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Optimal green technology investment and lot-sizing decision under carbon tax and cap-and-trade regulations considering planned shortages, outsourced repair and batch shipments

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A B S T R A C T

In recent years, various issues such as industrial waste and emissions of greenhouse gases have led to serious environmental pollution. Industrial managers nowadays need to regard cutting carbon emissions as one of their principal responsibilities in relation to the environment, as industry is a major source of carbon emissions. Two prominent regulatory approaches to reducing carbon emissions from operations are the carbon tax and the cap-and-trade system. The existing literature on inventory studies has often considered the market-expanding effects of greening efforts. Nevertheless, a number of additional factors exert influence on greening efforts, with the cost reduction effect representing a critical one. This paper develops an inventory system in which each time a lot of items is received, a proportion of items are found to be of imperfect quality; to identify these, the retailer carries out a 100% inspection of goods received. Following this inspection, the saleable items are added to the inventory in the warehouse in batches of equal size, rather than one by one, and the retailer allows backordering to meet demand. Carbon emissions are incurred at every stage, including ordering, purchasing, repairing, transporting, and holding, so advanced green technology is employed to reduce them. Imperfect products can be sold to a second-hand market or sent to a repair shop. The model discussed in this paper calculates, for both options, the most cost-effective lot size for orders, shortage quantity, scale of green investment and number of batches.

1. Introduction

Because of the growing impact of climate change and global warming, many countries have introduced environmental laws and standards that penalise companies for excessive releases of polluted water and air into their surroundings. These regulations come with challenges for both companies and governments. Companies will follow the most profitable path they can take within their governments' regulations, systems, and incentives. Governments, for their part, are responsible for validating and tracing carbon emissions from each energy consumer. In addition, governments may establish mechanisms for carbon emission trading or incentives to promote green investment that reduces carbon emission.

The main policies suggested in the literature to reduce carbon emissions are carbon taxes, and cap-and-trade systems. Cap-and-trade specifies the permissible level of emissions, while concurrently letting the market decide the cost of cutting emissions to that level. Companies whose emissions are below the stated limit, are allowed to sell or trade their surplus allowance to companies that are unable to sufficiently reduce their emissions. For example, the emissions trading system of the European Union, which was launched in 2005 as the first such system in the world, is part of a suite of key policies. It offers crucial experience of building and running a cap-and-trade system across an international economy. Carbon tax is the other side of cap and trade: it defines a price for carbon emissions. Companies subject to a cap can balance the cost of cutting their emissions against the cost of tax for continuing their current emissions (Konstantaras et al., 2021). The carbon tax, which has been implemented in countries including Japan, Denmark, Finland, and Ireland, imposes a levy per unit of emissions

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released on companies whose activities result in pollution (Turken et al., 2020). Furthermore, companies can also invest in green technology, which is becoming cheaper than paying for permits (Liu & Zhu, 2024). The integration of green technology and carbon reduction can contribute significantly to mitigating climate change and help in reaching sustainability goals. The aim of green technology is to incorporate environment-friendly methods and technology into processes, not only for manufacturing and repairing, but also for ordering and transportation, so as to minimize the ecological footprint. Carbon reduction, in contrast, directly targets the cutting of carbon emissions to limit climate change. With these approaches, businesses can reduce waste, increase output, and decrease carbon emissions from their operations while achieving the targeted dimensions of sustainability (He et al., 2015; Gupta & Khanna, 2024).

Any enterprise needs to constantly consider how to optimise its inventory, and the effectiveness of the way it consumes and replenishes its stocks is critically influential in the company's financial condition and competitiveness. It also facilitates meeting or exceeding consumer expectations by maintaining sufficient stocks of each product, leading to the maximum net profit. An inventory is needed because real-world contexts never give a perfect balance of supply and demand. There are two basic reasons for careful management of inventory levels: to maximise sales by meeting demand, and to minimise total inventory costs. An inventory control system has the fundamental purpose of resolving two issues: when to place a replenishment order and the size of that order. The models developed for inventory control aim to provide solutions to these questions. The first known inventory control model is the economic order quantity (EOQ) model proposed by Harris (1913). The economic production quantity model, developed by Taft (1918), obtains the optimal production cycle time by assuming a finite production rate. Nevertheless, these models are limited by certain assumptions that restrict their application to real-life situations.

One of these assumptions is that a production process is used to produce products of perfect quality; product quality, in reality, is not always perfect. A percentage of items received, which we may call p, is likely to be of imperfect quality. Inspection is essential to ensure acceptable quality of items ordered or produced. The primary objectives of inspection are to confirm that the product meets the specifications and to determine if a non-conforming product can still be utilised in any way. After completing the inspection routine, the non-conforming items can be dealt with in several ways: they can be allowed to sell at a reduced price, repaired, sent back to the supplier, or removed from the system. Imperfect products within production and inventory systems are a common issue in the automotive industry (Bahety et al., 2018; Albalooshi et al., 2021). Another assumption of classical inventory management models is that stockouts (shortages) are not allowed. However, it is normal business practice for companies to maintain backlog orders (Cárdenas-Barrón, 2009; Sepehri & Gholamian, 2023).

The majority of studies in the field of green inventory management have employed market expansion-based demand models to investigate the impact of greening on inventory systems. A significant proportion of the theoretical foundations underlying these consumer demand models are predicated upon the assumption that the introduction of environmentally-friendly initiatives leads to an increase in the effective demand for the relevant product (Dash et al., 2023). Porter and Van der Linde's (1995) contribution was among the first to identify the consequences of green initiatives. Their work delineated three main effects: cost reduction, demand expansion, and the price premium. The price premium effect quantifies the additional revenue generated by a firm's capacity to charge a premium price for its environmentally friendly products in comparison to the price charged by its competitors (Ghosh et al., 2020). The impact of green initiatives on business operations frequently yields operational enhancements, primarily through a reduction in unit production costs. A significant proportion of these operational improvements can be attributed to innovation-led programmes. These include the reduction of emissions through the utilisation of end-of-pipeline technologies, an increase in production line efficiencies, which has resulted in a greater throughput of material and a reduction in wastage, innovative approaches to the reuse and recycling of waste materials, which have enabled their integration back into production processes, and a reduction in package sizes, which has led to a decrease in logistics costs (Genc & De Giovanni, 2020). In a similar vein, Kuiti et al. (2019) examine a dyadic supply chain in which the manufacturer implements product and process modifications, resulting in cost savings for both the retailer and manufacturer. Our paper delves into the beneficial impact of such innovation-driven sustainability initiatives.

Environmental variability and uncertain consumption conditions creates a need for mathematical instruments that can determine economically viable inventory levels, considering minimisation of the total costs, and, correspondingly, maximal profit through the creation and maintenance of stocks. The quality with which the resulting system is constructed influences not only the extent of customer satisfaction with the level of service they receive, but also the profitability of the entire system (Savchenko & Grygorak, 2019). Hence, a compromise is constantly necessary between a sufficient inventory level, the level of shortage that can be allowed, and avoiding excess inventory. This needs an inventory management system to be created which can respond quickly and accurately to developments in the external environment, while maintaining quality and efficient customer service (Savchenko & Grygorak, 2019).

The world now confronts many environmental issues that have developed over the years. Among them are the need to deal with greenhouse gases that have accumulated in the atmosphere from past emissions, how to reduce current emissions, and how to deal with resources and waste that have fulfilled their purpose and been abandoned in landfill sites or elsewhere, where they may or may not decay. There is also potential for many more problems in the near future. The environment contains large quantities of lead and other heavy metals, derived from industrial processes, and from fuels containing anti-knock lead additives (Bonney & Jaber, 2011). Environmental problems are a cause of steadily growing concern, and this paper explores the relationship between inventory management and the environment; especially the possibility of developing an inventory planning system that is environmentally responsible.

In this paper, we propose an economic order quantity model featuring an infinite planning horizon, together with cost reduction effect, shortages, outsourced repair, investment in reducing carbon emissions, and carbon tax and cap-and-trade enforcements. The manufacturer or supplier delivers lots of equal sizes to the retailer. Each of these lots is assumed to include a percentage of items of substandard quality, which cannot be used to fulfil demand. The retailer conducts full inspection of the lot received, since letting a substandard item go to the end user could have severe consequences. The acceptable items, i.e., items of good quality, are added to the working inventory in equal-size batches. That is, items that meet the required standards are sent to the working inventory in batches rather than on a unit-by-unit basis. The substandard items are subtracted from inventory as a single lot when the inspection process is complete, either for sale at a reduced price, or to be shipped to a third-party repairer to be brought to as-new condition, and then returned to the inventory. The retailer's system accrues costs including the usual expenses for ordering, purchasing, holding, repairing, and shortage, as well as a carbon emission tax and investment in green technology. In this analysis, we focus on the two most commonly used forms of emission regulations: (1) emission taxes and (2) cap-and-trade regulations. The carbon tax system aims to minimise emissions, while investment in green technology and carbon cap-and-trade contribute to more efficient management of emissions from the system. Subject to this regulatory mechanism and investment in carbon emission reduction, we aim to determine the inventory decisions of the optimal size of order lots, shortage quantity, and number of batches, as well as the level of investment in green technology to minimize the total cost of the retailer's system. In particular, we answer the following questions:

RQ1: What are the strategic lot sizing and green technology investment decisions for the retailer in the proposed inventory system?

RQ2: What is the optimal strategy for imperfect quality products? To what extent might this contribute to a reduction in carbon emissions? What are the financial and environmental benefits to the retailer?

RQ3: What is the impact of unit cost reduction and different environmental regulations on the structure of the inventory system?

A numerical example is given and the results are discussed. The developed model's behaviour is also investigated for varying values of parameters to stress their importance in determining the retailer's costs and the scale of carbon emissions.

The article is organized as follows. Section 2 provides a review of the literature on inventory models, with special attention to papers that consider imperfect quality items, shortages, and carbon emissions. Section 3 defines the problem and specifies the assumptions and notations employed in the inventory model. Section 4 details the mathematical model, the technical details of the optimization procedure, and the solution. Section 5 illustrates the model with a numerical example, and highlights some managerial insights. Section 6 summarises the findings of the study and suggests directions for future research.

2. Literature review

This study draws primarily on two major areas of research, namely carbon emissions and reduction, and imperfect production systems. In order to gain a comprehensive understanding of the existing literature in these fields, we will undertake a thorough review of the existing studies. We will subsequently highlight the unique findings and contributions of this study to these two domains. Carbon emissions are the leading cause of global warming, leading to the current focus of some governments on reducing carbon emissions, which mainly come from human activities such as the use of fossil fuels in industry, electricity generation and internal combustion engines. In the past decade, growing attention has been paid to including carbon emissions in research on inventory control. A study by El Saadany et al. (2011) gave insight into the inclusion of environmental concerns in inventory performance metrics. They stressed the need for modern producers to incorporate environmentally friendly inventory systems and the necessity for inventory models to be developed that can account for the costs involved in environmental impact. A number of studies have been conducted within the framework of the EOQ, a basic concept in inventory management. In 2011, Bonney and Jaber provided an extended EOQ model that takes into account the costs of emissions associated with landfill and transportation. Hua et al. (2011) proposed a model incorporating carbon cap-and-trade considerations and determined the most efficient quantity to order. They also examined how emissions, order quantities, and overall cost were impacted by variations in the carbon cap and price. Taking a range of environmental restrictions into consideration, Chen et al. (2013) examined the optimum ordering sizes and reached the conclusion that carbon emissions could be limited without a significant increase in costs. Jaber et al. (2013) devised an integrated inventory model featuring a coordination mechanism and taking action on the manufacturing side to address greenhouse gas emissions. Their model minimized the combined expenses from inventory-related costs and emission costs, including penalties for carbon emissions that exceeded the set limits. Battini et al. (2014) proposed a green EOQ model comprising three parts: carbon emission costs from warehousing, based on storage volume; inventory ordering and scrapping; and transportation, according to quantity and distance. They included sustainability factors in their ordering decisions and evaluated their effect. Toptal et al. (2014) assessed the effect on traditional EOQ models of various different carbon emissions regulations, including carbon emission limits, carbon taxes, and carbon trading. The retailer was able to invest in green technology to assist in ordering, storage, and purchasing. Hovelaque and Bironneau (2015) examined a model that relies on economic order quantities, where the demand rate for a product is determined by the price and carbon dioxide emissions. Moreover, He et al. (2015) analysed the influence of manufacturing and regulatory parameters on optimal batch size and emissions through an EOQ inventory modelling. Bazan et al. (2015) presented two integrated inventory models taking account of energy usage and greenhouse gas emissions resulting from the combined production and transportation operations of both vendor and buyer in the context of emission taxation. Taleizadeh et al.'s (2018) study of a production inventory model considered the costs of carbon emission, with stock outs permitted. Tao and Xu (2019) investigated how regulation measures and lack of consumer perceptions of carbon issues affect optimal order quantities, emission volumes and total costs, employing an EOQ inventory framework. Huang et al. (2020) examined the impacts of carbon policies and green technologies on an integrated inventory model, taking carbon emissions into account during production, transportation, and storage. They proposed a model to determine optimal quantities for production and delivery, with an ideal amount of green investment, aiming to minimize the costs under each of two carbon emission policies: cap-and-trade and carbon taxation. Konstantaras et al. (2021) proposed a model involving an inventory system with two storage locations and a finite planning horizon comprising two periods. During the first period, the demand of customers is met with newly manufactured lots of unequal sizes. During the second period, demand is fulfilled from remanufactured items, again of unequal sizes. Both production and remanufacturing processes produce imperfect items, which are reworked and restored to good quality. The processes of production, remanufacturing, repair, and collection of used products all generate carbon emissions. Gupta and Khanna's (2024) recent study devised a mathematical model intended to achieve maximum total profit by finding the optimal levels of price, environmental capital investment and volumes produced. The model includes items of imperfect quality, treating them in two ways, either by salvaging or reworking. They also examined the effect of carbon emission policies on production, with a focus on the issue of sustainability and the balance between protecting the environment and maintaining economic expansion. Based on the studies outlined above, in this paper, we consider green technology investment, and explore the optimal inventory management strategy to reduce carbon emissions while balancing environmental protection against the need to achieve a profit.

Schrady (1967) presented an EOQ model based on timely manufacture and repair rates and with zero disposal. Rosenblatt and Lee (1986) investigated the effect of process deterioration during production, resulting in defective items. Porteus (1986) investigated the impact of incorporating imperfect items on the quantities that can be produced economically. Zhang and Gerchak (1990) investigated a combined policy for order lot sizing and inspection, based on an EOQ model where the proportion of defective items was random: they were able to determine the optimal order quantity and the proportion of items to be inspected. Salameh and Jaber (2000) studied a policy for optimal lot sizing based on EOQ modeling. They assumed 100% inspection and that the defective items found were sold, as one batch, to secondary markets when the process was complete. Hayek and Salameh (2001) also investigated a model with items of imperfect quality. In their model the density function of the percentage of imperfect items is known, there is a finite production rate, and shortages are fully backordered. Chiu (2003) developed an extension to Hayek and Salameh`s model to include the reworking of some imperfect items to perfect quality, with others sold at a discounted price. Jamal et al. (2004) devised a production-inventory model where all imperfect products were reworked to good quality during the identical manufacturing cycle. Wee et al. (2007) made the implicit assumption that backordered items were delivered on receipt of the products and before completion of the screening process, while Eroglu and Ozdemir (2007) investigated an EOQ problem on the assumption that imperfect items could not immediately eliminate backorders. The model devised by Konstantaras et al. (2007) examined an inventory system where a proportion of each received lot of items was of imperfect quality. On completion of the inspection process, the high-quality (perfect) items were shipped to the working stock warehouse in equal-sized batches, rather than unit by unit. The model considers two alternatives for the imperfect products: they can be sold to a secondary market, or reworked in-house at a cost and then used as new items to fulfil demand. They devise a function for total profit, with optimal values for order lot size and the number of batches as decision variables. Cárdenas-Barrón (2009) adapted the model by Jamal et al. (2004) to allow planned backordering. Hasanov et al. (2012) developed a model for production, remanufacturing and waste disposal, in which repaired and newly manufactured items are not interchangeable, i.e., customers consider them to be different in quality and functionality. Wee et al. (2013) presented a production inventory model with a procedure for handling imperfect items and under constraints of shortage and screening. They developed a function for the total profit, using time intervals as decision variables. The model proposed by Jaber et al. (2014) investigated the shipment of imperfect quality items to a third party repairer to be restored to as-new condition, or selling them at a discount and replacing them with a corresponding number of perfect items purchased from a local supplier. Öztürk et al. (2015) examined an EOQ model for imperfect items, including a rework option where the defective items included a portion which could be reworked and the repair was carried out at the retailer's end. In the work of Taleizadeh et al. (2017) a production inventory model is reported with imperfect items and backordering of demand. They assumed that following the screening period all imperfect items would be withdrawn and sent for repair; after the repair process was completed, the items repaired were returned to the factory and taken into inventory. Ahmed et al. (2021) devised a synergic inventory model to achieve the maximum profit by incorporating an allowance for reworking, partial backordering, and a policy of multi-period delays in payments. The mathematical models devised by Gautam et al. (2022) considered demand that is sensitive to both green issues and price, involving carbon emissions and with two options for handling imperfect items, i.e., salvage and rework. Sepehri and Gholamian (2023) presented an inventory model analysing the effect of shortages in an EOQ model where the proportion of imperfect items produced can be reduced by employing quality improvement technologies, and where investment in green technology reduces emissions.

This paper builds on the work of Konstantaras et al. (2007) by investigating a more realistic inventory system that allows backordering of demand. In addition to this extension, it makes the assumption that imperfect items are shipped by the retailer to the manufacturer or to a third-party facility for inspection and later repairs. As well as inventory-related costs, it considers carbon emissions during ordering, purchasing, transportation, repair and storage through a tax on emissions. The study also includes investment in green technology that provides environmental benefits, cost savings, a competitive edge, and compliance with regulations. This helps to cut carbon emissions and enables organizations to assess the cleanness of their practices and their environmental efforts. Finally, we examine the cost-reduction impact of green initiatives. To the best of our knowledge, these factors have not been considered in the context of green inventory management.

3. Problem definition, assumptions and notation

The main problem examined in this paper is to optimize the cost of a retailer. The retailer's annual demand is D units, and regular orders are placed with a fixed size of y. In each order received, a percentage of items are of imperfect quality. When the order arrives at the retailer's establishment, the lot is 100% screened. The screening rate per unit time is fixed at x units, with the screened items being classified as good quality or imperfect. As the inspection proceeds, batches of good items are added to the working inventory, for use in meeting the demand. The batches are of equal size q, distributed over n times at equal time intervals t_S. The items begin to be used immediately the first batch of good-quality items reaches the working inventory. Meanwhile, the imperfect items are retained until the received lot has been fully screened, at which point they are deducted from the inventory. They may be sold at a lower price than the unit purchase cost or sent to a workshop for repair. Repaired items are returned to the retailer and re-inspected to ensure that all the quality requirements are now met and the products are not subject to any defects. When this is verified, they are returned to the working inventory to meet demand. The sustainable inventory models constructed in this study is illustrated in Fig. 1. It is further assumed shortages are allowed in the inventory system and are fully backordered. As well as covering the unit price of the product, the retailer is responsible for additional costs, including for ordering, screening, holding stocks, repairing, time-dependent backorders, and batch transportation. Other assumptions in formulating the model are as follows:

a) The model covers a single product, for which the demand rate is constant.
b) A lot of size y includes items of imperfect quality.
c) Screening is conducted at a constant rate which exceeds the demand rate.
d) The retailer's inspection is error-free and identifies all the imperfect items. In the real world, companies often store less inventory so as to optimize the cost of holding inventory and the system consequently faces stock shortages. The system may also face shortages because of eliminating items of imperfect quality. For these reasons, shortages are allowed, with a linear time-dependent cost applied for backordering.
e) Both initially good items and repaired items are used to satisfy demand.
f) Carbon emissions are generated throughout the entirety of the inventory system, from the initial ordering and purchasing stages, through to the transportation, storage and repair of the imperfect products.
g) It is not possible to achieve complete reduction in carbon emissions through green investment. The effect can be quantified and defined as a quadratic function based on historical data.
h) Inventory replenishment takes place instantaneously, with an infinite time horizon.

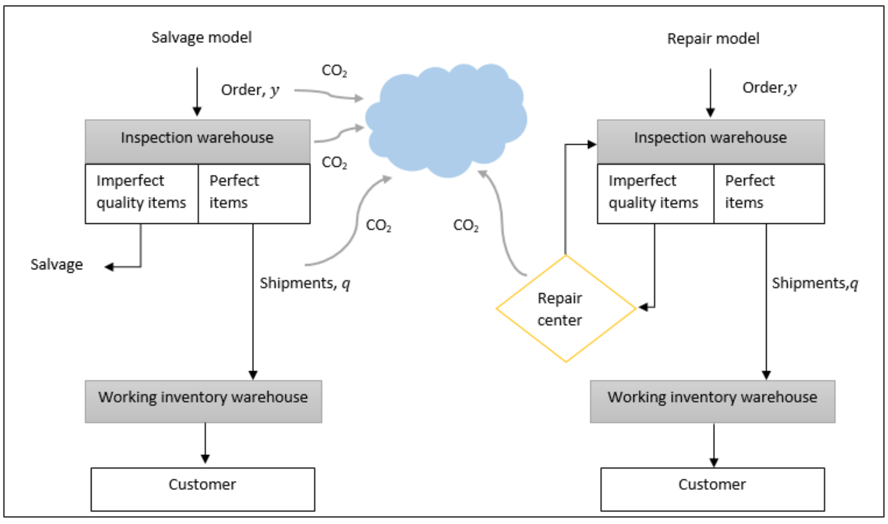

Fig. 1. Structures of Salvage model and Repair model.

The following notations are used through the model formulation:

Parameters:

D	Demand rate per unit of time
X	Inspection rate
K	Retailer's ordering cost
c	Purchasing cost per unit
h	Holding cost
b_1	Shortage cost
d	Screening cost
S	Repair setup cost
A	Transportation fixed cost
c_T	Unit transportation cost
c_m	Unit material cost
h'	Unit holding cost
w	Repair rate
t_T	Total transport time
m	Markup percentage
v	Transportation cost of a batch of good items to the working inventory
α	The percentage of the backordered demand for the last batch in shortage period
p	The percentage of imperfect quality items
L	Distance between warehouses
L_1	Distance between retailer and third party facility
E_h	The carbon emissions from holding inventory
$E_{h'}$	Carbon emissions from holding inventory in third part facility
E_T	The carbon emissions from sending a batch of good items to the inventory
E_P/E_R	The carbon emissions from purchasing/repairing a unit product
E_O/E_S	The carbon emissions from ordering/repair setup product
C_1	The carbon tax charged per unit of carbon emission
C_2	The price of carbon trading per unit of carbon emission
g_c	The benefit of green to the cost of purchase
θ	The efficiency factor of carbon emissions reduction
β	The offset factor of carbon emissions reduction
Decision variables:	
n	Number of batches of good items sent to the working inventory warehouse
y	Order lot size
B	Shortage quantity
G	The green technology investment amount
Dependent variables:	
q	The batch size of good items sent to the working inventory warehouse
TCU	The total cost of the inventory system
TCU_c	The total cost of the inventory system considering carbon emissions
TPU	The total profit of the inventory system

4. Model formulation

4.1. Benchmark models

This section builds upon the work of Konstantaras et al. (2007) to incorporate planned shortages. This section presents a mathematical model to analyse two scenarios for the handling of items with imperfect quality. In order to incorporate the concept of sustainability, the initial model posits that imperfect quality items are salvaged at the conclusion of the inspection process. In contrast, the second model postulates that imperfect quality items are repaired and subsequently employed to meet demand. Fig. 2 and Fig. 3 illustrate, for both models, the behaviour of inventory levels over time. They show that stock levels at the inspection stage decrease by q units at a time, while the working inventory levels simultaneously rise by q units. Note that $y - iq$ units remain at the inspection stage after $i \times t_S$ time units or after sending i batches of good items to the working inventory, but these are not all uninspected units. When $t = y/x$, t is the time needed to inspect the complete lot of y units. Setting the number of shipments in each time cycle to n and with p as the percentage of imperfect items, the batch size of good-quality items added to the working inventory is shown by $q = (1 - p)y/n$. Meanwhile, the time interval t_S between consecutive transfers of good-quality items can be expressed as $t_S = t/n = y/nx$. During t_S, the retailer receives demand from customers equivalent to $D \times t_S$. If there is sufficient stock in the working inventory, this demand can be met: if not, there will be a backorder shortage which will accumulate until it reaches B units. Hence, on the nth shipment, the quantity consumed over the previous $n-1$ shipments is calculated as $(n-1)Dt_S$. Thus, the maximum level of the working inventory can be obtained by Eq. (1).

$$I_{max} = y-(n-1)Dt_S - py - B = (1-p)y - \frac{D(n-1)y}{nx} - B. \tag{1}$$

4.1.1. Salvage model

The py items of imperfect quality are kept in stock for sale as one batch at a reduced price on completion of the inspection.

The time needed to compensate for the shortage is given by Eq. (2) as

$$t_1 = \frac{(B-\alpha q)t_S}{q-Dt_S} = \frac{(Bn-\alpha(1-p)y)(y/nx)}{n((1-p)y/n-Dy/nx)} = \frac{Bn-\alpha(1-p)y}{((1-p)x-D)n}. \tag{2}$$

The reason for this is that, at any interval in this period, the inventory is re-stocked at a rate of $q-Dt_S$. Thus, the amount by which the inventory is re-stocked can be calculated as $(q-Dt_S)t_1$. The first addition to the working inventory takes place at $t=0$, so it is possible to compensate q out of B units of shortage in the inventory at $t=0$. When the next batches of good-quality items are added to the working inventory, the system will receive demand at rate D. Since the next batches of good-quality units will arrive after an interval of t_S, the amount $D\times t_S$ is added to the shortage. Hence, during any time interval between the receipt of two batches of good-quality items, the inventory will increase by $q-Dt_S$. Hence, during period t_1 when the system is in shortage, at each time interval, $q-Dt_S$ units are subtracted from the total amount of shortage, till the shortage is completely compensated. Therefore, the amount of shortage, which equals $B-\alpha q$, based on these figures, should be established as $(q-Dt_S)(t_1/t_S)$. Following the same procedure, the time needed to reach the maximum inventory level can be calculated by Eq. (3):

$$t_2 = \frac{(I_{max}-(1-\alpha)q)(t_S)}{q-Dt_S} = \frac{(1-p)y}{(1-p)x-D} - \frac{D(n-1)y}{xn\big((1-p)x-D\big)} - \frac{B}{(1-p)x-D} - \frac{(1-\alpha)(1-p)y}{n\big((1-p)x-D\big)}. \tag{3}$$

Period t_3 is the stage when the greatest amount of inventory I_{max} is consumed. It is calculated as:

$$t_3 = \frac{I_{max}}{D} = \frac{(1-p)y}{D} - \frac{(n-1)y}{nx} - \frac{B}{D}. \tag{4}$$

According to Fig. 2, the amount of time the system suffers from the shortage can be calculated by Eq. (5) as

$$t_4 = \frac{B}{D}. \tag{5}$$

The duration of the inventory cycle is T, which equals $t_1+t_2+t_3+t_4$. By substituting t_i, $i=1,2,3,4$, then T is calculated as:

$$T = t_1+t_2+t_3+t_4 = \frac{(1-p)y}{D}. \tag{6}$$

Please refer to Fig. 2 once more. This figure illustrates the movement of the retailer's inventory over the course of a replenishment cycle. The thick line denotes the inventory position, while the shaded areas are used to calculate the average holding and backorder costs. The holding cost per cycle is paid during periods t, t_2 and t_3, and can be derived from Eq. (7):

$$\begin{aligned} HC = {} & \left(nyt_S - \frac{n(n-1)qt_S}{2}\right)h + \left(\frac{(t_2+t_3)\left(\left(\frac{t_2}{t_S}\right)q+(1-\alpha)q\right)}{2} - \frac{\left(\frac{t_2}{t_S}\right)\left(\frac{t_2}{t_S}+1\right)(qt_S)}{2}\right)h = \left(\frac{y^2}{x} - \frac{(n-1)(1-p)y^2}{2xn}\right)h \\ & + \left(\frac{(1-p)xy^2}{2D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2 - \left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]\right. \\ & + \frac{(1-p)y^2}{2n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right) - \frac{\alpha}{n}\left(\frac{n-1}{x} - \frac{\alpha(1-p)}{(1-p)x-D}\right)\right] + \frac{(1-p)xyB}{2D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n} - 2\left(1-p-\frac{D(n-1)}{nx}\right)\right] \\ & \left. + \frac{(1-p)yB}{2n}\left(\frac{\alpha}{(1-p)x-D} - \frac{1-\alpha}{D}\right) + \frac{(1-p)xB^2}{2D\big((1-p)x-D\big)}\right)h. \end{aligned} \tag{7}$$

This combines the holding cost from the items under inspection, Fig. 2, top, and from those in the working inventory, Fig. 2, bottom. The time-weighted inventory in the inspection warehouse can be measured by calculating the areas of stacked rectangles. It can be calculated by using the area of rectangles with height y minus the area of rectangles with height q. Note the number of the rectangles with height y and the number of rectangles with height q are n and $n(n-1)/2$, respectively. The time-weighted inventory in the working inventory is the stair-like and triangular areas above the horizontal axis. It can be calculated by using the area of triangle with base $(t_2 + t_3)$ minus the area of rectangles with base t_S. The number of rectangles with base t_S is $\left(\frac{t_2}{t_S}\right)\left(\frac{t_2}{t_S}+1\right)/2$.

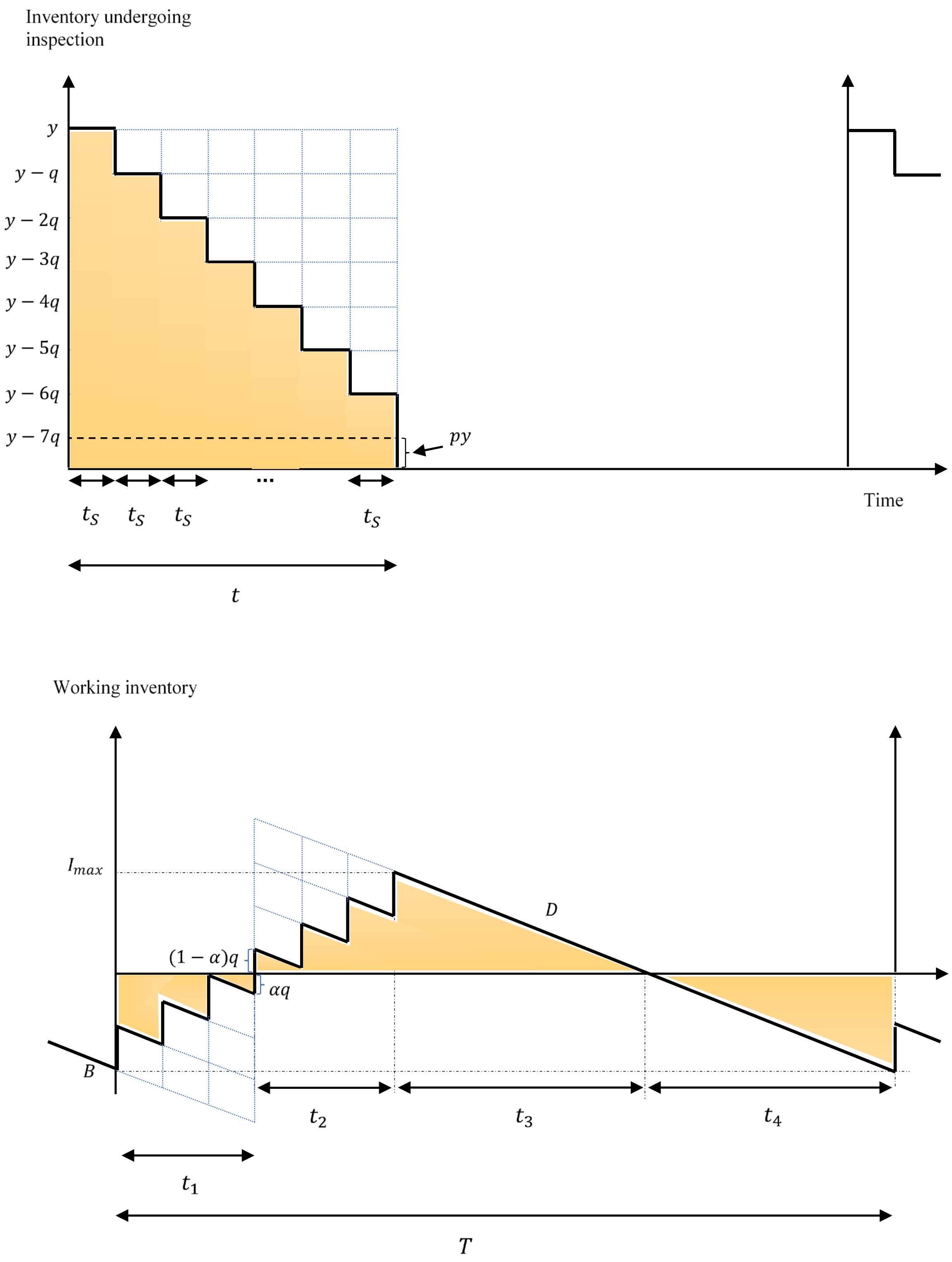

Fig. 2. Inventory level for Salvage model, where $n = 7$

The holding cost per cycle (HC) is divided by the cycle length $T = (1-p)y/D$, to derive the annual inventory holding cost. To simplify the mathematical derivations, some new notations compacted by other notations are used in this section and are given in Appendices.

$$HCU = \frac{hDy}{(1-p)x} - \frac{hD(n-1)y}{2xn} + \frac{hxy}{2\big((1-p)x-D\big)}U_1 + \frac{hDy}{2n}U_2 + \frac{hxB}{2\big((1-p)x-D\big)}U_3 + \frac{hDB}{2n}U_4 + \frac{hxB^2}{2\big((1-p)x-D\big)y} \tag{8}$$

The cost of shortage per cycle (SC) can be derived from Eq. (9):

$$SC = \left(\frac{\left(\left(\frac{t_1}{t_S}q+\alpha q\right)+B\right)t_1}{2} - \frac{\left(\frac{t_1}{t_S}\right)\left(\frac{t_1}{t_S}+1\right)qt_S}{2}\right)b_1 + \left(\frac{B(t_4)}{2}\right)b_1 = \left(\frac{B^2}{2\big((1-p)x-D\big)} - \frac{(1-p)yB}{2n\big((1-p)x-D\big)} - \frac{(1-p)^2(\alpha-1)\alpha y^2}{2n^2\big((1-p)x-D\big)}\right)b_1 + \frac{B^2}{2D}b_1\ . \tag{9}$$

Note that the accumulated shortage cost, which is time-dependent, is determined on the basis of t_1 and t_4, which are the periods where shortage occurs. In the lower part of Fig. 2, the time-weighted backorder level can be measured by calculating the areas below the horizontal axis. It can be calculated by using area of rectangle with height t_1 and the area of triangle with height B minus the area of the rectangles with height t_S. The number of the rectangles with height t_S is $\left(\frac{t_1}{t_S}\right)\left(\frac{t_1}{t_S}+1\right)/2$. The cost of shortage per cycle (SC) is divided by the cycle length $T=(1-p)y/D$, to calculate the annual inventory shortage cost:

$$SCU = \frac{b_1xB^2}{2y\big((1-p)x-D\big)} - \frac{b_1DB}{2n\big((1-p)x-D\big)} - \frac{(1-p)(\alpha-1)b_1\alpha Dy}{2n^2\big((1-p)x-D\big)}\ . \tag{10}$$

The retailer's total cost combines the annual ordering cost, and costs for purchasing, inspection, transferring batches, holding in inventory, shortage, and is therefore given by:

$$\begin{aligned} TCU(n,y,B) = {} & \frac{KD}{(1-p)y} + \frac{(c+d)D}{1-p} + \frac{vnD}{(1-p)y} + \frac{hDy}{(1-p)x} - \frac{hD(n-1)y}{2xn} + \frac{hxy}{2\big((1-p)x-D\big)}U_1 + \frac{hDy}{2n}U_2 \\ & + \frac{hxB}{2\big((1-p)x-D\big)}U_3 + \frac{hDB}{2n}U_4 - \frac{b_1DB}{2n\big((1-p)x-D\big)} - \frac{(1-p)(\alpha-1)b_1\alpha Dy}{2n^2\big((1-p)x-D\big)} + \frac{(h+b_1)xB^2}{2\big((1-p)x-D\big)y} \end{aligned} \tag{11}$$

For a fixed number of shipments for a batch, the order quantity and the maximum shortage quantity would be:

$$y = \sqrt{\frac{[D(K+vn)]/(1-p)}{U_5 - \frac{\big((1-p)x-D\big)U_6^2}{2x(h+b_1)}}}\ , \tag{12}$$

$$B = \frac{\big((1-p)x-D\big)U_6y}{(h+b_1)x}\ . \tag{13}$$

The above expressions would be iterated to find an optimal number of shipments per batch (n) using the following algorithm:

Step 1. Assume $n=2$ and $TotalCost = +\infty$.
Step 2. Compute the order quantity and shortage quantity using Equations (12) and (13), respectively.
Step 3. Compute the annual cost of the inventory system using Equation (11).
Step 4. If $TCU(n,y,B) < TotalCost$, set $n=n+1$ and repeat Steps 1 through 4. Else Stop.

4.1.2. Repair model

Imperfect-quality items are held until the inspection period, t, is completed, when they are transferred to a workshop for repair. The renovated items are sent back for re-inspection before the working inventory of good items is fully consumed. A 100% re-inspection takes place at a rate X (where $X > D$) during the time period, t_{SR} (where $t_{SR} = py/X$), and are then added to the working inventory as a single batch. In this way, renovated items are added to the working inventory after t_R units of time, including the time needed for transportation, repair and re-inspection.

In repairing the py items, the repair shop incurs the costs of $S + 2A + py\big(c_m + 2c_T + h'(t_R - t_{SR})\big)$, where S is the cost of repair setup, A is the fixed cost of transportation, c_m is the combined cost of materials and labour for repair of each item, c_T is the unit cost of transportation between the inventory system and the repair shop (both ways), and h' is the cost of holding items at the repair shop. Hence, the cost of each repaired item is c_R, which can be expressed as a unit cost multiplied by a markup percentage, as follows:

$$c_R = (1+m)\left(\frac{S+2A}{py} + c_m + 2c_T + h'(t_R - t_{SR})\right), \tag{14}$$

where m is the repair workshop's markup percentage, $t_R = py/w + t_T + t_{SR}$, t_T is the total time needed to transport py units between the inventory system and the repair shop (both ways), w is the repair rate (where $w > D$). Unlike the previous model, the equation for t_3 is

$$t_3 = \frac{I_{max} - Dt_R + py}{D}\,. \tag{15}$$

The holding cost comprises the cost of holding items during the inspection stage, Fig. 3, top, and the cost of holding items in the working inventory, Fig. 3, bottom. Thus, the holding cost per cycle is as follows:

$$\begin{aligned}
HC &= \left(n(yt_S) - \left(\frac{n(n-1)}{2}\right)(qt_S) + pyt_{SR}\right)h \\
&+ \left(\frac{(t_2+t_R+t_3)\left(\left(\frac{t_2}{t_S}\right)q + py + (1-\alpha)q\right)}{2} - \frac{\left(\frac{t_2}{t_S}\right)\left(\frac{t_2}{t_S}+1\right)(qt_S)}{2} - \left(\frac{t_2}{t_S}\right)(pyt_S) - pyt_R\right)h \\
&= \left(\frac{y^2}{x} - \frac{(n-1)(1-p)y^2}{2xn}\right)h \\
&+ \left(\frac{(1-p)xy^2}{2D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2 - \left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]\right. \\
&+ \frac{(1-p)y^2}{2n}\left[(1-\alpha)\left(\frac{1-p}{D} - \frac{n-1}{nx}\right) - \frac{\alpha}{n}\left(\frac{n-1}{x} - \frac{\alpha(1-p)}{(1-p)x-D}\right)\right] \\
&+ \frac{py^2}{2}\left[\frac{1}{D} + \left(\frac{1}{w}+\frac{1}{X}\right)\left(1-p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right) - \frac{\alpha(1-p)}{n((1-p)x-D)} - \frac{2(n-1)}{nx} - \frac{p}{w}\right] \\
&+ \frac{pyB}{2((1-p)x-D)}\left[2 - (1-p)x\left(\frac{1}{w}+\frac{1}{X}+\frac{1}{D}\right)\right] \\
&+ \frac{(1-p)xyB}{2D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n} - 2\left(1-p-\frac{D(n-1)}{nx}\right)\right] \\
&+ \frac{(1-p)yB}{2n}\left(\frac{\alpha}{(1-p)x-D} - \frac{1-\alpha}{D}\right) + \frac{(1-p)xB^2}{2D((1-p)x-D)} \\
&\left.+ \frac{y}{2}\left[t_T\left(1-3p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right)\right] - \frac{(1-p)xt_TB}{n((1-p)x-D)}\right)h\,.
\end{aligned} \tag{16}$$

The time-weighted inventory in the inspection warehouse can be calculated by using the area of rectangles with height y and the area of rectangle with height py minus the area of rectangles with height q. Note the number of the rectangles with height y and the number of rectangles with height q are n and $n(n-1)/2$, respectively. In the lower part of Fig. 3, the time-weighted inventory is the stair-like area above the horizontal axis. It can be calculated by using the area of triangle with base $(t_2 + t_R + t_3)$ minus the area of rectangles with base q, the area of rectangles with base py and the area of rectangle with height t_R. The number of rectangles with base q is $\left(\frac{t_2}{t_S}\right)\left(\frac{t_2}{t_S}+1\right)/2$ and the number of rectangles with base py is $\left(\frac{t_2}{t_S}\right)$. The holding cost per cycle (HC) is divided by the cycle length $T = y/D$, so the annual cost of holding inventory can be calculated as:

$$\begin{aligned}
HCU &= \frac{hDy}{x} - \frac{hD(n-1)(1-p)y}{2xn} + \frac{hx(1-p)y}{2((1-p)x-D)}U_1 + \frac{hD(1-p)y}{2n}U_2 + \frac{hDpy}{2}U_7 + \frac{hDpB}{2((1-p)x-D)}U_8 + \frac{h(1-p)xB}{2((1-p)x-D)}U_3 + \\
&\frac{hD(1-p)B}{2n}U_4 + \frac{h(1-p)xB^2}{2((1-p)x-D)y} + \frac{hDt_T}{2}U_9 - \frac{hD(1-p)xt_TB}{((1-p)x-D)ny}.
\end{aligned} \tag{17}$$

As in the previous model, the accumulated value of the shortage cost, which is time-dependent, is determined on the basis of t_1 and t_4, which are the periods when shortage occurs. Similarly, the backordering level can be measured by calculating the areas of stair-like and triangle below the horizontal axis. Thus, with the shortage cost per cycle SC divided by the cycle length $T = y/D$, the annual cost of inventory shortage is:

$$SCU = \frac{b_1(1-p)xB^2}{2y((1-p)x-D)} - \frac{(1-p)b_1DB}{2n((1-p)x-D)} - \frac{(1-p)^2(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)}\,. \tag{18}$$

The retailer's total cost is the combination of the annual costs for ordering (K/T), purchasing (cy/T), inspection (dy/T), batch transfers ($v(n+1)/T$), repair of imperfect items ($c_R py/T$), holding inventory, and shortage, which is consequently given by:

$$\begin{aligned}
TCU &= \frac{KD}{y} + (c+d)D + \frac{v(n+1)D}{y} + Dp(1+m)\left(\frac{S+2A}{py} + c_m + 2c_T + h'(t_R - t_{SR})\right) \\
&+ \frac{hDy}{x} - \frac{hD(n-1)(1-p)y}{2xn} + \frac{hx(1-p)y}{2((1-p)x-D)}U_1 + \frac{hD(1-p)y}{2n}U_2 + \frac{h(1-p)xB}{2((1-p)x-D)}U_3 \\
&+ \frac{hD(1-p)B}{2n}U_4 + \frac{hDpy}{2}U_7 + \frac{hDpB}{2((1-p)x-D)}U_8 + \frac{hDt_T}{2}U_9 - \frac{hD(1-p)xt_TB}{((1-p)x-D)ny}
\end{aligned}$$

$$-\frac{(1-p)b_1DB}{2n\big((1-p)x-D\big)}-\frac{(1-p)^2(\alpha-1)b_1\alpha Dy}{2n^2\big((1-p)x-D\big)}+\frac{(h+b_1)(1-p)xB^2}{2\big((1-p)x-D\big)y}. \tag{19}$$

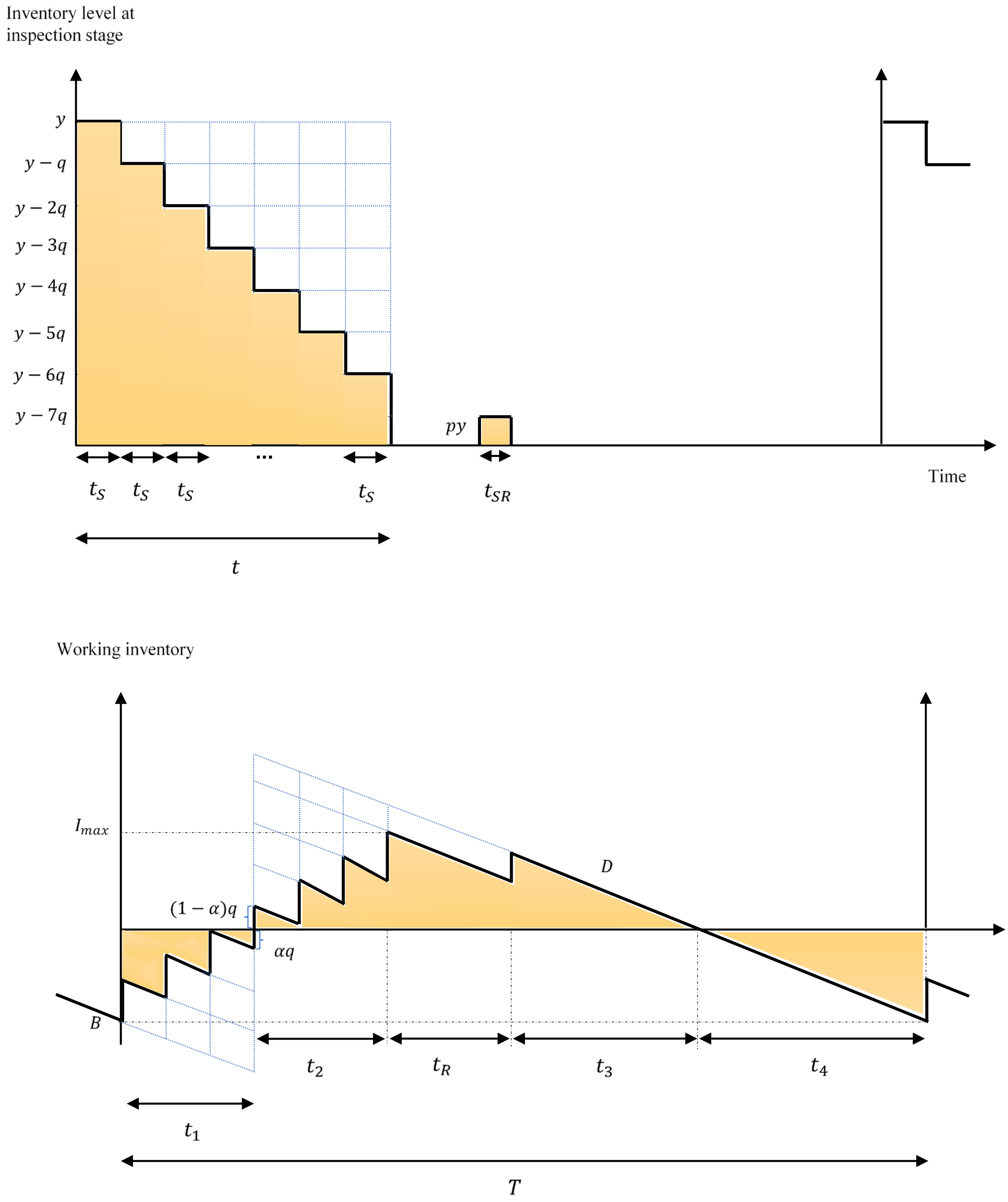

Fig. 3. Inventory level for Repair model, where $n=7$

For a fixed number of shipments for a batch, the order quantity and the maximum shortage quantity would be:

$$y=\sqrt{\frac{D[K+v(n+1)+(1+m)(S+2A)]-\frac{h^2D^2t_T^2x(1-p)}{2n^2(h+b_1)\big((1-p)x-D\big)}}{U_{10}-\frac{\big((1-p)x-D\big)U_{11}^2}{2x(1-p)(h+b_1)}}}, \tag{20}$$

$$B = \frac{((1-p)x-D)U_{11}y + \frac{hD(1-p)xt_T}{n}}{(h+b_1)(1-p)x}. \tag{21}$$

4.2. Green technology investment, cost reduction effect and carbon regulations

Let us suppose that the carbon emissions resulting from the purchase of a single unit of a given product are represented by E_P. At the outset of each inventory cycle, the retailer is obliged to prepare an order, which will inevitably give rise to carbon emissions, E_O. The distance travelled and the size of the transportation lot have a considerable impact on the carbon emissions generated during the transportation process. The carbon emissions from the transportation process can be obtained by multiplying the carbon emissions from a unit product, E_T, by the delivery distance, L. Furthermore, carbon emissions may also result from the storage of undelivered or unsold products, which may be attributed to product characteristics or other factors. The yearly carbon emissions from product storage can be calculated by multiplying the carbon emissions from storing a unit product, E_h, by the sum of the average inventory from the retailer. The carbon emissions resulting from the repair of a single unit of product are designated as E_R. The repair centre is required to set up production equipment at the commencement of each inventory cycle, which would result in the emission of carbon, E_S. The carbon emissions from the transportation process can be obtained by multiplying the carbon emissions from a unit distance, E_T, by the delivery distance, L_1. In the event that the quantity of carbon emissions generated by the retail system exceeds the permitted threshold, rather than incurring a penalty cost, it is possible to invest in green technology with the objective of reducing emissions. The carbon reduction function for this technology is given by $CR(G) = \theta G - \beta G^2$, where θ denotes the carbon reduction efficiency factor and β denotes the offsetting carbon reduction factor. This indicates that as the retailer invests the green cost, G, in the green technology, a reduction of θG in carbon emissions can be achieved. However, it should be noted that the use of the green technology may also result in an increase in energy consumption, which is represented by βG^2. The values of θ and β can be obtained by fitting the historical data of carbon emissions reduction and the amount of green investment (Huang et al., 2020). The retailer's purchasing cost, denoted by $c \geq 0$, is a constant per product. Furthermore, the retailer may implement green initiatives with the objective of reducing the purchasing cost, thus enabling process innovation investments. In particular, the purchasing cost can be decreased by $g_c > 0$. This is the objective of a process innovation, namely, reducing the impact of operations by reducing the purchasing cost. Consequently, the purchasing cost function can be expressed as follows: $c(G) = c - g_c G$. It should be noted that the term $g_c G$ represents the operational benefits that innovation-led programs provide (Genç and DeGiovanni, 2020). Furthermore, the reduction in unit cost can be defined as a margin enhancement effect of greening (Ghosh et al., 2020). In considering the carbon emissions from the ordering and purchasing processes, transporting, and holding, together with investment in green technology and cost reduction effect, the total cost of inventory and the amount of carbon emissions for Salvage model are calculated by:

$$TCU(n,y,B,G) = \frac{KD}{(1-p)y} + \frac{(c-g_cG+d)D}{1-p} + \frac{vnD}{(1-p)y} + \frac{hDy}{(1-p)x} - \frac{hD(n-1)y}{2xn} + \frac{hxy}{2((1-p)x-D)}U_1 + \frac{hDy}{2n}U_2 + \frac{hxB}{2((1-p)x-D)}U_3 + \frac{hDB}{2n}U_4 - \frac{b_1DB}{2n((1-p)x-D)} - \frac{(1-p)(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)} + \frac{(h+b_1)xB^2}{2((1-p)x-D)y} + G, \tag{22}$$

And

$$CE = \frac{DE_P}{1-p} + \frac{DE_0}{(1-p)y} + \frac{DLE_Tn}{(1-p)y} + \left(\frac{Dy}{(1-p)x} - \frac{D(n-1)y}{2xn} + \frac{xy}{2((1-p)x-D)}U_1 + \frac{Dy}{2n}U_2 + \frac{xB}{2((1-p)x-D)}U_3 + \frac{DB}{2n}U_4 + \frac{xB^2}{2((1-p)x-D)y}\right)E_h - (\theta G - \beta G^2), \tag{23}$$

respectively. To take into account the carbon emissions from ordering and purchasing, transportation, storage, and repair, as well as the cost of investment in green technology, the total cost of inventory for the Repair model is derived as follows:

$$\begin{aligned} TCU(n,y,B,G) &= \frac{KD}{y} + (c - g_cG + d)D + \frac{v(n+1)D}{y} + Dp(1+m)\left(\frac{S+2A}{py} + c_m + 2c_T + h'\left(\frac{py}{w} + t_T\right)\right) \\ &+ \frac{hDy}{x} - \frac{hD(n-1)(1-p)y}{2xn} + \frac{hx(1-p)y}{2((1-p)x-D)}U_1 + \frac{hD(1-p)y}{2n}U_2 + \frac{h(1-p)xB}{2((1-p)x-D)}U_3 \\ &+ \frac{hD(1-p)B}{2n}U_4 + \frac{hDpy}{2}U_7 + \frac{hDpB}{2((1-p)x-D)}U_8 + \frac{hDt_T}{2}U_9 - \frac{hD(1-p)xt_TB}{((1-p)x-D)ny} \\ &- \frac{(1-p)b_1DB}{2n((1-p)x-D)} - \frac{(1-p)^2(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)} + \frac{(h+b_1)(1-p)xB^2}{2((1-p)x-D)y} + G, \end{aligned} \tag{24}$$

while carbon emission amount is:

$$\begin{aligned} CE &= DE_P + \frac{DE_0}{y} + \frac{DLE_T(n+1)}{y} + E_RDp + \frac{E_SD}{y} + \frac{2DL_1E_T}{y} + \left(Dp\left[\frac{py}{w} + t_T\right]\right)E_{h'} \\ &+ \left(\frac{Dy}{x} - \frac{D(n-1)(1-p)y}{2xn} + \frac{x(1-p)y}{2((1-p)x-D)}U_1 + \frac{D(1-p)y}{2n}U_2 + \frac{(1-p)xB}{2((1-p)x-D)}U_3 + \frac{D(1-p)B}{2n}U_4 \right. \end{aligned}$$

$$+\frac{Dpy}{2}U_7+\frac{DpB}{2((1-p)x-D)}U_8+\frac{Dt_T}{2}U_9-\frac{D(1-p)xt_TB}{((1-p)x-D)ny}+\frac{(1-p)xB^2}{2((1-p)x-D)y}\Bigg)E_h-(\theta G-\beta G^2). \tag{25}$$

4.2.1. Carbon tax regulation

A carbon tax imposes a charge for each unit of carbon dioxide emitted, incentivizing companies and individuals to cut their emissions. This is a simple approach by which the government fixes a price for each unit of carbon emissions and when companies emit carbon, they have to pay that price; the higher the emissions, the more tax they pay. The purpose is to encourage the use of cleaner, more efficient technologies by making carbon-emitting processes more expensive. In this model, the carbon tax is set at C_1 for a per-unit carbon emission, increasing linearly as the volume of carbon dioxide emissions increases. The retailer can mitigate the tax payments by making investments in environmental projects that lower carbon dioxide emissions. The inventory model costs comprise the retailer's cost for order processing, the cost of transporting products, cost of holding inventory, shortage cost, carbon tax payment, and expenditure on environmentally friendly investments. By totalling the quantity of carbon emitted during ordering and purchasing, transporting product, and holding inventory, then deducting the effectiveness of carbon emission reduction from the environmental cost, we can obtain the overall quantity of carbon dioxide emissions.

4.2.1.1. Salvage model under carbon tax regulation

The total cost of carbon taxation is calculated as:

$$\begin{aligned}TCU(n,y,B,G)&=TCU(n,y,B,G)+C_1(CE)\\
&=\frac{D(K+vn+C_1E_0+C_1E_TLn)}{(1-p)y}+\frac{D(c-g_cG+d+C_1E_P)}{1-p}+\frac{(h+C_1E_h)Dy}{2}\Bigg(\frac{2}{(1-p)x}-\frac{(n-1)}{xn}\\
&+\frac{x}{D((1-p)x-D)}U_1+\frac{1}{n}U_2\Bigg)+\frac{(h+C_1E_h)DB}{2}\Bigg(\frac{x}{D((1-p)x-D)}U_3+\frac{1}{n}U_4\Bigg)\\
&-\frac{b_1DB}{2n((1-p)x-D)}-\frac{(1-p)(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)}+\frac{(h+b_1+C_1E_h)xB^2}{2((1-p)x-D)y}-C_1(\theta G-\beta G^2)+G.\end{aligned} \tag{26}$$

Proposition 1. Under the carbon tax regulation, the total cost function of the retailer is convex in y for given values of n, B and G and the optimal order quantity for the retailer is given by

$$y=\sqrt{\frac{[D(K+vn+C_1E_0+C_1E_TLn)]/(1-p)}{U_{12}-\frac{((1-p)x-D)U_{13}^2}{2x(h+b_1+C_1E_h)}}}. \tag{27}$$

Proof. Partial differentiation of Equation (26) with respect to y gives

$$\begin{aligned}\frac{\partial TCU(n,y,B,G)}{\partial y}&=-\frac{D(K+vn+C_1E_0+C_1E_TLn)}{(1-p)y^2}+\frac{(h+C_1E_h)D}{2}\Bigg(\frac{2}{(1-p)x}-\frac{(n-1)}{xn}+\frac{x}{D((1-p)x-D)}U_1+\frac{1}{n}U_2\Bigg)\\
&-\frac{(1-p)(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)}-\frac{(h+b_1+C_1E_h)xB^2}{2((1-p)x-D)y^2},\end{aligned} \tag{28}$$

$$\frac{\partial^2 TCU(n,y,B,G)}{\partial y^2}=\frac{2D(K+vn+C_1E_0+C_1E_TLn)}{(1-p)y^3}+\frac{(h+b_1+C_1E_h)xB^2}{((1-p)x-D)y^3}. \tag{29}$$

From Eq. (29), it is observed that all the terms in the right-hand side are positive which implies $\partial^2TCU_t(n,y,B,G)/\partial y^2>0$. So, the total cost function of the retailer is convex in y for given n, B and G. From the first-order optimality condition, i.e. by solving $\partial TCU_t(n,y,B,G)/\partial y=0$ for y, we obtain the optimal order quantity as

$$y^*=\sqrt{[D(K+vn+C_1E_0+C_1E_TLn)/(1-p)]/\left[U_{12}-\frac{((1-p)x-D)U_{13}^2}{2x(h+b_1+C_1E_h)}\right]}.$$

Proposition 2. Under the carbon tax regulation, the total cost function of the retailer is convex in B for given values of n, y and G and the optimal shortage quantity for the retailer is given by

$$B=\frac{((1-p)x-D)U_{13}y}{(h+b_1+C_1E_h)x}. \tag{30}$$

Proof. Partial differentiation of Equation (26) with respect to B gives

$$\frac{\partial TCU(n,y,B,G)}{\partial B}=\frac{(h+C_1E_h)D}{2}\left(\frac{x}{D\big((1-p)x-D\big)}U_3+\frac{1}{n}U_4\right)-\frac{b_1D}{2n\big((1-p)x-D\big)}+\frac{(h+b_1+C_1E_h)xB}{\big((1-p)x-D\big)y}, \tag{31}$$

$$\frac{\partial^2 TCU(n,y,B,G)}{\partial B^2}=\frac{(h+b_1+C_1E_h)x}{\big((1-p)x-D\big)y}, \tag{32}$$

It is clear from Eq. (32) that $\partial^2 TCU_t(n,y,B,G)/\partial B^2>0$. Therefore, the total cost function of the retailer is convex in B and then by solving $\partial TCU_t(n,y,B,G)/\partial B=0$ for B, we obtain the optimal shortage quantity as $B^*=\big[\big((1-p)x-D\big)U_{13}y\big]/[(h+b_1+C_1E_h)x]$.

Proposition 3. Under the carbon tax regulation, the total cost function of the retailer is convex in G for given values of n,y and B and the optimal green investment amount for the retailer is given by

$$G=\frac{C_1\theta+\frac{Dg_c}{1-p}-1}{2C_1\beta}. \tag{33}$$

Proof. Partial differentiation of Equation (26) with respect to G gives

$$\frac{\partial TCU(n,y,B,G)}{\partial G}=-\frac{Dg_c}{1-p}-C_1(\theta-2\beta G)+1\,. \tag{34}$$

$$\frac{\partial^2 TCU(n,y,B,G)}{\partial G^2}=2C_1\beta\,. \tag{35}$$

Clearly, the total cost function of the retailer is convex in G and then by solving $\partial TCU_t(n,y,B,G)/\partial G=0$ for G, we obtain the optimal green investment amount as $G^*=\left[C_1\theta+\frac{Dg_c}{1-p}-1\right]/[2C_1\beta]$.

Solution algorithm

The aim is to calculate the value of n that reduces $TCU(n,y,B,G)$ to a minimum. The number of batches of good items, n, is a discrete variable, so the optimal value of n can be found through the following algorithm:

Step 1. Assume $n=2$ and $TotalCost=+\infty$.
Step 2. Compute the order and shortage quantities and green investment amount using Eq. (27), Eq. (30) and Eq. (33), respectively.
Step 3. Compute the annual cost of the inventory system using Eq. (26).
Step 4. If $TCU(n,y,B,G)<TotalCost$, set $n=n+1$ and repeat Steps 1 through 4. Else Stop.
Step 5. Having obtained the optimal value of n (say n^*), the optimal batch size of good items q^* is obtained from $q^*=(1-p)y^*/n^*$.

4.2.1.2. Repair model under carbon tax regulation

The total cost of carbon taxation is derived as:

$$\begin{aligned}TCU(n,y,B,G)&=TCU(n,y,B,G)+C_1(CE)\\&=\frac{D(K+v(n+1)+C_1E_0+C_1E_TL(n+1)+C_1E_S+2C_1L_1E_T)}{y}+D(c-g_cG+d+C_1E_P+C_1E_Rp)\\&+Dp(1+m)\left(\frac{S+2A}{py}+c_m+2c_T+h'\left(\frac{py}{w}+t_T\right)\right)+C_1E_{h'}Dp\left[\frac{py}{w}+t_T\right]\\&+\frac{(h+C_1E_h)Dy}{2}\left(\frac{2}{x}-\frac{(n-1)(1-p)}{xn}+\frac{(1-p)x}{D\big((1-p)x-D\big)}U_1+\frac{1-p}{n}U_2+pU_7\right)\\&+\frac{(h+C_1E_h)DB}{2}\left(\frac{(1-p)x}{D\big((1-p)x-D\big)}U_3+\frac{1-p}{n}U_4+\frac{p}{\big((1-p)x-D\big)}U_8\right)\end{aligned}$$

$$+\frac{(h+C_1E_h)Dt_T}{2}U_9-\frac{(h+C_1E_h)D(1-p)xt_TB}{((1-p)x-D)ny}-\frac{(1-p)b_1DB}{2n((1-p)x-D)}$$
$$-\frac{(1-p)^2(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)}+\frac{(h+b_1+C_1E_h)(1-p)xB^2}{2((1-p)x-D)y}-C_1(\theta G-\beta G^2)+G\ . \tag{36}$$

The following results are similar; hence their proof is omitted here.

Proposition 4. Under the carbon tax regulation, the total cost function of the retailer is convex in y for given values of n, B and G and the optimal order quantity for the retailer is given by

$$y=\sqrt{\frac{D[K+v(n+1)+C_1E_0+C_1E_TL(n+1)+C_1E_S+2C_1L_1E_T+(1+m)(S+2A)]-\frac{D^2t_T^2x(h+C_1E_h)^2(1-p)}{2((1-p)x-D)(h+b_1+C_1E_h)n^2}}{U_{14}-\frac{((1-p)x-D)U_{15}^2}{2x(1-p)(h+b_1+C_1E_h)}}}\ . \tag{37}$$

Proposition 5. Under the carbon tax regulation, the total cost function of the retailer is convex in B for given values of n, y and G and the optimal shortage quantity for the retailer is given by

$$B=\frac{((1-p)x-D)U_{15}y+\frac{(h+C_1E_h)(1-p)Dxt_T}{n}}{(h+b_1+C_1E_h)(1-p)x}\ . \tag{38}$$

Proposition 6. Under the carbon tax regulation, the total cost function of the retailer is convex in G for given values of n, y and B and the optimal green investment amount for the retailer is given by

$$G=\frac{C_1\theta+Dg_c-1}{2C_1\beta}. \tag{39}$$

4.2.2. Cap-and-trade regulation

The cap-and-trade policy establishes a regulatory framework for the total amount of carbon emissions permitted from the retailer. In the event that the total amount of carbon emissions does not exceed the upper limit, designated as U, the surplus may be sold at the price of C_2 per unit, thereby offsetting the anticipated costs. Conversely, in the event that carbon emissions exceed the upper limit, the firm is obliged to purchase allowances from other entities or invest in green costs in order to comply with the regulations pertaining to the limitation of carbon emissions. We may therefore posit that the surplus is valid only in the current period, irrespective of whether it is sold or purchased, and that the carbon trading price, C_2 represents the average price in the market. Furthermore, it is assumed that there is sufficient availability of carbon emissions allowances for purchase in the market. The surplus, $CE-U$, can be obtained by subtracting the sum of carbon emissions from the production activity, product transportation, and inventory holding of both parties from the upper limit of carbon emissions. Additionally, the carbon emissions reduction effectiveness must be subtracted from the investment in green technologies.

4.2.2.1. Salvage model under cap-and-trade regulation

The total cost of the inventory system is given by

$$TCU(n,y,B,G)=TCU(n,y,B,G)+C_2(CE-U)$$
$$=\frac{D(K+vn+C_2E_0+C_2E_TLn)}{(1-p)y}+\frac{D(c-g_cG+d+C_2E_P)}{1-p}+\frac{(h+C_2E_h)Dy}{2}\Big(\frac{2}{(1-p)x}-\frac{(n-1)}{xn}$$
$$+\frac{x}{D((1-p)x-D)}U_1+\frac{1}{n}U_2\Big)+\frac{(h+C_2E_h)DB}{2}\Big(\frac{x}{D((1-p)x-D)}U_3+\frac{1}{n}U_4\Big)$$
$$-\frac{b_1DB}{2n((1-p)x-D)}-\frac{(1-p)(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)}+\frac{(h+b_1+C_2E_h)xB^2}{2((1-p)x-D)y}-C_2U-C_2(\theta G-\beta G^2)+G. \tag{40}$$

Proposition 7. Under the cap-and-trade regulation, the total cost function of the retailer is convex in y for given values of n, B and G and the optimal order quantity for the retailer is given by

$$y = \sqrt{\frac{[D(K+vn+C_2E_0+C_2E_TLn)]/(1-p)}{U_{16}-\frac{((1-p)x-D)U_{17}^2}{2x(h+b_1+C_2E_h)}}}\,. \tag{41}$$

Proof. Partial differentiation of Eq. (40) with respect to y gives

$$\frac{\partial TCU(n,y,B,G)}{\partial y} = -\frac{D(K+vn+C_2E_0+C_2E_TLn)}{(1-p)y^2} + \frac{(h+C_2E_h)D}{2}\left(\frac{2}{(1-p)x} - \frac{(n-1)}{xn} + \frac{x}{D((1-p)x-D)}U_1 + \frac{1}{n}U_2\right)$$
$$-\frac{(1-p)(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)} - \frac{(h+b_1+C_1E_h)xB^2}{2((1-p)x-D)y^2}\,, \tag{42}$$

$$\frac{\partial^2 TCU(n,y,B,G)}{\partial y^2} = \frac{2D(K+vn+C_1E_0+C_1E_TLn)}{(1-p)y^3} + \frac{(h+b_1+C_1E_h)xB^2}{((1-p)x-D)y^3}\,. \tag{43}$$

From Eq. (43), it is observed that all the terms in the right-hand side are positive which implies $\partial^2 TCU(n,y,B,G)/\partial y^2 > 0$. So, the total cost function of the retailer is convex in y for given n, B and G. From the first-order optimality condition, i.e. by solving $\partial TCU_c(n,y,B,G)/\partial y = 0$ for y, we obtain the optimal order quantity as $y^* = \sqrt{[D(K+vn+C_2E_0+C_2E_TLn)/(1-p)]/\left[U_{16}-\frac{((1-p)x-D)U_{17}^2}{2x(h+b_1+C_2E_h)}\right]}$.

Proposition 8. Under the cap-and-trade regulation, the total cost function of the retailer is convex in B for given values of n, y and G and the optimal shortage quantity for the retailer is given by

$$B = \frac{((1-p)x-D)U_{17}y}{(h+b_1+C_1E_h)x}\,. \tag{44}$$

Proof. Partial differentiation of Equation (40) with respect to B gives

$$\frac{\partial TCU(n,y,B,G)}{\partial B} = \frac{(h+C_2E_h)D}{2}\left(\frac{x}{D((1-p)x-D)}U_3 + \frac{1}{n}U_4\right) - \frac{b_1D}{2n((1-p)x-D)} + \frac{(h+b_1+C_2E_h)xB}{((1-p)x-D)y}, \tag{45}$$

$$\frac{\partial^2 TCU(n,y,B,G)}{\partial B^2} = \frac{(h+b_1+C_2E_h)x}{((1-p)x-D)y}\,, \tag{46}$$

It is clear from Equation (46) that $\partial^2 TCU(n,y,B,G)/\partial B^2 > 0$. Therefore, the total cost function of the retailer is convex in B and then by solving $\partial TCU(n,y,B,G)/\partial B = 0$ for B, we obtain the optimal shortage quantity as $B^* = \left[((1-p)x-D)U_{17}y\right]/[(h+b_1+C_2E_h)x]$.

Proposition 9. Under the cap-and-trade regulation, the total cost function of the retailer is convex in G for given values of n, y and B and the optimal green investment amount for the retailer is given by

$$G = \frac{C_2\theta + \frac{Dg_c}{1-p} - 1}{2C_2\beta}. \tag{47}$$

Proof. Partial differentiation of Eq. (40) with respect to G gives

$$\frac{\partial TCU_c(n,y,B,G)}{\partial G} = -\frac{Dg_c}{1-p} - C_2(\theta - 2\beta G) + 1\,, \tag{48}$$

$$\frac{\partial^2 TCU_c(n,y,B,G)}{\partial G^2} = 2C_2\beta\,. \tag{49}$$

Clearly, the total cost function of the retailer is convex in G and then by solving $\partial TCU_t(n,y,B,G)/\partial G = 0$ for G, we obtain the optimal green investment amount as $G^* = [C_2\theta + (Dg_c/1-p) - 1]/[2C_2\beta]$.

Solution algorithm

The aim is to calculate the value of n that reduces $TCU(n, y, B, G)$ to a minimum. The number of batches of good items, n, is a discrete variable, so the optimal value of n can be found through the following algorithm:

Step 1. Assume $n = 2$ and $TotalCost = +\infty$.
Step 2. Compute the order and shortage quantities and green investment amount using Equations (41), (44) and (47), respectively.
Step 3. Compute the annual cost of the inventory system using Equation (40).
Step 4. If $TCU(n, y, B, G) < TotalCost$, set $n = n + 1$ and repeat Steps 1 through 4. Else Stop.
Step 5. Having obtained the optimal value of n (say n^*), the optimal batch size of good items q^* is obtained from $q^* = (1-p)y^*/n^*$.

4.2.2.2. Repair model under cap-and-trade regulation

The total cost of the inventory system is given by

$$\begin{aligned}
&TCU(n, y, B, G) = TCU(n, y, B, G) + C_2(CE - U) \\
&= \frac{D(K+v(n+1)+C_2E_0+C_2E_TL(n+1)+C_2E_S+2C_2L_1E_T)}{y} + D(c - \mathrm{g}_cG + d + C_2E_P + C_2E_Rp) \\
&+Dp(1+m)\left(\frac{S+2A}{py} + c_m + 2c_T + h'\left(\frac{py}{w} + t_T\right)\right) + C_2E_{h'}Dp\left[\frac{py}{w} + t_T\right] \\
&+\frac{(h+C_2E_h)Dy}{2}\left(\frac{2}{x} - \frac{(n-1)(1-p)}{xn} + \frac{(1-p)x}{D((1-p)x-D)}U_1 + \frac{1-p}{n}U_2 + pU_7\right) \\
&+\frac{(h+C_2E_h)DB}{2}\left(\frac{(1-p)x}{D((1-p)x-D)}U_3 + \frac{1-p}{n}U_4 + \frac{p}{((1-p)x-D)}U_8\right) \\
&+\frac{(h+C_2E_h)Dt_T}{2}U_9 - \frac{(h+C_2E_h)D(1-p)xt_TB}{((1-p)x-D)ny} - \frac{(1-p)b_1DB}{2n((1-p)x-D)} \\
&-\frac{(1-p)^2(\alpha-1)b_1\alpha Dy}{2n^2((1-p)x-D)} + \frac{(h+b_1+C_2E_h)(1-p)xB^2}{2((1-p)x-D)y} - C_2U - C_2(\theta G - \beta G^2) + G \,. \qquad (50)
\end{aligned}$$

The following results are similar; hence their proof is omitted here.

Proposition 10. Under the cap-and-trade regulation, the total cost function of the retailer is convex in y for given values of n, B and G and the optimal order quantity for the retailer is given by

$$y = \sqrt{\frac{\begin{array}{c}D[K+v(n+1)+C_2E_0+C_2E_TL(n+1)+C_2E_S+2C_2L_1E_T+(1+m)(S+2A)] \\ -\frac{D^2xt_T^2(h+C_2E_h)^2(1-p)}{2n^2((1-p)x-D)(h+b_1+C_2E_h)}\end{array}}{U_{18}-\frac{((1-p)x-D)U_{19}^2}{2x(1-p)(h+b_1+C_2E_h)}}}\,. \qquad (51)$$

Proposition 11. Under the cap-and-trade regulation, the total cost function of the retailer is convex in B for given values of n, y and G and the optimal shortage quantity for the retailer is given by

$$B = \frac{((1-p)x-D)U_{19}y + \frac{(h+C_2E_h)(1-p)Dxt_T}{n}}{(h+b_1+C_2E_h)(1-p)x}\,. \qquad (52)$$

Proposition 12. Under the cap-and-trade regulation, the total cost function of the retailer is convex in G for given values of n, y and B and the optimal green investment amount for the retailer is given by

$$G = \frac{C_2\theta + D\mathrm{g}_c - 1}{2C_2\beta}. \qquad (53)$$

5. An illustrative example

This section presents a numerical example with a sensitivity analysis to indicate the extent to which the proposed model is applicable and reliable. The model will be explained by solving this numerical example, originated by Konstantaras et al.

(2007) and Huang et al. (2020), to which values are added for the new parameters introduced in this model. The example considers an inventory situation where a retailer purchases a certain medical product, replenishing its stock instantly with y units, which are not all of the required quality. The estimated demand for the product is 50000 units. The cost of processing each order is \$100, and the retailer's cost for holding inventory is \$5 per unit. The inspection rate is 175,200 units per year; 50,000 units can be repaired per year, and it costs \$100 to set up the repair facility for each cycle. The holding cost during repair is \$4 per unit. The distance from the retailer to the repair workshop is roughly 20 km. Processing the retailer's order creates 10 units of carbon emissions, while purchasing a product results in 2 carbon units of carbon. Delivery of products from inspection to the working inventory results in 5 units of carbon emissions per kilometre. Annual storage of the product creates 4 units of carbon emissions. The retailer's carbon reduction has an efficiency factor of 15, with an offsetting carbon reduction factor of 0.01. Details of the data used in the example are presented below.

Demand rate $D = 50000$ units/year,
Inspection rate $X = 175200$ units/year,
Retailer's ordering cost $K = 100$ \$/cycle,
Purchasing cost $c = 25$ \$/unit,
Holding cost $h = 5$ \$/unit/year,
Transportation cost of a batch of good items to the working inventory $v = 5$ \$/batch,
Screening cost $d = 0.5$ \$/unit,
Shortage cost $b1 = 3$ \$/unit,
Shortage rate $\alpha = 0.80$
The percentage of imperfect quality items $p = 0.02$
Repair setup cost $S = 100$ \$
Transportation fixed cost $A = 200$ \$
Unit transportation cost $c_T = 0.5$ \$/unit
Unit material cost $c_m = 1$ \$/unit
Unit holding cost $h' = 1$ \$/unit/year
Repair rate $w = 50000$ units/year
Total transport time $t_T = 2/220$ year
Markup percentage $m = 0.10$
Distance between warehouses $L = 0.5$ km
Distance between retailer and repair shop $L_1 = 20$ km
Carbon emissions from holding a unit product $E_h = 4$,
Carbon emissions from holding a unit product in repair shop $E_{h'} = 4$,
Carbon emissions from sending a batch of good items to the working inventory $E_T = 5$,
Carbon emissions from purchasing/repairing a unit product $E_P = E_R = 2$,
Carbon emissions from ordering/repair setup product $E_O = E_S = 10$,
The carbon tax of unit carbon emission $C_1 = \$1.2$,
The carbon trading price of unit carbon emission $C_2 = \$1.8$
The benefit of green to the cost of purchase $g_c = 0.001$,
The efficiency factor of carbon emissions reduction $\theta = 15$,
The offset factor of carbon emissions reduction $\beta = 0.01$,
Cap on carbon emissions in the cap-and-trade policy $U = 160000$

Table 1
Optimal solutions for both models under cost reduction effect and different carbon regulations

Decision variables and cost	Salvage model	Repair model	Models under cost reduction effect and carbon tax regulation		Models under cost reduction effect and cap-and-trade regulation	
			Salvage	Repair	Salvage	Repair
Number of batches (n)	5	9	4	7	4	6
Order lot size (y)	2000.47	4839.61	1627.22	4955.41	1553.83	5191.91
Shortage quantity (B)	925.56	2260.53	923.43	2850.19	921.72	3149.12
Green investment amount (G)	-	-	2834.18	2791.67	2139.46	2111.11
Carbon emissions (CE)	-	-	141916.15	144949.66	117714.21	121789.57
Purchase cost per unit (c)	25	25	22.17	22.21	22.86	22.89
Total cost	1307396.46	1292614.79	1336108.26	1330086.10	1124547.63	1121020.57

The optimal solutions based on developed models are shown in Table 1. The results show that in the context of outsourcing repair of imperfect quality products, carbon regulations lead to increased quantities of products ordered and backorders and a decrease in the number of shipments. In addition, the difference in carbon regulations changes investment in green technologies. More importantly, carbon regulations provide an alternative way for retailers to reap the benefits of emissions reductions in the tax and cap-and-trade carbon market. As a result, the cost of a carbon tax exceeds the cost of cap-and-trade regulation, encouraging retailers to increase lot sizes and increase investment in green technologies. Although retailers have no control over the quantities of defective products, they can offset the cost increase resulting from the repair process and the significant investment in research and development to reduce emissions by adjusting outsourcing costs. It is clear that carbon regulations create differences in retailers' operational activities. As a result, a cap-and-trade regulation provides cost savings for retailers by allowing them to achieve emissions reductions with greater investment.

The results emphasize that carbon regulations lead to a decrease in the number of shipments and the quantity of products ordered and backordered in the Salvage model. Expanding on above results, which demonstrates that carbon regulations cause a decline in the quantity of ordered products, Savage model addresses this decline and the substantial green technology investments. In contrast, the Repair model maintains demand increase, and carbon regulations result in a remanufacturing for imperfect quality products. This implies that retailers can capture carbon regulations benefits through third-party repair facilities, ensuring beneficial cost savings. According to results, the carbon regulations prompts retailers to decrease order lot size in the Salvage model. In the Salvage model, retailers only sell good products, while in the Repair model, they offer both good and repaired products concurrently. Consequently, in the Salvage Model, retailers cannot ignore the demand erosion caused by imperfect quality products, adding pressure when determining the quantity of good products. Furthermore, retailers show increased interest in investment of emission reduction technology in the Salvage Model. Advanced emission reduction technology has the potential to significantly enhance the profitability of repaired products, incentivizing retailers to increase their output. In the Repair model, retailers directly observe the impact of investments in emission reduction technology on the quantity ordered and cost effectiveness of such products. To maximize the benefits associated with emission reduction technology, retailers become more actively used in carbon regulations and increase their investment efforts. The results assert that the Repair Model will be more profitable. Firstly, retailers can precisely control outsourcing costs to manage the quantity of repaired products, while relying on salvage value. Referring to the above results, retailers possess combined dominance in repairing and sales, enabling them to effectively maximize profits. Furthermore, in the Salvage model, retailers are more affected by the price of carbon regulations and emission reduction technologies compared to the Repair model. While retailers may have the option of increasing profits by outsourcing the repair of defective products, any alterations to the carbon cap-and-trade market or investment obstacles can have a considerable influence on the retailer's operational decision-making.

In the case of the carbon tax regulation, it was determined that the purchase cost of the savage model was less than that of the repair model. It should be recalled that imperfect quality products are sold at a discounted price in the Salvage model, whereas these products are repaired at an external facility in the Repair model. The outcome is of interest in that, despite the financial benefits to the retailer of reduced costs resulting from the greening initiative and the advantages gained from synergies in decision-making processes, the savage model results in an increased financial burden for the retailer. In the cap-and-trade regulation, it is evident that the savage model incurs lower costs than the repair model, as a result of the cost reduction effect. The collective findings indicate that a cost-reduction initiative yields benefits for customers. In both carbon regulations, the cost reduction effect allows for a lower costing strategy to be employed. It is clear that greater investment in green technology will also benefit end consumers. A comparative analysis of costs indicates that the financial burden on the retailer in the context of a carbon tax regulation is greater than it would be in the absence of such a tax and associated investment. This is despite the implementation of a cost-reduction strategy. These findings may be attributed to the benefits of the cap-and-trade scheme, which result in reduced greening costs and a cost reduction for the retailer, and increased benefits associated with higher greening efforts, leading to higher gross margins. It is noteworthy that the carbon cap-and-trade effect, coupled with the reduction in product costs resulting from greening efforts, has led to an increase in margins. The findings indicate that outsourcing repair can be advantageous for the retailer in the context of green technology investment and cost reduction. For the sake of convenience, we shall henceforth designate the four models under consideration in this study as {t-salvage, c-salvage, t-repair, c-repair}. In this context, the expressions {t-salvage, c-salvage} correspond to the two models that consider the salvage of imperfect products, while {t-repair, c-repair} represent the two models that consider the outsourcing of repair for imperfect products. Additionally, the subscripts t and c signify the tax and cap-and-trade, respectively.

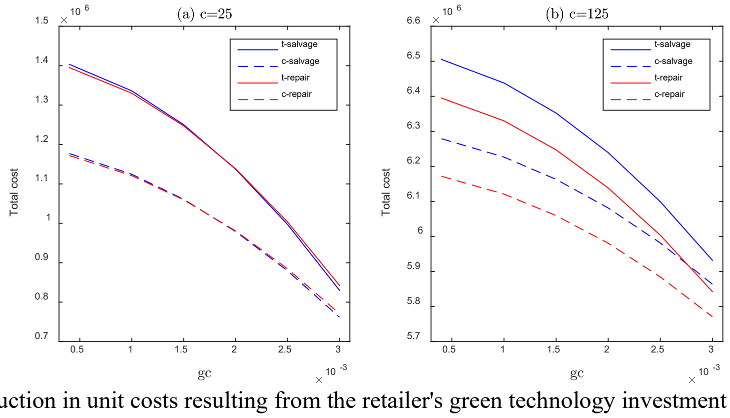

Fig. 4. Impact of the reduction in unit costs resulting from the retailer's green technology investment on the retailer's total costs.

Fig. 4 demonstrates a distinct negative correlation between the retailer's optimal cost and the reduction in unit costs resulting from the retailer's level of green technology investment across all models. Furthermore, the impact of the retailer's purchasing cost on cost savings is dependent upon the cost of the purchase in question (c). When the purchasing cost is high (e.g., $c = 125$), if the reduction in unit cost due to green technology investment is below the relevant threshold, the retailer will save more from repairing imperfect products than from selling them at a discounted price. Conversely, if the reduction in unit cost resulting from green technology investment exceeds a certain threshold, the retailer will consider repairing imperfect products. Therefore, for a retailer aiming to reduce costs with low purchasing costs, it would be advantageous to purchase when the reduction in unit cost due to green technology investment is high. Fig. 5 examines the impact of the reduction in unit cost resulting from the retailer's investment in green technology on green technology investment across all models. The results demonstrate that in all models, the retailer's green technology investment is significantly and positively influenced by this increase. This effect is even more pronounced in models that take into account the carbon tax regulation. While the decrease in unit cost due to the retailer's green technology investment is approximately 0.0004, the difference between the green technology investments of different models under the same carbon regulation is relatively minor. Subsequently, the discrepancy between the models increases significantly. Furthermore, the retailer's green technology investment is comparatively lower in the models that take into account the carbon cap-and-trade regulation. This finding demonstrates that a high purchase cost amplifies the impact of the retailer's repair of imperfect products on technology investment.

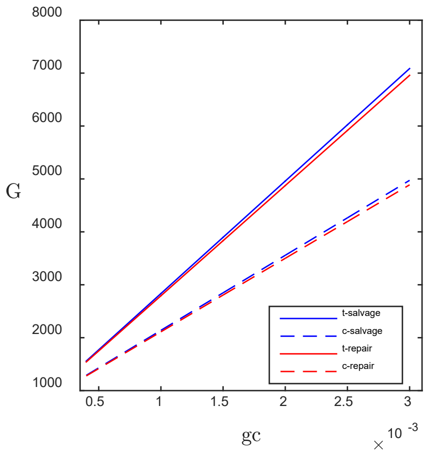

Fig. 5. Impact of the reduction in unit costs resulting from the retailer's green technology investment on green technology investment

Figure 6 illustrates the correlation between the retailer's cost and the percentage of imperfect quality products (p). Fig. 6 shows that an increase in the rate of imperfect quality products can lead to an increase in the retailer's costs and a change in the retailer's decision-making process regarding the evaluation of imperfect products, the outsourcing of repairs under carbon tax regulation, and the outsourcing of repairs under carbon cap-and-trade regulation. Consequently, the reduction of the imperfect product rate is of significant importance for the retailer's costs and the evaluation of imperfect products. It is recommended that the government should implement carbon regulation and make efforts to repair imperfect products in order to prevent the generation of waste.

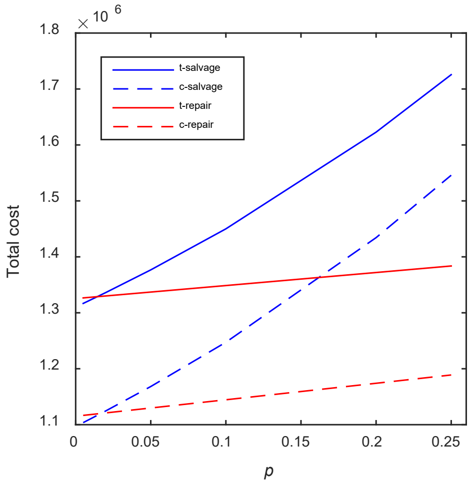

Fig. 6. Impact of imperfect quality products on the retailer's total costs

As illustrated in Fig. 7, a discernible inverse correlation is observed between the optimal lot size of the retailer and the proportion of imperfect products across repair models under both carbon regulations. Conversely, a positive correlation is evident between the optimal lot size of the retailer and the proportion of imperfect products across salvage models under both regulations. Moreover, when the unit backordering cost is high, the order lot size is reduced. The salvage of imperfect quality products has the potential to increase the level of order size, irrespective of the presence or absence of carbon regulation. As the percentage of imperfect quality products rises, the difference between the order levels in the salvage models widens, while narrowing in the repair models. This finding illustrates that a high level of percentage of imperfect products accentuates the impact of the salvage of imperfect quality products on order lot size.

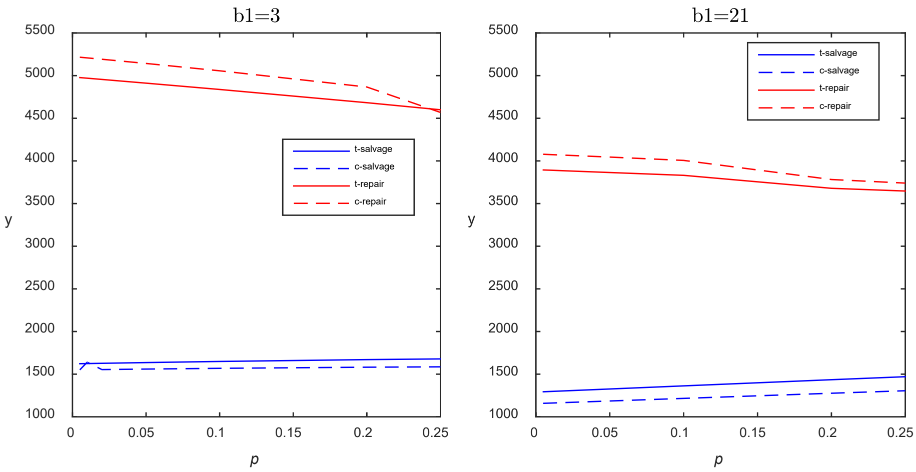

Fig. 7. Impact of imperfect quality products on the optimal lot size

The optimal total cost of the retailer with different caps and with different tax values is illustrated in Fig. 8. Figure 8(a) pertains to the repair of products of imperfect quality, whereas Figure 8(b) concerns the salvaging of such products. The straight lines in Fig. 8 represent the optimal costs of the retailer ($Cost_t$) under the carbon tax regulation with different tax values. The dashed line represents the optimal cost of the retailer ($Cost_c$) under the cap-and-trade regulation. As shown in Fig. 5, when $C_1 = 1.8$, $Cost_t$ is greater than $Cost_c$ over all levels of cap U. If C_1 is equal to 0.4, then $Cost_t$ is less than $Cost_c$ when the cap U is less than 145400 and 144100, respectively. Therefore, an increase in the carbon cap will prompt the retailer to alter its stance on carbon regulation.

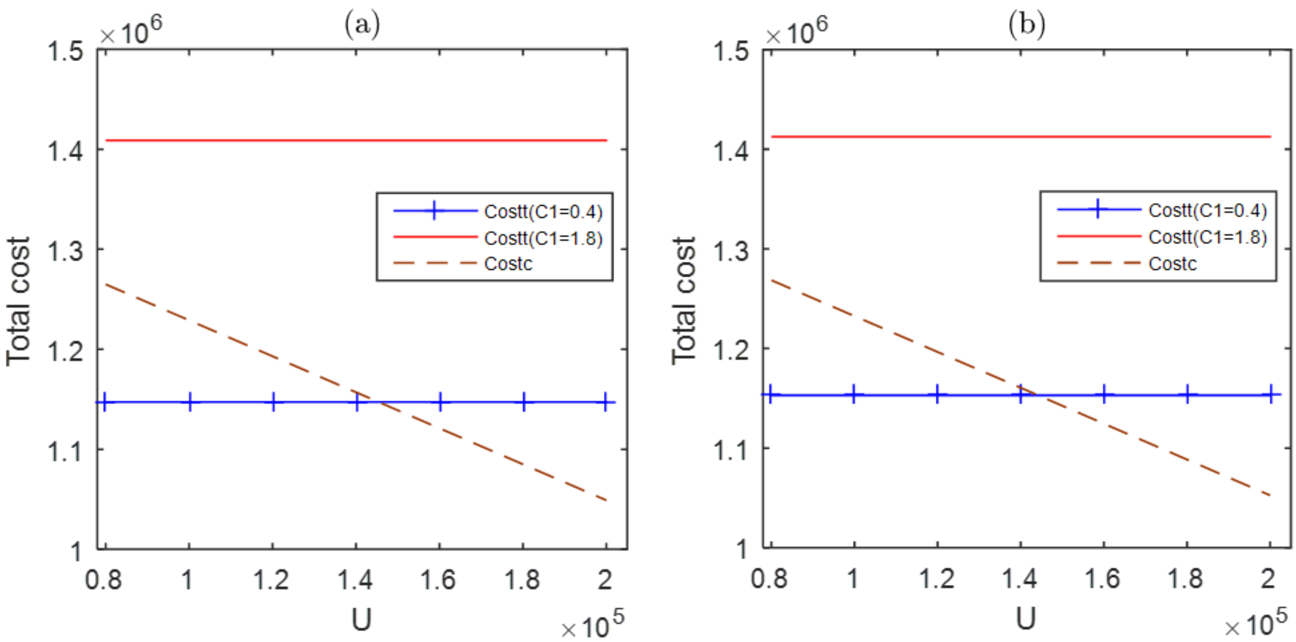

Fig. 8. The optimal total cost under the two regulations at different carbon cap levels.

A comparison of all models in Figs. 4-8 reveals that the model which incorporates the repair of imperfect products via a carbon cap-and-trade regulation produces superior results and is therefore more advantageous for the inventory system. The total cost and investment levels of the carbon cap-and-trade regulation are lower than those of the carbon tax regulation. This result demonstrates the significance of carbon regulation in terms of the economic and environmental sustainability of inventory systems. Furthermore, as the reduction in the unit cost resulting from the retailer's green technology investment level increases, it becomes evident that all models are more sustainable in both economic and environmental dimensions.

6. Conclusions

Governments impose regulations to oblige firms to make their inventory systems sustainable. In modelling such systems, therefore, additional costs need to be considered, other than those of holding, purchasing, and ordering, such as the costs of carbon emissions, investment in green technology, and repairing. This paper addresses the order lot-sizing and green investment decisions of a firm featuring monopoly competition under cap-and-trade and carbon tax regulations based on an inventory model with imperfect quality items. Each lot received is allowed to contain a percentage of imperfect products. The retailer conducts a 100% inspection of each lot so as to identify items that meet the required standards. Two cases are considered for items of imperfect quality: they can be repaired by a third party and then used as new items to meet demand, or sold to secondary markets as a single lot at a reduced price. Good-quality items are transferred from the inspection stage to the working inventory in batches of equal size. Carbon emissions are incurred at every stage, including ordering, purchasing, repairing, transporting, and holding. We characterize the optimal order size, shortage quantity, number of shipments and corresponding investment under each regulation, and compare the performance of the firm with respect to the optimal carbon emissions, green investment and total cost under the two regulations. This study highlights the importance of considering the cost-benefit analysis of such investments. Our main conclusions are the following.

Under carbon regulations, retailers are recommended to invest in emission reduction technologies. Compared to scenarios without carbon regulations, retailers' investment in green technologies reduces the number of shipments, increases the order lot size and shortage quantity for outsourced repairs, but reduces the order lot size and number of shipments for salvage.

The outsourced repairs demonstrate a higher quantity of products ordered, emission reductions, and retailer profits in comparison to the salvage option, which involves a lower quantity of products. While the outsourced repair has been more economically profitable, this does not necessarily correspond with an increase in the levels of emissions reductions. When repair costs and tax prices are high, repair increases the cost. A lack of incentives may prevent retailers from investing more in emissions reductions.

The following are the most significant findings of this study in relation to reality:

When introducing carbon regulations, policymakers need to consider retailers' carbon emissions intensity and tax and trade pricing. Policymakers should facilitate collaboration between retailers and third-party companies on the reduction of carbon emissions. Furthermore, outsourcing repair of defective products has the potential to stimulate the development of the repair industry and increase investments in the reduction of emissions within the inventory system.

Retailers' investments in emission reduction technology can enable them to reap the benefits of outsourcing repairs. Cooperation with the government on the repair of defective products and the involvement of third-party companies in the chain is required to grow the industry and reduce emissions. Under carbon regulations, the retailers should prioritize outsourced repairs to maximize profits. Naturally, our research work is not free of limitations. All imperfect quality items are assumed to be repaired and treated as new items. However, not all imperfect items are of the same quality, so it would be interesting for future research to consider differences in quality terms among the imperfect items. Alternatively, it could include pricing decisions or other types of policies to reduce carbon emissions. Another possible way to extend this model would be through partial backordering, even when there are random imperfections in the process. This study can also be extended in the form of a game strategic supply chain.

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Appendix

$$U_1 = \left(1-p-\frac{D(n-1)}{nx}\right)^2 - \left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n},$$

$$U_2 = (1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right) - \frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right),$$

$$U_3 = \frac{(1-\alpha)(1-p)}{n} - 2\left(1-p-\frac{D(n-1)}{nx}\right),$$

$$U_4 = \frac{\alpha}{(1-p)x-D} - \frac{1-\alpha}{D},$$

$$U_5 = \frac{hD}{2}\left(\frac{2}{(1-p)x} - \frac{n-1}{xn}\right.$$
$$+ \frac{x}{D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2 - \left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]$$
$$\left.+\frac{1}{n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right) - \frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right)\right]\right) - \frac{(1-p)(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)},$$

$$U_6 = -\frac{hD}{2}\left(\frac{x}{D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n} - 2\left(1-p-\frac{D(n-1)}{nx}\right)\right]\right.$$
$$\left.+\frac{1}{n}\left(\frac{\alpha}{(1-p)x-D}-\frac{1-\alpha}{D}\right)\right) + \frac{b_1 D}{2n((1-p)x-D)},$$

$$U_7 = \left[\frac{1}{D} + \left(\frac{1}{w}+\frac{1}{X}\right)\left(1-p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right) - \frac{\alpha(1-p)}{n((1-p)x-D)} - \frac{2(n-1)}{nx} - \frac{p}{w}\right],$$

$$U_8 = \left[2-(1-p)x\left(\frac{1}{w}+\frac{1}{X}+\frac{1}{D}\right)\right],$$

$$U_9 = \left(1-3p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right),$$

$$U_{10} = \frac{hD}{2}\left(\frac{2}{x} - \frac{(n-1)(1-p)}{xn}\right.$$
$$+ \frac{(1-p)x}{D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2 - \left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]$$
$$+\frac{(1-p)}{n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right) - \frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right)\right]$$
$$\left.+p\left[\frac{1}{D}+\left(\frac{1}{w}+\frac{1}{X}\right)\left(1-p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right) - \frac{\alpha(1-p)}{n((1-p)x-D)} - \frac{2(n-1)}{nx} - \frac{p}{w}\right]\right) + \frac{h'Dp^2(1+m)}{w} -$$
$$\frac{(1-p)^2(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)},$$

$$U_{11}=-\frac{hD}{2}\left(\frac{(1-p)x}{D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n}-2\left(1-p-\frac{D(n-1)}{nx}\right)\right]\right.$$
$$\left.+\frac{1}{n}\left(\frac{\alpha}{(1-p)x-D}-\frac{1-\alpha}{D}\right)+\frac{p}{((1-p)x-D)}\left[2-(1-p)x\left(\frac{1}{w}+\frac{1}{X}+\frac{1}{D}\right)\right]\right)+\frac{b_1D(1-p)}{2n((1-p)x-D)},$$

$$U_{12}=\frac{(h+C_1E_h)D}{2}\left(\frac{2}{(1-p)x}-\frac{n-1}{xn}\right.$$
$$+\frac{x}{D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2-\left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]$$
$$\left.+\frac{1}{n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right)-\frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right)\right]\right)-\frac{(1-p)(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)}$$

$$U_{13}=-\frac{(h+C_1E_h)D}{2}\left(\frac{x}{D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n}-2\left(1-p-\frac{D(n-1)}{nx}\right)\right]\right.$$
$$\left.+\frac{1}{n}\left(\frac{\alpha}{(1-p)x-D}-\frac{1-\alpha}{D}\right)\right)+\frac{b_1D}{2n((1-p)x-D)}.$$

$$U_{14}=\frac{(h+C_1E_h)D}{2}\left(\frac{2}{x}-\frac{(n-1)(1-p)}{xn}\right.$$
$$+\frac{(1-p)x}{D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2-\left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]$$
$$+\frac{1-p}{n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right)-\frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right)\right]$$
$$\left.+p\left[\frac{1}{D}+\left(\frac{1}{w}+\frac{1}{X}\right)\left(1-p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right)-\frac{\alpha(1-p)}{n((1-p)x-D)}-\frac{2(n-1)}{nx}-\frac{p}{w}\right]\right)$$
$$-\frac{(1-p)^2(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)}+\frac{h'Dp^2(1+m)}{w}+\frac{C_1E_h{}'Dp^2}{w},$$

$$U_{15}=-\frac{(h+C_1E_h)D}{2}\left(\frac{(1-p)x}{D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n}-2\left(1-p-\frac{D(n-1)}{nx}\right)\right]\right.$$
$$\left.+\frac{1-p}{n}\left[\frac{\alpha}{(1-p)x-D}-\frac{1-\alpha}{D}\right]+\frac{p}{((1-p)x-D)}\left[2-(1-p)x\left(\frac{1}{w}+\frac{1}{X}+\frac{1}{D}\right)\right]\right)$$
$$+\frac{(1-p)b_1D}{2n((1-p)x-D)},$$

$$U_{16}=\frac{(h+C_2E_h)D}{2}\left(\frac{2}{(1-p)x}-\frac{n-1}{xn}\right.$$
$$+\frac{x}{D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2-\left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]$$
$$\left.+\frac{1}{n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right)-\frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right)\right]\right)-\frac{(1-p)(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)},$$

$$U_{17}=-\frac{(h+C_2E_h)D}{2}\left(\frac{x}{D((1-p)x-D)}\left[\frac{(1-\alpha)(1-p)}{n}-2\left(1-p-\frac{D(n-1)}{nx}\right)\right]+\frac{1}{n}\left(\frac{\alpha}{(1-p)x-D}-\frac{1-\alpha}{D}\right)\right)$$
$$+\frac{b_1D}{2n((1-p)x-D)},$$

$$U_{18}=\frac{(h+C_2E_h)D}{2}\left(\frac{2}{x}-\frac{(n-1)(1-p)}{xn}\right.$$
$$+\frac{(1-p)x}{D((1-p)x-D)}\left[\left(1-p-\frac{D(n-1)}{nx}\right)^2-\left(1-p-\frac{D(n-1)}{nx}\right)\frac{(1-\alpha)(1-p)}{n}\right]$$
$$+\frac{1-p}{n}\left[(1-\alpha)\left(\frac{1-p}{D}-\frac{n-1}{nx}\right)-\frac{\alpha}{n}\left(\frac{n-1}{x}-\frac{\alpha(1-p)}{(1-p)x-D}\right)\right]$$
$$\left.+p\left[\frac{1}{D}+\left(\frac{1}{w}+\frac{1}{X}\right)\left(1-p+\frac{D\alpha(1-p)}{n((1-p)x-D)}\right)-\frac{\alpha(1-p)}{n((1-p)x-D)}-\frac{2(n-1)}{nx}-\frac{p}{w}\right]\right)$$
$$-\frac{(1-p)^2(\alpha-1)b_1\alpha D}{2n^2((1-p)x-D)}+\frac{h'Dp^2(1+m)}{w}+\frac{C_2E_h{}'Dp^2}{w}$$

and

$$U_{19} = -\frac{(h+C_2E_h)D}{2}\left(\frac{(1-p)x}{D\left((1-p)x-D\right)}\left[\frac{(1-\alpha)(1-p)}{n} - 2\left(1-p-\frac{D(n-1)}{nx}\right)\right]\right.$$
$$\left.+\frac{1-p}{n}\left[\frac{\alpha}{(1-p)x-D} - \frac{1-\alpha}{D}\right] + \frac{p}{\left((1-p)x-D\right)}\left[2-(1-p)x\left(\frac{1}{w}+\frac{1}{X}+\frac{1}{D}\right)\right]\right)$$
$$+\frac{(1-p)b_1D}{2n\left((1-p)x-D\right)}\,.$$