its profit through the optimal fixed fee for technology licensing, it must ensure that the downstream firm is willing to accept the take-it-or-leave-it licensing contract. In other words, the fixed fee contract is profitable to ensure that the downstream firm's profit is not less than that in the absence of licensing. Thus, the downstream firm's decision to accept or reject the license contract makes no difference when $U_d^{FL} = U_d^{*N}$. As a result, we can determine the optimal fixed fee for licensing and derive Corollary 3.

Corollary 3: *In the FL model, the optimal fixed fee for the licensing is given by:*

$F^{*FL} = \frac{(c\theta+er\lambda)[2-c(4+\theta)-\lambda e(4+r)]}{4(2-s)^2}$.

Corollary 3 carries a significant implication: when determining the fixed fee for technology licensing, the upstream firm takes into account the impact of factors such as the difference in production costs, the difference in carbon emissions, the carbon trading price, and the shareholding. Accordingly, we derive the following proposition.

Proposition 2: $\frac{\partial F^{*FL}}{\partial r} > 0, \frac{\partial F^{*FL}}{\partial \theta} > 0, \frac{\partial F^{*FL}}{\partial s} > 0$, *and* $\frac{\partial F^{*FL}}{\partial \lambda} > (\leq) 0$ *if* $\lambda < (\geq) \frac{r(1-2c-\theta)-2c\theta}{er(4+r)}$.

Proposition 2 implies that as both the production cost differential and carbon emission differential increase, the upstream firm will charge a higher fixed fee payment for technology licensing. This is attributed to the superior efficacy of carbon neutrality technology in reducing carbon emissions and production costs. Furthermore, Proposition 2 demonstrates that a larger shareholding by the upstream firm leads to a higher fixed fee. However, if the carbon trading price is sufficiently low, an increase in the carbon trading price drives the upstream firm to opt for a higher fixed fee. If not, the upstream firm offers a lower fixed fee licensing agreement.

Using the optimal fixed fee, we can derive the following corollary.

Corollary 4: *In the FL model, the equilibrium results are given by:*

$$w^{*FL} = \frac{1-s(1-c-\lambda e)}{2-s}, \quad p^{*FL} = \frac{3-2s+2(c+\lambda e)}{2-s}, \quad q^{*FL} = \frac{1-2(c+\lambda e)}{2(2-s)},$$
$$U_u^{*FL} = \frac{3[1-2(c+\lambda e)]^2+4K\lambda(2-s)^2(1+s)-(1-s)[1-c(2+\theta)-\lambda e(2+r)]^2}{4(2-s)^2},$$
$$U_d^{*FL} = \frac{(1-s)\{[1-c(2+\theta)-\lambda e(2+r)]^2+4K\lambda(2-s)^2\}}{4(2-s)^2},$$
$$U_c^{*FL} = U_u^{*FL} + U_d^{*FL} = \frac{3[1-2(c+\lambda e)]^2+8K\lambda(2-s)^2}{4(2-s)^2}, \quad CS^{*FL} = \frac{1}{2}(q^{*FL})^2 = \frac{[1-2(c+\lambda e)]^2}{8(2-s)^2}.$$

Proposition 3: $w^{*FL} > w^{*N}, p^{*FL} < p^{*N}, q^{*FL} > q^{*N}, U_u^{*FL} > U_u^{*N}, U_d^{*NL} = U_d^{*N}, U_c^{*FL} > U_c^{*N}$, *and* $CS^{*FL} > CS^{*N}$.

Proposition 3 demonstrates that licensing carbon neutrality technology through a fixed fee contract motivates the upstream firm to raise the wholesale price and the downstream firm to lower the market price, resulting in increased consumer demand and consequently higher consumer surplus and higher profits for the upstream firm and supply chain system. However, under fixed fee licensing, the upstream firm can capture all the incremental profit generated by the technology licensing while ensuring the downstream firm's compliance with the agreement.

4.4 The RL model

Suppose that the upstream firm licenses its carbon neutrality technology to the downstream firm in exchange for a royalty payment. The royalty rate per unit of product is denoted as $\delta(> 0)$. Similarly, we can describe the initial profit functions of the upstream and downstream firms under royalty licensing as follows (the superscript "RL" denotes the royalty licensing model):

$$\pi_u^{RL} = (w-c)q - \lambda(eq-K) + \delta q \qquad (16)$$
$$\pi_u^{RL} = (w-c)q - \lambda(eq-K) + \delta \qquad (17)$$

By Eq. (16) and Eq. (17), we can characterize the decision problems of the supply chain firms as follows:

$$\pi_u^{RL} = (w-c)q - \lambda(eq-K) + \delta q \qquad (18)$$
$$\max_p U_d^{RL} = (1-s)\pi_d^{RL} \qquad (19)$$

We solve the Stackelberg game by backward induction. First, according to the downstream firm's optimal market price decision to maximize profit, solving $\frac{\partial U_d^{RL}}{\partial p} = 0$ yields $p = \frac{1}{2}(1+c+w+\delta+e\lambda)$. Then, according to the optimal wholesale price decision of the upstream firm for maximizing profit, solving $\frac{\partial U_u^{RL}}{\partial w} = 0$ obtains $w = \frac{1-2\delta+s(1-c-\delta-e\lambda)}{2-s}$. Accordingly, at the royalty licensing stage, the profit functions of the supply chain entities can be rewritten as:

$$U_u^{RL} = \frac{[1-2(c+\lambda e)]^2 + 4K\lambda(2-s)(1+s)}{4(2-s)} \qquad (20)$$
$$U_d^{RL} = \frac{(1-s)\{[1-2(c+\lambda e)]^2 + 4K\lambda(2-s)^2\}}{4(2-s)^2} \qquad (21)$$

Eq. (20) and Eq. (21) indicate that the profits of both the upstream and downstream firms are independent of the royalty rate. The rationale behind this is that the upstream firm adjusts the wholesale price and fully reimburses the royalties paid by the downstream firm for utilizing the licensed technology. Of course, when royalty payments are countered by a lower wholesale price, the downstream firm doesn't have any incentive to raise the market price, with the result that consumer demand is unaffected by the royalty, and so are the supply chain firms' profits.

Following that, we can deduce all of the equilibrium results for the royalty licensing setting and obtain the following corollary.

Corollary 5: *In the RL model, the equilibrium results are given by:*

$$w^{*RL}=\frac{1-2\delta+s(1-c-\delta-e\lambda)}{2-s},\quad p^{*RL}=\frac{3-2s+2(c+\lambda e)}{2-s},\quad q^{*RL}=\frac{1-2(c+\lambda e)}{2(2-s)},$$
$$U_u^{*RL}=\frac{[1-2(c+\lambda e)]^2+4K\lambda(2-s)(1+s)}{4(2-s)},\ U_d^{*RL}=\frac{(1-s)\{[1-2(c+\lambda e)]^2+4K\lambda(2-s)^2\}}{4(2-s)^2},$$
$$U_c^{*RL}=U_u^{*RL}+U_d^{*RL}=\frac{3[1-2(c+\lambda e)]^2+8K\lambda(2-s)^2}{4(2-s)^2},\ CS^{*RL}=\frac{1}{2}(q^{*SL})^2=\frac{[1-2(c+\lambda e)]^2}{8(2-s)^2}.$$

Proposition 4: $w^{*RL}\geq(<)w^{*N}$ *if* $\delta\leq(>)\frac{(1-s)(c\theta+er\lambda)}{2-s}$, $p^{*RL}<p^{*N}$, $q^{*RL}>q^{*N}$, $U_u^{*RL}>U_u^{*N}$, $U_d^{*RL}>U_d^{*N}$, $U_c^{*RL}>U_c^{*N}$, *and* $CS^{*RL}>CS^{*N}$.

Proposition 4 demonstrates that whether the wholesale price is greater or lower in royalty licensing, compared to the benchmark model, is primarily determined by the value of the royalty. Specifically, if the royalty rate is sufficiently low (high), the upstream firm decides on a greater (lower) wholesale price in the royalty licensing scenario. Nevertheless, the downstream firm tends to lower the market price due to reduced carbon trading and production costs attributed to technology licensing. Without a doubt, this contributes to consumer demand, resulting in larger profits for supply chain firms and the supply chain system, as well as increased consumer surplus. Therefore, royalty licensing is a viable and acceptable strategy for both supply chain firms.

4.5 The SL model

The SL model considers the case where the upstream firm licenses its carbon neutrality technology to the downstream firm through a revenue-sharing contract. Assuming that the revenue-sharing rate is $\xi(>0)$. Correspondingly, under revenue-sharing licensing, the initial profit functions of the upstream and downstream firms are characterized by (the superscript "SL" denotes the revenue-sharing licensing model):

$$\pi_u^{SL}=(w-c)q-\lambda(eq-K)+\xi pq \tag{22}$$
$$\pi_d^{SL}=(p-w-c)q-\lambda(eq-K)-\xi pq \tag{23}$$

By Eq. (22) and Eq. (23), the decision problems of the upstream and downstream firms are described as:

$$\max_{\xi,w}U_u^{SL}=\pi_u^{SL}+s\pi_d^{SL} \tag{24}$$
$$\max_{p}U_d^{SL}=(1-s)\pi_d^{SL} \tag{25}$$

We employ backward induction to solve the Stackelberg game. First, the downstream firm chooses its optimal market price to maximize profit. According to $\frac{\partial U_d^{SL}}{\partial p}=0$, we derive $p=\frac{1+c+w+\lambda e-\xi}{2(1-\xi)}$. Following that, the upstream firm determines the optimal wholesale price to maximize profit. According to $\frac{\partial U_u^{SL}}{\partial w}=0$, we obtain $w=\frac{1-(2+c+e\lambda)\xi+\xi^2-s(1-\xi)(1-c-e\lambda-\xi)}{2-s(1-\xi)-\xi}$. Therefore, the corresponding profits of the upstream and the downstream firms are rewritten as follows:

$$U_u^{SL}=\frac{[1-2(c+e\lambda)]^2+4K\lambda(2-s)(1+s)-4K\lambda\xi(1+s)(1-s)}{4[2-s-\xi(1-s)]} \tag{26}$$

$$U_d^{SL}=\frac{\begin{bmatrix}(1-s)\{[1-2(c+e\lambda)]^2+4K\lambda(2-s)^2\}\\-\xi(1-s)\{[1-2(c+e\lambda)]^2+8K\lambda(2-s)(1-s)-4K\lambda\xi(1-s)^2\}\end{bmatrix}}{4[2-s-\xi(1-s)]^2} \tag{27}$$

At the revenue-sharing licensing stage, the upstream firm chooses the optimal revenue-sharing rate to maximize profit while also ensuring the downstream firm's incentive to accept the revenue-sharing contract for technology licensing. As a result, by Eq. (26) and Eq. (27), the upstream firm's decision problem on the revenue-sharing rate can be rewritten as follows:

$$\max_{\xi}U_u^{SL}=\frac{[1-2(c+e\lambda)]^2+4K\lambda(2-s)(1+s)-4K\lambda\xi(1+s)(1-s)}{4[2-s-\xi(1-s)]} \tag{28}$$
$$s.t.\,U_d^{SL}\geq U_d^{*N}$$

It is easy to see, from Eq. (26) and Eq. (27), that as the revenue-sharing rate increases, the upstream firm's profit increases while the downstream firm's profit decreases. Therefore, we can deduce the optimal revenue-sharing rate from $U_d^{SL} = U_d^{*N}$ and obtain the following corollary.

Corollary 6: *In the SL model, the optimal revenue-sharing rate for licensing is given by:*

$$\xi^{*SL} = \frac{2(2-s)(1-s)[1-c(2+\theta)-\lambda e(2+r)]^2-(2-s)^2(1-2c-2\lambda e)^2}{2(1-s)^2[1-c(2+\theta)-\lambda e(2+r)]^2} + \frac{(2-s)(1-2c-2\lambda e)\sqrt{(2-s)^2(1-2c-2\lambda e)^2-4(1-s)[1-c(2+\theta)-\lambda e(2+r)]^2}}{2(1-s)^2[1-c(2+\theta)-\lambda e(2+r)]^2}.$$

The result of Corollary 6 implies that when determining the revenue-sharing rate, the upstream firm will take into account factors such as the difference in carbon emissions, the difference in production costs, the carbon trading price, and the shareholding. However, due to the complexity of expressing the optimal revenue-sharing rate, we conduct a numerical study to examine the influence of these parameters and derive the following proposition.

Observation 1: $\frac{\partial \xi^{*SL}}{\partial \theta} > 0$, $\frac{\partial \xi^{*SL}}{\partial r} > 0$, $\frac{\partial \xi^{*SL}}{\partial \lambda} > 0$, and $\frac{\partial \xi^{*SL}}{\partial s} < 0$.

Observation 5 summarizes the empirical results presented in Table 1. These results show that as the differences in unit carbon emissions and unit production costs increase, the downstream firm is contractually required to allocate a greater share of its revenue to the upstream firm. This is due to the *i* increased efficacy of carbon neutrality technology in reducing the downstream firm's costs. Moreover, a higher carbon trading price implies that technology licensing proves more advantageous in reducing the downstream firm's carbon trading expenses, thus prompting the upstream firm to demand a greater revenue share. However, if the upstream firm holds a larger ownership stake in the downstream firm, it opts for a lower revenue-sharing rate. This strategic choice aims to stimulate the downstream firm to lower the market price, thereby boosting consumer demand. Naturally, this creates an opportunity for the upstream firm to benefit more from the downstream firm's increased profit.

Table 1
The impact of relevant parameters on the optimal revenue-sharing rate
$(K = 0.01, c = 0.1, e = 0.1, \theta = 0.05, r = 0.03, \lambda = 0.2, s = 0.3)$

θ	ξ^{*SL}	r	ξ^{*SL}	λ	ξ^{*SL}	s	ξ^{*SL}
0.1	0.129	0.1	0.089	0.1	0.068	0.1	0.148
0.3	0.279	0.3	0.129	0.3	0.078	0.2	0.102
0.5	0.381	0.5	0.166	0.5	0.089	0.3	0.073
0.7	0.459	0.7	0.198	0.7	0.101	0.4	0.055
0.9	0.522	0.9	0.228	0.9	0.114	0.5	0.042

Corollary 7: *In the SL model, the equilibrium results are given by:*

$$w^{*SL} = \frac{1-s(1-c-e\lambda)-\xi^{*SL}[4-\xi^{*SL}(1-s)-(1+s)(2-c-e\lambda)]}{2-s-\xi^{*SL}(1-s)}, \quad p^{*SL} = \frac{3-2s+2(c+\lambda e)-2\xi^{*SL}(1-s)}{2[2-s-\xi^{*SL}(1-s)]}$$

$$q^{*SL} = \frac{1-2(c+\lambda e)}{2[2-s-\xi^{*SL}(1-s)]}, \quad U_u^{*SL} = \frac{[1-2(c+\lambda e)]^2+4K\lambda(2-s)(1+s)-4K\lambda\xi^{*SL}(1+s)(1-s)}{4[2-s-\xi^{*SL}(1-s)]},$$

$$U_d^{*SL} = \frac{(1-s)\{[1-2(c+\lambda e)]^2+4K\lambda(2-s)^2\}-\xi^{*SL}(1-s)\{[1-2(c+\lambda e)]^2+8K\lambda(2-s)(1+s)-4K\lambda\xi^{*SL}(1-s)^2\}}{4[2-s-\xi^{*SL}(1-s)]^2},$$

$$U_c^{*SL} = U_u^{*SL} + U_d^{*SL} = \frac{[1-2(c+e\lambda)]^2[3-2s(1-\xi^{*SL})-2\xi^{*SL}]+9K\lambda[2-s-\xi^{*SL}(1-s)]^2}{4[2-s-\xi^{*SL}(1-s)]^2},$$

$$CS^{*SL} = \frac{1}{2}(q^{*SL})^2 = \frac{[1-2(c+e\lambda)]^2}{9[2-s-\xi^{*SL}(1-s)]^2}.$$

Table 2
Comparison of equilibrium wholesale price
$(K = 0.01, c = 0.1, e = 0.1, \theta = 0.05, r = 0.03, \lambda = 0.2)$

s	w^{*SL}	w^{*N}
0.1	0.365	0.477
0.2	0.381	0.455
0.3	0.379	0.431
0.4	0.366	0.403
0.5	0.346	0.371

Following that, we compare the equilibrium results of the SL model with the benchmark and derive the following proposition. It should be highlighted that as the computational challenges and complexities, the comparative analysis of equilibrium wholesale prices is conducted by a numerical study, as illustrated in Table 2.

Proposition 5: $w^{*SL} < w^{*N}$, $p^{*SL} < p^{*N}$, $q^{*SL} > q^{*N}$, $U_u^{*SL} > U_u^{*N}$, $U_d^{*SL} = U_d^{*N}$, $U_c^{*SL} > U_c^{*N}$, *and* $CS^{*SL} > CS^{*N}$.

Proposition 5 shows that compared to the benchmark scenario, the upstream firm decides on a relatively lower wholesale price under revenue-sharing licensing. This differs significantly from free licensing and fixed fee licensing, where the upstream firm typically raises the wholesale price as a result of technology licensing. Obviously, through revenue-sharing licensing, the upstream firm can increase the downstream firm's profit by choosing a lower wholesale price, thereby maximizing its own revenue share. Consequently, the lower wholesale price, coupled with reduced carbon trading and production costs, empowers the downstream firm to determine a lower market price. This, in turn, leads to heightened consumer demand, increased consumer surplus, and improved profits for the upstream firm and the entire supply chain system. However, the profit of the downstream firm remains unchanged.

5. Comparative statics

In this section, we primarily investigate the impact of the difference in carbon emissions, the difference in production costs, carbon trading price, carbon emission quota, and shareholding on the operational decisions and profits of supply chain firms under both non-licensing and licensing models.

5.1 Effect of the difference in carbon emissions

Proposition 6: *(1) under the benchmark,* $\frac{\partial w^{*N}}{\partial r} < 0$, $\frac{\partial p^{*N}}{\partial r} > 0$, $\frac{\partial q^{*N}}{\partial r} < 0$, $\frac{\partial U_u^{*N}}{\partial r} < 0$, $\frac{\partial U_d^{*N}}{\partial r} < 0$, $\frac{\partial U_c^{*N}}{\partial r} < 0$, *and* $\frac{\partial CS^{*N}}{\partial r} < 0$*;*
(2) under the NL model, $\frac{\partial w^{*NL}}{\partial r} = 0$, $\frac{\partial p^{*NL}}{\partial r} = 0$, $\frac{\partial q^{*NL}}{\partial r} = 0$, $\frac{\partial U_u^{*NL}}{\partial r} = 0$, $\frac{\partial U_d^{*NL}}{\partial r} = 0$, $\frac{\partial U_c^{*NL}}{\partial r} = 0$, *and* $\frac{\partial CS^{*NL}}{\partial r} = 0$*;*
(3) under the FL model, $\frac{\partial w^{*FL}}{\partial r} = 0$, $\frac{\partial p^{*FL}}{\partial r} = 0$, $\frac{\partial q^{*FL}}{\partial r} = 0$, $\frac{\partial U_u^{*FL}}{\partial r} > 0$, $\frac{\partial U_d^{*FL}}{\partial r} < 0$, $\frac{\partial U_c^{*FL}}{\partial r} = 0$, *and* $\frac{\partial CS^{*FL}}{\partial r} = 0$*;*
(4) under the RL model, $\frac{\partial w^{*RL}}{\partial r} = 0$, $\frac{\partial p^{*RL}}{\partial r} = 0$, $\frac{\partial q^{*RL}}{\partial r} = 0$, $\frac{\partial U_u^{*RL}}{\partial r} = 0$, $\frac{\partial U_d^{*RL}}{\partial r} = 0$, $\frac{\partial U_c^{*RL}}{\partial r} = 0$, *and* $\frac{\partial CS^{*RL}}{\partial r} = 0$.

Indeed, an increase in the difference in carbon emissions implies that the downstream firm emits more carbon emissions, resulting in higher carbon trading costs. Consequently, as Proposition 6 shows in the benchmark model, the upstream firm opts to lower the wholesale price to mitigate the downstream firm's purchasing cost, while the downstream firm chooses to raise the market price due to increased carbon trading costs. As a result of the increased market price, consumer demand decreases, resulting in lower profits for both supply chain firms as well as lower consumer surplus and supply chain system profit. However, the difference in carbon emissions does not exert any influence on the operational decisions and corresponding profits of the supply chain under the NL and RL models. It is intuitive that if the difference in carbon emissions is bridged through technology licensing of carbon neutrality technology, then neither firm would consider the difference in their operational decisions. In contrast to the NL and RL models, the results in the FL model show that as the difference in carbon emissions increases, the upstream firm's profit increases while the downstream firm's profit decreases. This is due to the upstream firm taking into account the difference in carbon emissions when formulating the technology licensing agreement. As the difference increases, the upstream firm charges a higher fixed fee for technology licensing, resulting in increased profit for the upstream firm and decreased profit for the downstream firm. It is evident that the fixed fee serves as a mechanism for allocating supply chain system profit between the upstream and downstream firms, thereby keeping supply chain system profit unaltered by the difference in carbon emissions.

Next, as the complexity of calculations, we conduct a numerical study to examine the impact of the difference in carbon emissions on the equilibrium results in the SL model. Table 3 summarizes the main results, from which we can draw the following observations.

Table 3
Impact of r on equilibrium results under SL model
($K = 0.01, c = 0.1, e = 0.1, \theta = 0.05, \lambda = 0.2, s = 0.3$)

r	w^{*SL}	p^{*SL}	q^{*SL}	U_u^{*SL}	U_d^{*SL}	U_c^{*SL}	CS^{*SL}
0.1	0.368	0.768	0.232	0.0908	0.0357	0.1265	0.0269
0.3	0.339	0.764	0.236	0.0923	0.0354	0.1277	0.0279
0.5	0.314	0.760	0.240	0.0938	0.0350	0.1288	0.0288
0.7	0.291	0.757	0.243	0.0951	0.0346	0.1297	0.0296
0.9	0.271	0.753	0.247	0.0963	0.0343	0.1306	0.0304

Observation 2: *Under the SL model, as the difference in carbon emissions (r) increases, the wholesale price (w^{*SL}) decreases, the market price (p^{*SL}) decreases, the consumer demand (q^{*SL}) increases, the upstream firm's profit (U_u^{*SL}) increases, the downstream firm's profit (U_d^{*SL}) decreases, the supply chain system profit (U_c^{*SL}) increases, the consumer surplus (CS^{*SL}) increases.*

It is worth noting that as the difference in carbon emissions increases, the upstream firm sets a higher revenue-sharing rate, allowing it to benefit more from the downstream firm's sales revenue. This prompts the upstream firm to reduce the wholesale price accordingly. On the other hand, the greater the difference in carbon emissions, the more the downstream firm can save on carbon trading costs through technology licensing, motivating the downstream firm to lower the market price and, as a result, increasing consumer demand, consumer surplus, and the profits of both the upstream firm and the supply chain system. However, the downstream firm's profit is diminished due to the fact that it has to split more profit with the upstream firm.

5.2 Effect of the difference in production costs

Proposition 7: *(1) under the benchmark,* $\frac{\partial w^{*N}}{\partial \theta} < 0$, $\frac{\partial p^{*N}}{\partial \theta} > 0$, $\frac{\partial q^{*N}}{\partial \theta} < 0$, $\frac{\partial U_u^{*N}}{\partial \theta} < 0$, $\frac{\partial U_d^{*N}}{\partial \theta} < 0$, $\frac{\partial U_c^{*N}}{\partial \theta} < 0$, *and* $\frac{\partial CS^{*N}}{\partial \theta} < 0$*;*
(2) under the NL model, $\frac{\partial w^{*NL}}{\partial \theta} = 0$, $\frac{\partial p^{*NL}}{\partial \theta} = 0$, $\frac{\partial q^{*NL}}{\partial \theta} = 0$, $\frac{\partial U_u^{*NL}}{\partial \theta} = 0$, $\frac{\partial U_d^{*NL}}{\partial \theta} = 0$, $\frac{\partial U_c^{*NL}}{\partial \theta} = 0$, *and* $\frac{\partial CS^{*NL}}{\partial \theta} = 0$*;*
(3) under the FL model, $\frac{\partial w^{*FL}}{\partial \theta} = 0$, $\frac{\partial p^{*FL}}{\partial \theta} = 0$, $\frac{\partial q^{*FL}}{\partial \theta} = 0$, $\frac{\partial U_u^{*FL}}{\partial \theta} > 0$, $\frac{\partial U_d^{*FL}}{\partial \theta} < 0$, $\frac{\partial U_c^{*FL}}{\partial \theta} = 0$, *and* $\frac{\partial CS^{*FL}}{\partial \theta} = 0$*;*
(4) under the RL model, $\frac{\partial w^{*RL}}{\partial \theta} = 0$, $\frac{\partial p^{*RL}}{\partial \theta} = 0$, $\frac{\partial q^{*RL}}{\partial \theta} = 0$, $\frac{\partial U_u^{*RL}}{\partial \theta} = 0$, $\frac{\partial U_d^{*RL}}{\partial \theta} = 0$, $\frac{\partial U_c^{*RL}}{\partial \theta} = 0$, *and* $\frac{\partial CS^{*RL}}{\partial \theta} = 0$.

Comparing the results in Proposition 7 to those in Proposition 6, it is evident that the effects of the difference in production costs on the equilibrium results across the benchmark, NL, FL, and RL models align entirely with the results outlined in Proposition 6. This consistency can be attributed to a straightforward rationale. A rise in the difference in production costs indicates that the downstream firm must invest more in its production processes. Similarly, a larger difference in carbon emissions corresponds to increased expenses associated with carbon trading. Therefore, the difference in production costs exerts the same impact on the operational decisions and corresponding profits of the supply chain as the difference in carbon emissions.

Next, we delve into the influence of the difference in production costs on the equilibrium results under the SL model through a numerical study. Table 4 shows the numerical results, which are also summarized in Observation 3. Notably, these results align closely with those in Observation 2, allowing us to explain them in a similar manner.

Table 4
Impact of θ on equilibrium results under SL model
($K = 0.01, c = 0.1, e = 0.1, r = 0.03, \lambda = 0.2, s = 0.3$)

θ	w^{*SL}	p^{*SL}	q^{*SL}	U_u^{*SL}	U_d^{*SL}	U_c^{*SL}	CS^{*SL}
0.1	0.34	0.764	0.236	0.092	0.035	0.128	0.028
0.3	0.24	0.747	0.253	0.099	0.034	0.132	0.032
0.5	0.17	0.735	0.265	0.103	0.032	0.135	0.035
0.7	0.12	0.724	0.276	0.107	0.030	0.138	0.038
0.9	0.09	0.715	0.285	0.111	0.029	0.139	0.041

Observation 3: *Under the SL model, as the difference in unit production costs (θ) increases, the wholesale price (w^{*SL}) increases, the market price (p^{*SL}) decreases, the consumer demand (q^{*SL}) increases, the upstream firm's profit (U_u^{*SL}) increases, the downstream firm's profit (U_d^{*SL}) decreases, the supply chain system profit (U_c^{*SL}) increases, the consumer surplus (CS^{*SL}) increases.*

5.3 Effect of carbon trading price

Denote $s_1 = \frac{3(1-2c-2\lambda e)-(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(1-2c-2\lambda e)}$, $K_1 = \frac{e(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(1+s)(2-s)}$, $K_2 = \frac{e(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2}$, $K_3 = \frac{e(2+r)(3-2s)[1-c(2+\theta)-\lambda e(2+r)]}{4(2-s)^2}$, $K_4 = \frac{e(1-2c-2\lambda e)}{(1+s)(2-s)}$, $K_5 = \frac{e(1-2c-2\lambda e)}{(2-s)^2}$, $K_6 = \frac{e(3-2s)(1-2c-2\lambda e)}{2(2-s)^2}$, and $K_7 = \frac{2e(3-2s)(1-2c-2\lambda e)-e(1-s)(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(1+s)(2-s)^2}$.

Proposition 8: *(1) under the benchmark,* $\frac{\partial w^{*N}}{\partial \lambda} \geq (<) 0$ *if* $r \leq (>) \frac{s}{1-s}$, $\frac{\partial p^{*N}}{\partial \lambda} > 0$, $\frac{\partial q^{*N}}{\partial \lambda} < 0$, $\frac{\partial CS^{*N}}{\partial \lambda} < 0$*; however, as for the profits,* $\frac{\partial U_u^{*N}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*N}}{\partial \lambda} < 0$, *and* $\frac{\partial U_c^{*N}}{\partial \lambda} < 0$ *if* $K < K_2$*;* $\frac{\partial U_u^{*N}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*N}}{\partial \lambda} \geq 0$, *and* $\frac{\partial U_c^{*N}}{\partial \lambda} < 0$ *if* $K_2 \leq K < K_3$*;* $\frac{\partial U_u^{*N}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*N}}{\partial \lambda} > 0$, *and* $\frac{\partial U_c^{*N}}{\partial \lambda} \geq 0$ *if* $K_3 \leq K < K_1$*;* $\frac{\partial U_u^{*N}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*N}}{\partial \lambda} > 0$, *and* $\frac{\partial U_c^{*N}}{\partial \lambda} > 0$ *if* $K \geq K_1$.
(2) under the NL model, $\frac{\partial w^{*NL}}{\partial \lambda} > 0$, $\frac{\partial p^{*NL}}{\partial \lambda} > 0$, $\frac{\partial q^{*NL}}{\partial \lambda} < 0$, $\frac{\partial CS^{*NL}}{\partial \lambda} < 0$*; however, as for the profits,* $\frac{\partial U_u^{*NL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*NL}}{\partial \lambda} < 0$, *and* $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ *if* $K < K_5$*;* $\frac{\partial U_u^{*NL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*NL}}{\partial \lambda} \geq 0$, *and* $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ *if* $K_5 \leq K < K_6$*;* $\frac{\partial U_u^{*NL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*NL}}{\partial \lambda} > 0$, *and* $\frac{\partial U_c^{*NL}}{\partial \lambda} \geq 0$ *if* $K_6 \leq K < K_4$*;* $\frac{\partial U_u^{*NL}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*NL}}{\partial \lambda} > 0$, *and* $\frac{\partial U_c^{*NL}}{\partial \lambda} > 0$ *if* $K \geq K_4$.
(3) under the FL model, $\frac{\partial w^{*FL}}{\partial \lambda} > 0$, $\frac{\partial p^{*FL}}{\partial \lambda} > 0$, $\frac{\partial q^{*FL}}{\partial \lambda} < 0$, $\frac{\partial CS^{*FL}}{\partial \lambda} < 0$*; however, as for the profit, ①when* $s \geq s_1$, $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, *and* $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ *if* $K < K_7$*;* $\frac{\partial U_u^{*FL}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, *and* $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ *if* $K_7 \leq K < K_6$*;* $\frac{\partial U_u^{*FL}}{\partial \lambda} > 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, *and*

$\frac{\partial U_c^{*NL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_2$; $\frac{\partial U_u^{*FL}}{\partial \lambda} > 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} > 0$ if $K \geq K_2$; ② when $s < s_1$, $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K < K_2$; $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K_2 \leq K < K_6$; $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} > 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_7$; $\frac{\partial U_u^{*FL}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} > 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} > 0$ if $K \geq K_7$.

(4) under the RL model, $\frac{\partial w^{*RL}}{\partial \lambda} > 0$, $\frac{\partial p^{*RL}}{\partial \lambda} > 0$, $\frac{\partial q^{*RL}}{\partial \lambda} < 0$, $\frac{\partial CS^{*RL}}{\partial \lambda} < 0$; however, as for the profits, $\frac{\partial U_u^{*RL}}{\partial \lambda} < 0$,$\frac{\partial U_d^{*RL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} < 0$ if $K < K_5$; $\frac{\partial U_u^{*RL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*RL}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} < 0$ if $K_5 \leq K < K_6$; $\frac{\partial U_u^{*RL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*RL}}{\partial \lambda} > 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_4$; $\frac{\partial U_u^{*RL}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*RL}}{\partial \lambda} > 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} > 0$ if $K \geq K_4$.

Proposition 8 highlights that the upstream firm's response to an increased carbon trading price in the benchmark depends on the difference in carbon emissions. If the difference in carbon emissions is small enough, indicating a low carbon emission per unit product for the downstream firm, the downstream firm's expenditure in purchasing extra carbon credits is low, or it may even benefit substantially from the sale of excess carbon quotas. As a result, the upstream firm is stimulated to raise the wholesale price. Otherwise, the upstream firm opts for a lower wholesale price. Nonetheless, under the NL, FL, and RL models, the upstream firm will undeviatingly choose to raise the wholesale price without factoring in the difference in carbon emissions. This intuitive outcome arises from the licensing of carbon neutrality technology, which promotes the downstream firm's carbon emission per unit product to the same level as the upstream firm.

Furthermore, both non-licensing and licensing models promote the downstream firm to invariably raise the market price in response to the increasing carbon trading price, leading to a decrease in both consumer demand and consumer surplus. Proposition 8 reveals an intriguing phenomenon: irrespective of additional costs or benefits associated with carbon emission quotas, supply chain firms are inclined to raise their pricing decisions as a strategy to cope with the increasing carbon trading price. However, a serious consequence is that consumer demand falls. Therefore, Proposition 8 implies an important managerial insight: regulators must implement effective policies and strategies to control rising carbon trading prices. Failure to do so would shake supply chain firms' confidence in product production and weaken consumers' motivation to make purchases.

Finally, Proposition 8 demonstrates whether the supply chain firms' profits increase or decrease with the carbon trading price is influenced by the carbon emission quotas. Specifically, when the carbon emission quota is sufficiently high or low, both supply chain firms and the overall system experience increases or decreases profits across the benchmark, NL, FL, and RL models. It is evident that a higher carbon emission quota corresponds to greater economic benefit from selling excess carbon quotas or reduced expenditures in purchasing extra carbon credits, both of which contribute to increased supply chain profits. Conversely, a lower carbon emission quota has the inverse effect. However, in the benchmark, NL, and RL models, if the carbon emission quota is moderate, the downstream firm's profit increases while the upstream firm's profit decreases. This indicates a decrease in the overall supply chain system profit initially, followed by a subsequent increase. This suggests that, although the upstream firm can benefit from profit sharing through ownership in the downstream firm, it faces a disadvantage when confronted with the rising carbon trading price. But it's not absolute under the FL model. The FL model demonstrates that when the upstream firm holds enough downstream firm shares and the carbon emission quota is moderate, the downstream firm's profit decreases while the upstream firm's profit increases as the carbon trading price rises.

Tables 5-7 provide a numerical analysis examining the impact of the carbon trading price on the equilibrium results in the SL model. The corresponding primary results are summarized in the following Observation 4.

Observation 4: *Under the SL model, as the carbon trading price (λ) increases, regardless of the carbon emissions quotas, both the wholesale price (w^{*SL}) and the market price (p^{*SL}) increase, but the consumer demand (q^{*SL}) and the consumer surplus (CS^{*SL}) decrease; when the carbon emission quota is low enough ($K = 0.01$), the profits of the supply chain firms (U_u^{*SL} and U_d^{*SL}) and the supply chain system (U_c^{*SL}) decrease; when the carbon emissions quota is high enough ($K = 0.08$), the profits of the supply chain firms (U_u^{*SL} and U_d^{*SL}) and the supply chain system (U_c^{*SL}) increase; however, when the carbon emissions quota is moderate ($K = 0.028$), the upstream firm's profit (U_u^{*SL}) decreases, the downstream firm's profit (U_d^{*SL}) increases, while the supply chain system profit (U_c^{*SL}) will decrease first and then increase.*

The numerical results for the SL model, similar to the NL, FL, and RL models, show that increasing the carbon trading price stimulates the upstream firm to raise both the wholesale price and the revenue-sharing rate, thereby incentivizing the downstream firm to increase the market price. This diminishes both consumer demand and surplus. In addition, the numerical results highlight the essential role of carbon emission quotas in shaping the influence of the carbon trading price on the profits of supply chain firms and the supply chain system. The results are equally completely compatible with the NL and RL models, which may be interpreted similarly.

Table 5
Impact of λ on equilibrium results under SL model ($K = 0.01$)

$(K = 0.01, c = 0.1, e = 0.1, \theta = 0.05, r = 0.03, s = 0.3)$

λ	w^{*SL}	p^{*SL}	q^{*SL}	U_u^{*SL}	U_d^{*SL}	U_c^{*SL}	CS^{*SL}
0.1	0.382	0.764	0.236	0.093	0.037	0.130	0.028
0.3	0.377	0.775	0.225	0.087	0.035	0.122	0.025
0.5	0.371	0.786	0.214	0.081	0.033	0.114	0.023
0.7	0.364	0.797	0.203	0.076	0.031	0.107	0.021
0.9	0.357	0.809	0.191	0.071	0.029	0.100	0.018

Table 6

Impact of λ on equilibrium results under SL model $(K = 0.028)$

$(K = 0.028, c = 0.1, e = 0.1, \theta = 0.05, r = 0.03, s = 0.3)$

λ	w^{*SL}	p^{*SL}	q^{*SL}	U_u^{*SL}	U_d^{*SL}	U_c^{*SL}	CS^{*SL}
0.1	0.382	0.764	0.236	0.0957	0.0383	0.1340	0.028
0.3	0.377	0.775	0.225	0.0941	0.0385	0.1326	0.025
0.5	0.371	0.786	0.214	0.0930	0.0389	0.1319	0.023
0.7	0.364	0.797	0.203	0.0923	0.0395	0.1319	0.021
0.9	0.357	0.809	0.191	0.0921	0.0403	0.1324	0.018

Table 7

Impact of λ on equilibrium results under SL model $(K = 0.08)$

$(K = 0.08, c = 0.1, e = 0.1, \theta = 0.05, r = 0.03, s = 0.3)$

λ	w^{*SL}	p^{*SL}	q^{*SL}	U_u^{*SL}	U_d^{*SL}	U_c^{*SL}	CS^{*SL}
0.1	0.382	0.764	0.236	0.102	0.042	0.144	0.028
0.3	0.377	0.775	0.225	0.114	0.049	0.164	0.025
0.5	0.371	0.786	0.214	0.127	0.057	0.184	0.023
0.7	0.364	0.797	0.203	0.140	0.065	0.205	0.021
0.9	0.357	0.809	0.191	0.153	0.073	0.226	0.018

5.4 Effect of shareholding

Proposition 9: *(1) under the benchmark,* $\frac{\partial w^{*N}}{\partial s} < 0, \frac{\partial p^{*N}}{\partial s} < 0, \frac{\partial q^{*N}}{\partial s} > 0, \frac{\partial U_u^{*N}}{\partial s} > 0, \frac{\partial U_d^{*N}}{\partial s} < 0, \frac{\partial U_c^{*N}}{\partial s} > 0,$ *and* $\frac{\partial CS^{*N}}{\partial s} > 0.$

(2) under the NL model, $\frac{\partial w^{*NL}}{\partial s} < 0, \frac{\partial p^{*NL}}{\partial s} < 0, \frac{\partial q^{*NL}}{\partial s} > 0, \frac{\partial U_u^{*NL}}{\partial s} > 0, \frac{\partial U_d^{*NL}}{\partial s} < 0, \frac{\partial U_c^{*NL}}{\partial s} > 0,$ *and* $\frac{\partial CS^{*NL}}{\partial s} > 0.$

(3) under the FL model, $\frac{\partial w^{*FL}}{\partial s} < 0, \frac{\partial p^{*FL}}{\partial s} < 0, \frac{\partial q^{*FL}}{\partial s} > 0, \frac{\partial U_u^{*FL}}{\partial s} > 0, \frac{\partial U_d^{*FL}}{\partial s} < 0, \frac{\partial U_c^{*FL}}{\partial s} > 0,$ *and* $\frac{\partial CS^{*FL}}{\partial s} > 0.$

(4) under the RL model, $\frac{\partial w^{*RL}}{\partial s} < 0, \frac{\partial p^{*RL}}{\partial s} < 0, \frac{\partial q^{*RL}}{\partial s} > 0, \frac{\partial U_u^{*RL}}{\partial s} > 0, \frac{\partial U_d^{*RL}}{\partial s} < 0, \frac{\partial U_c^{*RL}}{\partial s} > 0,$ *and* $\frac{\partial CS^{*RL}}{\partial s} > 0.$

Intuitively, an increase in the upstream firm's shareholding in the downstream firm signifies an increase in the dividends received by the former. Thus, Proposition 9 demonstrates that, regardless of whether the upstream firm licenses the carbon neutrality technology to the downstream firm, it is permitted to sell the product at a lower wholesale price. This naturally induces the downstream firm to reduce the market price, resulting in increased consumer demand and surplus. In addition to increasing dividends, the increased consumer demand contributes to higher profitability for the upstream firm. However, it is not profitable for the downstream firm since a significant portion of the generated profit is distributed to the upstream firm as dividends. Even so, the profit of the supply chain system increases due to the decrease in the double margin. Further, Table 8 shows the numerical results on the impact of the upstream firm's shareholding in the downstream firm under the SL model, and the main conclusions are presented as follows.

Table 8

Impact of s on equilibrium results under the SL model

$(K = 0.01, c = 0.1, e = 0.1, \theta = 0.05, r = 0.03, \lambda = 0.2)$

s	w^{*SL}	p^{*SL}	q^{*SL}	U_u^{*SL}	U_d^{*SL}	U_c^{*SL}	CS^{*SL}
0.1	0.366	0.785	0.215	0.084	0.0373	0.1212	0.0231
0.2	0.381	0.779	0.221	0.086	0.0367	0.1232	0.0244
0.3	0.380	0.770	0.230	0.090	0.0359	0.1260	0.0266
0.4	0.367	0.758	0.242	0.095	0.0345	0.1295	0.0294
0.5	0.346	0.743	0.257	0.101	0.0326	0.1333	0.0330

Observation 5: *Under the SL model, as the share owned by the upstream firm to the downstream firm (s) increases, the wholesale price* (w^{*SL}) *increases first and then decreases, the market price* (p^{*SL}) *decreases, the consumer demand* (q^{*SL}) *increases, the upstream firm's profit* (U_u^{*SL}) *increases, the downstream firm's profit* (U_d^{*SL}) *decreases, the supply chain system profit* (U_c^{*SL}) *increases, and the consumer surplus* (CS^{*SL}) *increases.*

A comparison between Table 8 and Proposition 9 reveals a significant distinction within the SL model: the upstream firm raises the wholesale price if its shareholding in the downstream firm increases by a small margin. With the result presented in

Proposition 5 that the upstream firm's optimal revenue-sharing rate decreases as its stake in the downstream firm increases, if the benefits from increased shareholding are not substantial enough to offset the loss resulting from decreased revenue sharing, the upstream firm prioritizes profitability by setting a higher wholesale price. Remarkably, the downstream firm is immune to the upstream firm's increased wholesale price decision and consistently opts to lower the market price, influencing consumer demand, supply chain profitability, and consumer surplus in alignment with the trends observed in Proposition 9.

5.5 Effect of carbon emission quota

The above Section 4 clearly demonstrates that the carbon emission quota primarily influences the profits of supply chain firms, rather than their operational decisions. Building upon this understanding, we proceed to derive the following proposition.

Proposition 10:
(1) under the benchmark, $\frac{\partial U_u^{*N}}{\partial K} > 0, \frac{\partial U_d^{*N}}{\partial K} > 0,$ *and* $\frac{\partial U_c^{*N}}{\partial K} > 0.$
(2) under the NL model, $\frac{\partial U_u^{*NL}}{\partial K} > 0, \frac{\partial U_d^{*NL}}{\partial K} > 0,$ *and* $\frac{\partial U_c^{*NL}}{\partial K} > 0.$
(3) under the FL model, $\frac{\partial U_u^{*FL}}{\partial K} > 0, \frac{\partial U_d^{*FL}}{\partial K} > 0,$ *and* $\frac{\partial U_c^{*FL}}{\partial K} > 0.$
(4) under the RL model, $\frac{\partial U_u^{*RL}}{\partial K} > 0, \frac{\partial U_d^{*RL}}{\partial K} > 0,$ *and* $\frac{\partial U_c^{*RL}}{\partial K} > 0.$
(5) under the SL model, $\frac{\partial U_u^{*SL}}{\partial K} > 0, \frac{\partial U_d^{*SL}}{\partial K} > 0,$ *and* $\frac{\partial U_c^{*SL}}{\partial K} > 0.$

Proposition 10 shows that as the carbon emission quota increases, so do the profits for both the supply chain firms and the overall system, regardless of whether the upstream firm licenses its carbon neutrality technology to the downstream firm. The underlying rationale behind this is straightforward. The higher the carbon emission quota, the more carbon emissions the supply chain firms can emit. Consequently, supply chain firms can benefit financially by selling more excess carbon emission quotas or buying fewer extra carbon emission quotas, contributing to higher supply chain profitability. Proposition 10 indicates that supply chain firms can profit from a carbon policy wherein a regulator sets higher carbon emission quotas or puts no restrictions. However, such a policy may lead to increased carbon emissions and environmental challenges. Conversely, if a regulator sets a lower carbon emissions quota that curtails supply chain profitability, it may incentivize supply chain firms to invest in carbon neutrality technology to reduce carbon emissions, thereby mitigating the negative effect of lower quotas and improving the ecological environment.

6. Optimal carbon neutrality technology licensing contracts

This section delves into the examination of optimal licensing contracts for carbon neutrality technology from the perspectives of supply chain firms, consumers, and society. This analysis enables supply chain firms to understand the most profitable technology licensing contract and guides regulators in formulating carbon neutrality technology licensing policies that support green development and societal well-being.

6.1 Optimal licensing contract from consumers' perspective

Proposition 11: $CS^{*SL} > CS^{*RL} = CS^{*FL} = CS^{*NL} > CS^{*N}$.

Proposition 11 illustrates that technology licensing contributes to improved consumer surplus. This result is intuitive as technology licensing incentivizes the downstream firm to lower the market price by reducing production costs, thereby increasing consumer demand. Secondly, there is no difference in consumer surplus among the NL, FL, and RL models. Corollaries 2, 4, and 5 demonstrate that the fixed fee in the FL model merely serves to reallocate the total profit between the upstream and downstream firms without influencing the pricing decisions and hence consumer demand. Similarly, in the RL model, the upstream firm can remit fully the downstream firm's royalty payment by decreasing the wholesale price, leaving market price and consumer demand unchanged. Naturally, in the NL, FL, and RL models, consumer demand and the corresponding consumer surplus remain equivalent. Finally, the revenue-sharing licensing contract emerges as the optimal choice for maximizing consumer surplus. Evidently, compared to other licensing contracts, revenue sharing in the SL model maximizes the upstream firm's incentive to lower the wholesale price, resulting in the lowest market price and the highest consumer surplus.

6.2 Optimal licensing contract from supply chain firms' perspective

Proposition 12: $U_u^{*SL} > U_u^{*FL} > U_u^{*RL} = U_u^{*NL} > U_u^{*N}$, $U_d^{*RL} = U_d^{*NL} > U_d^{*SL} = U_d^{*FL} = U_d^{*N}$, *and* $U_c^{*SL} > U_c^{*FL} = U_c^{*RL} = U_c^{*NL} > U_u^{*N}$.

Proposition 12 demonstrates that all technology licensing contracts are conducive to improving the upstream firm's profit, and revenue-sharing licensing is the most effective among them. While fixed fee licensing is comparatively disadvantaged when compared to revenue-sharing licensing, it still dominates over royalty licensing and free licensing because it allows the upstream firm to capture all the profit while leaving the downstream firm's profit unchanged. However, there is no difference between royalty licensing and free licensing in terms of improving the upstream firm's profit. Similarly, both royalty licensing and free licensing have the same effect on the downstream firm's profit but are preferred by the downstream firm. Interestingly, the downstream firm does not exhibit a preference for either fixed fee or revenue-sharing licensing compared to the non-licensing case. This is because the upstream firm captures all the incremental profits generated by these licensing contracts, resulting in the downstream firm's profit being equivalent to the non-licensing situation. Revenue-sharing licensing is also deemed optimal for maximizing the supply chain system profit, whereas fixed fee licensing, royalty licensing, and free licensing have no difference in terms of improving the supply chain system profit. In summary, Proposition 12 highlights that the revenue-sharing licensing contract is the best option for the upstream firm and the entire supply chain. However, it is worth noting that the downstream firm does not reap any financial benefits from this licensing contract, thus it may be challenging to successfully implement this technology licensing agreement. Therefore, it is essential to design a technology licensing agreement that optimizes the profitability of the entire supply chain and ensures that both supply chain firms experience higher profits, such as a licensing agreement that combines revenue sharing with a fixed fee.

6.3 Optimal licensing contract from the society's perspective

The previous subsections have primarily focused on investigating optimal technology licensing contracts based on consumer welfare and the economic benefits for supply chain firms. However, the environmental impact of technology licensing has yet to be considered. In this subsection, we aim to address this gap by examining the optimal technology licensing contracts from the perspective of social welfare, which encompasses the supply chain system profit, consumer surplus, and the environmental impact of carbon emissions. Specifically, we define social welfare as $SW = U_c + CS - \Delta T$, where Δ denotes the environmental impact of per unit carbon emission, and T represents the total carbon emissions of the product throughout its production processes within the supply chain.

Proposition 13: (1) $SW^{*RL} = SW^{*FL} = SW^{*NL}$; If $\Delta< (\geq)\Delta_1$, then $SW^{*SL} > (\leq) SW^{*NL}$.
(2) If $e \leq e_1$, or $e > e_1$ and $\Delta< \Delta_2$, then $SW^{*NL} > SW^{*N}$; if $e > e_1$ and $\Delta\geq \Delta_2$, then $SW^{*NL} \leq SW^{*N}$ if $\Delta\geq \Delta_2$.
(3) If $H(e,r) \leq 0$, or $H(e,r) > 0$ and $\Delta< \Delta_3$, then $SW^{*SL} > SW^{*N}$; if $H(e,r) > 0$ and $\Delta\geq \Delta_3$, then $SW^{*SL} \leq SW^{*N}$.

Proposition 13 demonstrates that whether carbon neutrality technology licensing improves social welfare and which technology licensing contract is optimal to maximize social welfare are dependent on carbon emissions per unit product (e) and the environmental impact of each unit of carbon emission (Δ). Firstly, Proposition 13 illustrates that social welfare remains constant across the RL, FL, and NL models. The intuition behind this is that, compared to the NL model, the royalty payment in the RL model does not affect the decisions of the supply chain firms and their corresponding profitability. Similarly, the fixed fee in the FL model only plays a role in reallocating supply chain system profit. Consequently, there is no difference in social welfare between royalty licensing, fixed-fee licensing, and free licensing.

Further, Proposition 13 reveals that, although revenue-sharing licensing surpasses royalty licensing, fixed fee licensing, and free licensing in improving supply chain system profit and consumer surplus, this advantage does not extend to social welfare. The pivotal factor is the environmental impact of each unit of carbon emission. Carbon emissions undoubtedly can have an adverse effect on the environment and, the greater the environmental impact, the lower the social welfare. If the environmental impact of per unit carbon emission is low enough, revenue-sharing licensing remains the preferred contract for optimizing social welfare. Otherwise, a sufficiently large environmental impact paired with the highest consumer demand will result in the revenue-sharing licensing being less effective than others.

Finally, Proposition 13 demonstrates that, compared to the non-licensing case, social welfare is improved by royalty licensing, fixed-fee licensing, and free licensing if the carbon emission per unit product is low enough, or the carbon emission per unit product is high enough while the environmental impact of per unit carbon emission is low enough. Conversely, higher carbon emission per unit product coupled with a higher environmental impact of each unit of carbon emission implies a greater adverse environmental impact, resulting in worse social welfare under royalty licensing, fixed-fee licensing, and free licensing than under the non-licensing case. The same holds true when comparing revenue-sharing licensing to non-licensing.

Overall, Proposition 13 highlights that the revenue-sharing licensing contract is the best choice for optimizing social welfare among all carbon neutrality technology licensing contracts only when carbon emissions per unit product and the environmental impact of per unit carbon emission are both sufficiently low. Otherwise, royalty licensing, fixed fee licensing, and free licensing may outperform revenue-sharing licensing in terms of improving social welfare. To be worse, if the carbon emissions per unit product and the environmental impact of per unit carbon emission are extremely high, all technology licensing contracts may result in worse social welfare than the non-licensing case.

7. Conclusions

This study considers supply chain technology licensing under a cap-and-trade policy, specifically when the upstream firm holds partial ownership of the downstream firm and possesses a carbon neutrality technology advantage. By characterizing the effectiveness of technology licensing in reducing carbon emissions and production costs for the downstream firm, and by analyzing the equilibrium results under different technology licensing models, this study investigates the impact of factors such as differences in unit carbon emissions and in production costs, carbon trading prices, carbon emission quotas, and shareholding on the operational decisions and corresponding profits of supply chain firms. More importantly, this study delves into the question of whether licensing carbon neutrality technology contributes to improving supply chain profitability, consumer surplus, and overall social welfare. It also evaluated the optimal licensing contracts from the perspectives of supply chain firms, consumers, and society at large, by comparing the equilibria achieved through different technology licensing models. The key findings of this study can be summarized as follows.

(1) Compared to the non-licensing models, all technology licensing contracts contribute to the profitability of the upstream firm and the supply chain system, as well as consumer surplus. However, the downstream firm's profit improves only in the NL and RL models, while remaining unchanged in the FL and SL models. Nevertheless, whether carbon neutrality technology licensing promotes social welfare is contingent on the carbon emissions per unit of product and the environmental impact of per unit of carbon emission. If both of them are sufficiently high, technology licensing may even result in a decrease in social welfare when compared to the scenario of no licensing.

(2) Revenue-sharing licensing emerges as the optimal choice for maximizing consumer surplus, upstream firm profit, and supply chain system profit. On the other hand, the downstream firm tends to prioritize royalty and free licensing contracts. Among all technology licensing contracts, the social welfare remains the same under free, fixed fee, and royalty licensing. However, when it comes to revenue-sharing licensing, the social welfare is higher (or lower) compared to the above three licensing contracts, depending on whether the environmental impact of carbon emissions per unit is sufficiently low or high.

(3) Contrary to the benchmark, where increased differences in unit carbon emissions and production costs lead to a decrease in consumer demand and lower profits for both supply chain firms, these differences have no impact on the equilibria in the NL and RL models. However, in the FL model, higher differences in unit carbon emissions and production costs can result in a redistribution of the supply chain system profit between the upstream and downstream firms. Interestingly, in the SL model, these differences can even prove beneficial for customers and the overall supply chain system.

(4) A higher carbon trading price, regardless of the technology licensing or non-licensing contracts, motivates the downstream firm to raise the market price of the product. This, in turn, leads to a decrease in consumer demand and consumer surplus. However, when the carbon emission quota is set at a sufficiently high (or low) level, the profits of supply chain firms and the overall supply chain system experience an increase (or decrease) as the carbon trading price rises.

(5) Higher carbon emission quotas do not impact the operational decisions of supply chain firms, customer demand, or consumer surplus. However, they do contribute to an improvement in the profitability of supply chain firms. On the other hand, an increase in the upstream firm's shareholding in the downstream firm prompts the downstream firm to reduce the market price of the product. This, in turn, leads to an increase in consumer demand, as well as consumer surplus, upstream firm profit, and supply chain system profit. However, the profit of the downstream firm decreases as a result. However, there are inevitably a few limitations to this study that could be potentially addressed in further research. First, our assumption is that the carbon neutrality technology is licensed by the upstream firm to the downstream firm. It would be interesting to consider the case where the downstream firm serves as the technology innovator and licenses the technology to the upstream firm. Second, we exclusively take into account technology licensing between vertically competing firms in a single supply chain. In actuality, however, supply chain structures are more complicated, such as those with multiple upstream firms or multiple downstream firms. Moreover, firms in the supply chain usually can choose to license the technology to their competitors. It is undoubtedly challenging to investigate technology licensing across vertically and horizontally competing firms in a supply chain network. Finally, we assume in our model that consumers are price-sensitive and that the licensing of carbon neutrality technology has no direct impact on consumer demand for products. Actually, many customers have a low-carbon preference and are prepared to pay a premium for low-carbon items. Therefore, a further investigation is warranted to incorporate low carbon-sensitive customer demand into our carbon neutrality technology licensing models.

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Appendix

Proof of Proposition 7: (1) Under the benchmark model, it holds that:

$$\frac{\partial w^{*N}}{\partial r} = -\frac{\lambda e(1-s)}{2-s} < 0,\quad \frac{\partial p^{*N}}{\partial r} = \frac{\lambda e}{2(2-s)} > 0,\quad \frac{\partial q^{*N}}{\partial r} = -\frac{\lambda e}{2(2-s)} < 0,$$

$$\frac{\partial U_u^{*N}}{\partial r} = -\frac{e\lambda[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)} < 0, \frac{\partial U_d^{*N}}{\partial r} = -\frac{\lambda e(1-s)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2} < 0,$$

$$\frac{\partial U_c^{*N}}{\partial r} = \frac{\partial U_u^{*N}}{\partial r} + \frac{\partial U_d^{*N}}{\partial r} < 0, \frac{\partial CS^{*N}}{\partial r} = q^{*N}\frac{\partial q^{*N}}{\partial r} < 0.$$

(2) Under the NL model, it holds that:

$$\frac{\partial w^{*NL}}{\partial r} = 0, \frac{\partial p^{*NL}}{\partial r} = 0, \frac{\partial q^{*NL}}{\partial r} = 0, \frac{\partial U_u^{*NL}}{\partial r} = 0, \frac{\partial U_d^{*NL}}{\partial r} = 0, \frac{\partial U_c^{*NL}}{\partial r} = 0, \frac{\partial CS^{*NL}}{\partial r} = 0.$$

(3) Under the FL model, it holds that:

$$\frac{\partial w^{*FL}}{\partial r} = 0, \frac{\partial p^{*FL}}{\partial r} = 0, \frac{\partial q^{*FL}}{\partial r} = 0, \frac{\partial U_c^{*FL}}{\partial r} = 0, \frac{\partial CS^{*FL}}{\partial r} = 0,$$

$$\frac{\partial U_u^{*FL}}{\partial r} = \frac{\lambda e(1-s)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2} > 0, \frac{\partial U_d^{*FL}}{\partial r} = -\frac{\lambda e(1-s)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2} < 0.$$

(4) Under the RL model, it holds that:

$$\frac{\partial w^{*RL}}{\partial r} = 0, \frac{\partial p^{*RL}}{\partial r} = 0, \frac{\partial q^{*RL}}{\partial r} = 0, \frac{\partial U_u^{*RL}}{\partial r} = 0, \frac{\partial U_d^{*RL}}{\partial r} = 0, \frac{\partial U_c^{*RL}}{\partial r} = 0, \frac{\partial CS^{*RL}}{\partial r} = 0.$$

The proof is completed.

Proof of Proposition 8: (1) Under the benchmark model, it holds that:

$$\frac{\partial w^{*N}}{\partial \theta} = -\frac{c(1-s)}{2-s} < 0,\quad \frac{\partial p^{*N}}{\partial \theta} = \frac{c}{2(2-s)} > 0,\quad \frac{\partial q^{*N}}{\partial \theta} = -\frac{c}{2(2-s)} < 0,$$

$$\frac{\partial U_u^{*N}}{\partial \theta} = -\frac{c[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)} < 0, \frac{\partial U_d^{*N}}{\partial \theta} = -\frac{c(1-s)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2} < 0,$$

$$\frac{\partial U_c^{*N}}{\partial \theta} = \frac{\partial U_u^{*N}}{\partial \theta} + \frac{\partial U_d^{*N}}{\partial \theta} < 0, \frac{\partial CS^{*N}}{\partial \theta} = q^{*N}\frac{\partial q^{*N}}{\partial \theta} < 0.$$

(2) Under the NL model, it holds that:

$$\frac{\partial w^{*NL}}{\partial \theta} = 0, \frac{\partial p^{*NL}}{\partial \theta} = 0, \frac{\partial q^{*NL}}{\partial \theta} = 0, \frac{\partial U_u^{*NL}}{\partial \theta} = 0, \frac{\partial U_d^{*NL}}{\partial \theta} = 0, \frac{\partial U_c^{*NL}}{\partial \theta} = 0, \frac{\partial CS^{*NL}}{\partial \theta} = 0.$$

(3) Under the FL model, it holds that:

$\frac{\partial w^{*FL}}{\partial \theta} = 0, \frac{\partial p^{*FL}}{\partial \theta} = 0, \frac{\partial q^{*FL}}{\partial \theta} = 0, \frac{\partial U_c^{*FL}}{\partial \theta} = 0, \frac{\partial CS^{*FL}}{\partial \theta} = 0,$
$\frac{\partial U_u^{*FL}}{\partial \theta} = \frac{c(1-s)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2} > 0, \frac{\partial U_d^{*FL}}{\partial \theta} = -\frac{c(1-s)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2} < 0.$
(4) Under the RL model, it holds that:
$\frac{\partial w^{*RL}}{\partial \theta} = 0, \frac{\partial p^{*RL}}{\partial \theta} = 0, \frac{\partial q^{*RL}}{\partial \theta} = 0, \frac{\partial U_u^{*RL}}{\partial \theta} = 0, \frac{\partial U_d^{*RL}}{\partial \theta} = 0, \frac{\partial U_c^{*RL}}{\partial \theta} = 0, \frac{\partial CS^{*RL}}{\partial \theta} = 0.$

The proof is completed.

Proof of Proposition 9: (1) Under the benchmark model, it holds that:

$\frac{\partial w^{*N}}{\partial \lambda} = \frac{e[s-r(1-s)]}{2-s}.$

We can calculate that $\frac{\partial w^{*N}}{\partial \lambda} \geq (<) 0$ if $r \leq (>) \frac{s}{1-s}$.
$\frac{\partial p^{*N}}{\partial \lambda} = \frac{e(2+r)}{2(2-s)} > 0,\ \frac{\partial q^{*N}}{\partial \lambda} = -\frac{e(2+r)}{2(2-s)} < 0. \frac{\partial CS^{*N}}{\partial \lambda} = q^{*N}\frac{\partial q^{*N}}{\partial \lambda} < 0.$
$\frac{\partial U_u^{*N}}{\partial \lambda} = \frac{2K(1+s)(2-s)-e(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)}, \frac{\partial U_d^{*N}}{\partial \lambda} = \frac{(1-s)\{2K(2-s)^2-e(2+r)[1-c(2+\theta)-\lambda e(2+r)]\}}{2(2-s)^2},$
$\frac{\partial U_c^{*N}}{\partial \lambda} = \frac{4K(2-s)^2-e(2+r)(3-2s)[1-c(2+\theta)-\lambda e(2+r)]\}}{2(2-s)^2}.$

Denote $K_1 = \frac{e(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(1+s)(2-s)}$, $K_2 = \frac{e(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2}$, and $K_3 = \frac{e(2+r)(3-2s)[1-c(2+\theta)-\lambda e(2+r)]}{4(2-s)^2}$.
We can calculate that $\frac{\partial U_d^{*N}}{\partial \lambda} < 0$, $\frac{\partial U_u^{*N}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*N}}{\partial \lambda} < 0$ if $K < K_2$; $\frac{\partial U_d^{*N}}{\partial \lambda} \geq 0$, $\frac{\partial U_u^{*N}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*N}}{\partial \lambda} < 0$ if $K_2 \leq K < K_3$; $\frac{\partial U_d^{*N}}{\partial \lambda} > 0$, $\frac{\partial U_u^{*N}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*N}}{\partial \lambda} \geq 0$ if $K_3 \leq K < K_1$; and $\frac{\partial U_d^{*N}}{\partial \lambda} > 0$, $\frac{\partial U_u^{*N}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*N}}{\partial \lambda} > 0$ if $K \geq K_1$.
(2) Under the NL model, it holds that:
$\frac{\partial w^{*NL}}{\partial \lambda} = \frac{es}{2-s} > 0,\ \frac{\partial p^{*NL}}{\partial \lambda} = \frac{e}{2-s} > 0, \frac{\partial q^{*NL}}{\partial \lambda} = -\frac{e}{2-s} < 0, \frac{\partial CS^{*NL}}{\partial \lambda} = q^{*NL}\frac{\partial q^{*NL}}{\partial \lambda} < 0.$
$\frac{\partial U_u^{*NL}}{\partial \lambda} = \frac{K(1+s)(2-s)-e(1-2c-2\lambda e)}{2-s}, \frac{\partial U_d^{*NL}}{\partial \lambda} = \frac{(1-s)[K(2-s)^2-e(1-2c-2\lambda e)]}{(2-s)^2},$
$\frac{\partial U_c^{*NL}}{\partial \lambda} = \frac{2K(2-s)^2-e(3-2s)(1-2c-2\lambda e)}{(2-s)^2}.$
Denote $K_4 = \frac{e(1-2c-2\lambda e)}{(1+s)(2-s)}$, $K_5 = \frac{e(1-2c-2\lambda e)}{(2-s)^2}$, and $K_6 = \frac{e(3-2s)(1-2c-2\lambda e)}{2(2-s)^2}$.
We can calculate that $\frac{\partial U_d^{*NL}}{\partial \lambda} < 0$, $\frac{\partial U_u^{*NL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K < K_5$; $\frac{\partial U_d^{*NL}}{\partial \lambda} \geq 0$, $\frac{\partial U_u^{*NL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K_5 \leq K < K_6$; $\frac{\partial U_d^{*NL}}{\partial \lambda} > 0$, $\frac{\partial U_u^{*NL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_4$; $\frac{\partial U_d^{*NL}}{\partial \lambda} > 0$, $\frac{\partial U_u^{*NL}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} > 0$ if $K \geq K_4$.
(3) Under the FL model, it holds that:
$\frac{\partial w^{*FL}}{\partial \lambda} = \frac{es}{2-s} > 0,\ \frac{\partial p^{*FL}}{\partial \lambda} = \frac{e}{2-s} > 0, \frac{\partial q^{*FL}}{\partial \lambda} = -\frac{e}{2-s} < 0, \frac{\partial CS^{*FL}}{\partial \lambda} = q^{*FL}\frac{\partial q^{*FL}}{\partial \lambda} < 0.$
$\frac{\partial U_u^{*FL}}{\partial \lambda} = \frac{2K(1+s)(2-s)^2-2e(3-2s)(1-2c-2\lambda e)+e(1-s)(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(2-s)^2},$
$\frac{\partial U_d^{*FL}}{\partial \lambda} = \frac{(1-s)\{2K(2-s)^2-e(2+r)[1-c(2+\theta)-\lambda e(2+r)]\}}{2(2-s)^2}, \frac{\partial U_c^{*NL}}{\partial \lambda} = \frac{2K(2-s)^2-e(3-2s)(1-2c-2\lambda e)}{(2-s)^2}.$
Denote $K_7 = \frac{2e(3-2s)(1-2c-2\lambda e)-e(1-s)(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(1+s)(2-s)^2}$ and $s_1 = \frac{3(1-2c-2\lambda e)-(2+r)[1-c(2+\theta)-\lambda e(2+r)]}{2(1-2c-2\lambda e)}$.
We can calculate that when $s \geq s_1$, it holds that $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K < K_7$; $\frac{\partial U_u^{*FL}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K_7 \leq K < K_6$; $\frac{\partial U_u^{*FL}}{\partial \lambda} > 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_2$; and $\frac{\partial U_u^{*FL}}{\partial \lambda} > 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} > 0$ if $K \geq K_2$.
When $s < s_1$, it holds that $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K < K_2$; $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} \geq 0$,and $\frac{\partial U_c^{*NL}}{\partial \lambda} < 0$ if $K_2 \leq K < K_6$; $\frac{\partial U_u^{*FL}}{\partial \lambda} < 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} > 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_7$; and $\frac{\partial U_u^{*FL}}{\partial \lambda} \geq 0$, $\frac{\partial U_d^{*FL}}{\partial \lambda} > 0$, and $\frac{\partial U_c^{*NL}}{\partial \lambda} > 0$ if $K \geq K_7$.
(4) Under the RL model, it holds that:
$\frac{\partial w^{*RL}}{\partial \lambda} = \frac{es}{2-s} > 0,\ \frac{\partial p^{*RL}}{\partial \lambda} = \frac{e}{2-s} > 0, \frac{\partial q^{*RL}}{\partial \lambda} = -\frac{e}{2-s} < 0, \frac{\partial CS^{*RL}}{\partial \lambda} = q^{*RL}\frac{\partial q^{*RL}}{\partial \lambda} < 0.$
$\frac{\partial U_u^{*RL}}{\partial \lambda} = \frac{K(1+s)(2-s)-e(1-2c-2\lambda e)}{2-s}, \frac{\partial U_d^{*RL}}{\partial \lambda} = \frac{(1-s)[K(2-s)^2-e(1-2c-2\lambda e)]}{(2-s)^2},$
$\frac{\partial U_c^{*RL}}{\partial \lambda} = \frac{2K(2-s)^2-e(3-2s)(1-2c-2\lambda e)}{(2-s)^2}.$
We can calculate that $\frac{\partial U_d^{*RL}}{\partial \lambda} < 0$, $\frac{\partial U_u^{*RL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} < 0$ if $K < K_5$; $\frac{\partial U_d^{*RL}}{\partial \lambda} \geq 0$, $\frac{\partial U_u^{*RL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} < 0$ if $K_5 \leq K < K_6$;

$\frac{\partial U_d^{*RL}}{\partial \lambda} > 0, \frac{\partial U_u^{*RL}}{\partial \lambda} < 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} \geq 0$ if $K_6 \leq K < K_4$; $\frac{\partial U_d^{*RL}}{\partial \lambda} > 0, \frac{\partial U_u^{*RL}}{\partial \lambda} \geq 0$, and $\frac{\partial U_c^{*RL}}{\partial \lambda} > 0$ if $K \geq K_4$.

The proof is completed.

Proof of Proposition 10: (1) Under the benchmark model, it holds that:

$$\frac{\partial w^{*N}}{\partial s} = -\frac{1-c(2+\theta)-\lambda e(2+r)}{(2-s)^2} < 0, \frac{\partial p^{*N}}{\partial s} = -\frac{1-c(2+\theta)-\lambda e(2+r)}{2(2-s)^2} < 0, \frac{\partial q^{*N}}{\partial s} = \frac{1-c(2+\theta)-\lambda e(2+r)}{2(2-s)^2} > 0,$$
$$\frac{\partial U_u^{*N}}{\partial s} = \frac{[1-c(2+\theta)-\lambda e(2+r)]^2+4k\lambda(2-s)^2}{4(2-s)^2} > 0, \frac{\partial U_d^{*N}}{\partial s} = -\frac{s[1-c(2+\theta)-\lambda e(2+r)]^2+4k\lambda(2-s)^3}{4(2-s)^3} < 0,$$
$$\frac{\partial U_c^{*N}}{\partial s} = \frac{(1-s)[1-c(2+\theta)-\lambda e(2+r)]^2}{2(2-s)^3} > 0, \frac{\partial CS^{*N}}{\partial s} = \partial q^{*N} \ \frac{\partial q^{*N}}{\partial s} > 0.$$

(2) Under the NL model, it holds that:
$$\frac{\partial w^{*NL}}{\partial s} = -\frac{1-2c-2e\lambda}{(2-s)^2} < 0, \frac{\partial p^{*NL}}{\partial s} = -\frac{1-2c-2e\lambda}{2(2-s)^2} < 0, \frac{\partial q^{*NL}}{\partial s} = \frac{1-2c-2e\lambda}{2(2-s)^2} > 0,$$
$$\frac{\partial U_u^{*NL}}{\partial s} = \frac{(1-2c-2\lambda e)^2+4k\lambda(2-s)^2}{4(2-s)^2} > 0, \frac{\partial U_d^{*NL}}{\partial s} = -\frac{s(1-2c-2\lambda e)^2+4k\lambda(2-s)^3}{4(2-s)^3} < 0,$$
$$\frac{\partial U_c^{*NL}}{\partial s} = \frac{(1-s)(1-2c-2e\lambda)^2}{2(2-s)^3} > 0, \frac{\partial CS^{*NL}}{\partial s} = q^{*NL}\frac{\partial q^{*NL}}{\partial s} > 0.$$
(3) Under the FL model, it holds that:
$$\frac{\partial w^{*FL}}{\partial s} = -\frac{1-2c-2e\lambda}{(2-s)^2} < 0, \frac{\partial p^{*FL}}{\partial s} = -\frac{1-2c-2e\lambda}{2(2-s)^2} < 0, \frac{\partial q^{*FL}}{\partial s} = \frac{1-2c-2e\lambda}{2(2-s)^2} > 0,$$
$$\frac{\partial U_u^{*FL}}{\partial s} = \frac{2(1-s)(1-2c-2\lambda e)^2+s[1-c(2+\theta)-\lambda e(2+r)]^2+4k\lambda(2-s)^3}{4(2-s)^3} > 0,$$
$$\frac{\partial U_d^{*FL}}{\partial s} = -\frac{s[1-c(2+\theta)-\lambda e(2+r)]^2+4k\lambda(2-s)^3}{4(2-s)^3} < 0, \frac{\partial U_c^{*FL}}{\partial s} = \frac{(1-s)(1-2c-2e\lambda)^2}{2(2-s)^3} > 0, \frac{\partial CS^{*FL}}{\partial s} = q^{*FL}\frac{\partial q^{*FL}}{\partial s} > 0.$$
(4) Under the RL model, it holds that:
$$\frac{\partial w^{*RL}}{\partial s} = -\frac{1-2c-2e\lambda}{(2-s)^2} < 0, \frac{\partial p^{*RL}}{\partial s} = -\frac{1-2c-2e\lambda}{2(2-s)^2} < 0, \frac{\partial q^{*RL}}{\partial s} = \frac{1-2c-2e\lambda}{2(2-s)^2} > 0,$$
$$\frac{\partial U_u^{*RL}}{\partial s} = \frac{(1-2c-2\lambda e)^2+4k\lambda(2-s)^2}{4(2-s)^2} > 0, \frac{\partial U_d^{*RL}}{\partial s} = -\frac{s(1-2c-2\lambda e)^2+4k\lambda(2-s)^3}{4(2-s)^3} < 0,$$
$$\frac{\partial U_c^{*RL}}{\partial s} = \frac{(1-s)(1-2c-2e\lambda)^2}{2(2-s)^3} > 0, \frac{\partial CS^{*RL}}{\partial s} = q^{*NL}\frac{\partial q^{*NL}}{\partial s} > 0.$$

The proof is completed.

Proof of Proposition 11: (1) Under the benchmark model, it holds that:
$$\frac{\partial U_u^{*N}}{\partial K} = \lambda(1+s) > 0, \ \frac{\partial U_d^{*N}}{\partial K} = \lambda(1-s) > 0, \frac{\partial U_c^{*N}}{\partial K} = 2\lambda > 0.$$

(2) Under the NL model, it holds that:
$$\frac{\partial U_u^{*NL}}{\partial K} = \lambda(1+s) > 0, \ \frac{\partial U_d^{*NL}}{\partial K} = \lambda(1-s) > 0, \frac{\partial U_c^{*NL}}{\partial K} = 2\lambda > 0.$$

(3) Under the FL model, it holds that:
$$\frac{\partial U_u^{*FL}}{\partial K} = \lambda(1+s) > 0, \ \frac{\partial U_d^{*FL}}{\partial K} = \lambda(1-s) > 0, \frac{\partial U_c^{*FL}}{\partial K} = 2\lambda > 0.$$

(4) Under the RL model, it holds that:
$$\frac{\partial U_u^{*RL}}{\partial K} = \lambda(1+s) > 0, \ \frac{\partial U_d^{*RL}}{\partial K} = \lambda(1-s) > 0, \frac{\partial U_c^{*RL}}{\partial K} = 2\lambda > 0.$$

(5) Under the SL model, it holds that:
$$\frac{\partial U_u^{*SL}}{\partial K} = \lambda(1+s) > 0, \ \frac{\partial U_d^{*SL}}{\partial K} = \lambda(1-s) > 0, \frac{\partial U_c^{*SL}}{\partial K} = 2\lambda > 0.$$

The proof is completed.

Proof of Proposition 12. By comparing consumer surpluses in the five alternative models, it derives:

$$CS^{*SL} - CS^{*RL} = \frac{\xi^{*SL}(1-s)(1-2c-2e\lambda)^2[4-s(2-\xi^{*SL})-\xi^{*SL}]}{8(2-s)^2(2-s-\xi^{*SL}+s\xi^{*SL})^2} > 0,$$
$$CS^{*RL} - CS^{*FL} = 0, CS^{*FL} - CS^{*NL} = 0, CS^{*NL} - CS^{*N} = \frac{(c\theta+er\lambda)[2-c(4+\theta)-\lambda e(4+r)]}{8(2-s)^2} > 0.$$
The proof is completed.

Proof of Proposition 13. (1) By comparing the upstream firm's profits in the five alternative models, it derives:
$U_u^{*NL} - U_u^{*N} = \frac{(c\theta+er\lambda)[2-c(4+\theta)-e\lambda(4+r)]}{4(2-s)} > 0$, $U_u^{*RL} - U_u^{*NL} = 0$,
$U_u^{*FL} - U_u^{*RL} = \frac{(1-s)(c\theta+er\lambda)[2-c(4+\theta)-\lambda e(4+r)]}{4(2-s)^2} > 0$,
$U_u^{*SL} - U_u^{*FL} = U_c^{*SL} - U_d^{*SL} - (U_c^{*FL} - U_d^{*FL}) = U_c^{*SL} - U_c^{*FL} = \frac{\xi(1-s)^2(1-2c-2e\lambda)^2(4-2s(1-\xi)-3\xi)}{4(2-s)^2(2-s-\xi+s\xi)^2} > 0.$
(2) By comparing the downstream firm's profits in the five alternative models, it derives:
$U_d^{*NL} - U_d^{*N} = \frac{(1-s)(c\theta+er\lambda)[2-c(4+\theta)-\lambda e(4+r)]}{4(2-s)^2} > 0$, $U_d^{*RL} - U_d^{*NL} = 0$, $U_d^{*FL} - U_d^{*N} = 0$, $U_d^{*SL} - U_d^{*FL} = 0$.
(3) By comparing the supply chain system profits in the five alternative models, it derives:
$U_c^{*NL} - U_c^{*N} = \frac{(3-2s)(c\theta+er\lambda)[2-c(4+\theta)-\lambda e(4+r)]}{4(2-s)^2} > 0$, $U_c^{*NL} = U_c^{*RL} = U_c^{*FL}$,
$U_c^{*SL} - U_c^{*NL} = \frac{\xi^{*SL}(1-s)^2(1-2c-2e\lambda)^2\left(4-2s\left(1-\xi^{*SL}\right)-3\xi^{*SL}\right)}{4(2-s)^2\left(2-s-\xi^{*SL}+s\xi^{*SL}\right)^2} > 0.$
The proof is completed.

Proof of Proposition 14. (1) By comparing SW^{*RL}, SW^{*FL}, and SW^{*NL}, it derives $SW^{*RL} = SW^{*FL} = SW^{*NL}$.
(2) By comparing SW^{*SL} and SW^{*NL}, it derives:
$SW^{*SL} - SW^{*NL} = \frac{\xi^{*SL}e(1-s)(1-2c-2\lambda e)(\Delta_1-\Delta)}{(2-s)(2-s-\xi^{*SL}+s\xi^{*SL})}$,
where $\Delta_1 = \frac{(1-2c-2\lambda e)[4-\xi^{*SL}-s(2-\xi^{*SL})]}{4e(2-s)[2-\xi^{*SL}-s(1-\xi^{*SL})]}$.
Thus, we have $SW^{*SL} > (\leq) SW^{*NL}$ if $\Delta < (\geq) \Delta_1$.

(3) By comparing SW^{*NL} and SW^{*N}, it derives:
$SW^{*NL} - SW^{*N} = \frac{(7-4s)(c\theta+\lambda er)[2-c(4+\theta)-\lambda e(4+r)]}{8(2-s)^2} + \Delta e \frac{r-2cr-2c\theta-cr\theta-\lambda er(4+r)}{2(2-s)}$.
We can calculate that $SW^{*NL} > SW^{*N}$ if $e \leq e_1$, where $e_1 = \frac{r-2cr-2c\theta-cr\theta}{\lambda r(4+r)}$.
When $e > e_1$, we have $SW^{*NL} > (\leq) SW^{*N}$ if $\Delta < (\geq) \Delta_2$, where $\Delta_2 = \frac{(7-4s)(c\theta+\lambda er)[2-c(4+\theta)-\lambda e(4+r)]}{4(2-s)[2cr+2c\theta+cr\theta+\lambda er(4+r)-r]}$.
(4) By comparing SW^{*SL} and SW^{*N}, it derives:

$$SW^{*SL} - SW^{*N} = \frac{\begin{Bmatrix}(7-4s)(c\theta+\lambda er)[2-c(4+\theta)-\lambda e(4+r)]\left(2-s-\xi^{*SL}+s\xi^{*SL}\right)\\ +2\xi^{*SL}(1-s)(1-2c-2\lambda e)^2\left[4-\xi^{*SL}-s\left(2-\xi^{*SL}\right)\right]\end{Bmatrix}}{8(2-s)^2\left(2-s-\xi^{*SL}+s\xi^{*SL}\right)^2} - \Delta e \frac{\begin{Bmatrix}2\xi^{*SL}(1-s)(1-2c-2\lambda e)\\ -(2-s-\xi^{*SL}+s\xi^{*SL})[r-2cr-2c\theta-cr\theta-\lambda er(4+r)]\end{Bmatrix}}{2(2-s)(2-s-\xi^{*SL}+s\xi^{*SL})}.$$

We can calculate that if $H(e,r) \leq 0$, then $SW^{*SL} > SW^{*N}$, where $H(e,r) = 2\xi^{*SL}(1-s)(1-2c-2\lambda e) - (2-s-\xi^{*SL}+s\xi^{*SL})[r-2cr-2c\theta-cr\theta-\lambda er(4+r)]$.

When $H(e,r) > 0$, we have $SW^{*SL} > (\leq) SW^{*N}$ if $\Delta < (\geq) \Delta_3$, where

$$\Delta_3 = \frac{\begin{Bmatrix}(7-4s)(c\theta+\lambda er)[2-c(4+\theta)-\lambda e(4+r)]\left(2-s-\xi^{*SL}+s\xi^{*SL}\right)\\ +2\xi^{*SL}(1-s)(1-2c-2\lambda e)^2\left[4-\xi^{*SL}-s\left(2-\xi^{*SL}\right)\right]\end{Bmatrix}}{4e(2-s)(2-s-\xi^{*SL}+s\xi^{*SL})\begin{Bmatrix}2\xi^{*SL}(1-s)(1-2c-2\lambda e)\\ -(2-s-\xi^{*SL}+s\xi^{*SL})[r-2cr-2c\theta-cr\theta-\lambda er(4+r)]\end{Bmatrix}}.$$

The proof is completed.