Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

# An improved black widow optimization (IBWO) algorithm for solving global optimization problems

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C H R O N I C L E

*Article history*:
Received February 12 2024
Received in Revised Format April 3 2024
Accepted April 27 2024
Available online April 27 2024

Keywords:
*Optimization approaches*
*Black widow optimization*
*Convergence*
*Benchmark functions*

A B S T R A C T

One of the primary goals of optimization approaches is to strike a balance between exploitation and exploration strategies, thereby enhancing the efficiency of the search process. To improve this balance, considerable research efforts have been directed towards refining these strategies. This paper introduces a novel exploration approach for the Black Widow Optimization (BWO) algorithm, termed Improved BWO (IBWO), aimed at achieving a robust equilibrium between global and local search strategies. The proposed approach tracks and remembers the effective research areas during the research iteration and uses them to direct the subsequent research process toward the most promising areas of the search space. Consequently, this method facilitates convergence towards optimal global solutions, leading to the generation of higher-quality solutions. To evaluate its performance, IBWO is compared with five optimization techniques, including BWO, GA, PSO, ABC, and BBO, across 39 benchmark functions. Simulation results demonstrate that IBWO consistently maintains precision in performance, achieving superior fitness values in 87.2%, 74.4%, and 69.2% of total trials across three distinct simulation settings. These outcomes underscore the efficacy of IBWO in effectively leveraging prior search space information to enhance the balance between exploitation and exploration capabilities. The proposed IBWO has broad applicability, addressing real-world optimization challenges in pilgrim crowd management and transportation during Hajj, supply chain logistics, and energy distribution optimization.



## 1. Introduction

Optimization studies have recently been actively published in the literature, including algorithms, applications, correlations, and analysis, due to their simplicity, powerful implementations, and high intensity. The optimization technique can be thought of as a way of reaching a minimum or maximum value of a problem in which the goal is to find the best solution based on a set of predefined criteria or restrictions. This procedure frequently uses an iterative algorithm to compare multiple findings until an ideal or suitable answer is discovered. Different sorts of optimization techniques are commonly used: deterministic and stochastic. Metaheuristic algorithms (MA) are known as stochastic computational approaches for optimizing and improving solutions. Such procedures are generally inspired by different natural origins and driven by the survival of the best (Abdel-Basset et al., 2018). They have effective utility over huge search space and tackle non-differentiable or multivariable challenges to get the global solution (Kalra & Singh, 2015).

Numerous meta-heuristic methods for addressing NP-hard and sophisticated nature optimization challenges have been designed over decades. Such methods can be categorized into distinct categories depending on certain characteristics, including stochastic, deterministic, population, and iterative-based methods. The term stochastic algorithm refers to an algorithm that attempts to improve a solution using randomized rules. On the other hand, population-based methods try to improve the performance of a group of solutions, while iterative approach algorithms try to find an optimal answer through

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ISSN 1923-2934 (Online) - ISSN 1923-2926 (Print)
2024 Growing Science Ltd.
doi: 10.5267/j.ijiec.2024.4.004

several iterations. Swarm intelligence and evolutionary methods, which rely on simulation theory with natural phenomena, are two prominent classes of population-based algorithms (Hussain & Muhammad, 2020)

Exploration vs. exploitation: what's the difference? Diversification and intensification (also known as divergence and convergence, respectively) are two ubiquitous and basic aspects of any optimization approach. These two characteristics are treated as pillars for successfully addressing an optimization issue (Crepinsek et al., 2013). Exploration is the capacity to broaden search throughout a large domain to discover previously unexplored areas, whereas exploitation is the ability to select promising regions (excellent solutions) to use and converge optimum using acquired search knowledge (Khajehzadeh et al., 2011). Many algorithms were devised and adjusted to strike a balance between these two features, ensuring that exploratory moves reached all places inside the search space. The algorithm, on the other hand, aims to converge rapidly without spending a lot of movement by utilizing extensive local search knowledge about the landscape and previous search experience (Yang et al., 2019).

Swarm intelligence (SI) approaches are meta-heuristics that take cues from nature and mimic the cooperative behavior of swarms of animals such as insects, fish, birds, and flocks of terrestrial animals (Rahmanifard & Plaksina, 2019). The particle swarm optimization (PSO) technique could be considered the most traditional SI algorithm since it models how birds seek food (Hu et al., 2023). Every member of the population constantly modifies their search patterns as a result of learning from their personal and also other individuals' experiences (Zhang et al., 2020). PSO has been frequently used in actual optimization assignments due to its straightforward structure and rapid convergence rate. Even though SI methods have produced positive results in variant disciplines such as image processing, feature selection, scheduling problems, and many others, there is still room for improvement in their ability to handle a variety of issues (Hu et al., 2023). As a result, incorporating modification tactics into the original SI algorithms seems to be a successful strategy for improving their performance. The primary goal of enhanced algorithms is to increase exploration and exploitation with high convergence speed. For instance, to boost the exploration and exploitation capacities of the GWO and prevent it from becoming trapped in local optimums, the Gaussian walks and Levy fly were employed (Khalilpourazari et al., 2021). To boost population variety, which improves the Marine Predators (MPA) algorithm's capacity for global exploration, Hu et al. introduced a combination of differential quasi-opposition strategy and evolution algorithm into MPA (Hu et al., 2020, 2021).

The success of SI algorithms depends on how well exploration and exploitation are balanced. Thus, the Black Widow Optimization (BWO) algorithm, a recent and revolutionary SI technique proposed by Hayyolalam and Kazem in 2020 (Hayyolalam & Pourhaji Kazem, 2020), is the focus of this work. It is a recent intelligent optimization method that draws inspiration from the unusual black widow breeding behavior. It has several benefits in multiple aspects, such as fast convergence and producing optimum outcomes, when assessed on different benchmark functions. The results of the actual case study challenges, on the other hand, demonstrate the BWO algorithm's efficacy in resolving challenges with uncertain and difficult spaces in the real world (Abu-Hashem et al., 2024; Shehab et al., 2024).

In this paper, an improved BWO (IBWO) algorithm is proposed to enhance the performance of the exploration search of the BWO algorithm. The proposed IBWO applied a new mutation mechanism to explore the promised region of the search space by exploiting the history of the algorithm search process. Three basic steps were employed in the IBWO algorithm: (i) Initializing a dynamic pool (DP) to hold values of good decision variables. (ii) Updating the DP with new values of accepted solutions during search iterations. (iii) Applying a modified mutation strategy by selecting the original operator of BWO or the proposed mutation strategy, with equal probability.

The following is how the rest of this work is organized: Section 2 presents recent publications in the field. Section 3 then goes through the specifics of the suggested methodology. The experimental data and analyses are then presented in Section 4. Finally, in Section 5, the conclusions are presented.

## 2. Related Works

There are numerous metaheuristic approaches available for tackling optimization problems, each striving to strike a balance between exploration and exploitation to achieve optimal results. Recognizing the limitations of existing methodologies, researchers have proposed numerous modifications to enhance the performance of metaheuristic algorithms (Shambour, 2018). These modifications aim to address the shortcomings of traditional approaches and improve their overall efficacy. Among these algorithms, the Black Widow Optimization (BWO) algorithm is notable for its potential but faces challenges in finding the right balance between exploration and exploitation. This section delves into some of the ongoing research efforts aimed at refining and optimizing the BWO algorithm to improve its exploration and exploitation capabilities.

The BWO method was introduced relatively recently (Hayyolalam & Pourhaji Kazem, 2020). It offers several benefits in diverse aspects, including early convergence and achieving superior results being assessed on various benchmark functions, as a revolutionary intelligent optimization method that was inspired by the distinctive reproductive activity of the black widow. To determine the appropriate set of thresholds, the BWO method has been used to solve the image segmentation issue (Houssein et al., 2021). The experiments show that, when compared to the other methods, the BWO-based method produces findings that are dependable and efficient. Although the BWO method is quite helpful in tackling optimization issues, there is still room for further enhancements to the algorithm's efficiency (Hu et al., 2022).

Indeed, the exploitation ability of the BWO algorithm may be limited, and it may require reactivation when it becomes stagnant during execution, as highlighted by Jabbar and Ku-Mahamud (Jabbar & Ku-Mahamud, 2021). Hence, several BWO algorithm enhancements have been proposed. Abbar and Ku-Mahamud (2021) suggested two improvements to the BWO method to address these issues and then employed the modified BWO for the Traveling Salesman Problem (TSP). The first improvement is the addition of dynamic neighborhood descent, which improves the exploitation operation by locating more nearby regions while the algorithm is running. The second improvement integrates a new convergence metric for the algorithm throughout the execution and online reactive search process, with an emphasis on the reactive search phase.

An improved algorithm called the Improved Black Widow-Bear Smell Search Algorithm (IBWBSA) has been proposed in (K. R & Ananthapadmanabha, 2021). This technique combines the Bear Smell Search Algorithm (BSSA) with BWO to create a multi-objective strategy for planning and operating distributed generators in distributed systems. By incorporating the BSSA exploration technique, IBWBSA aims to accelerate the convergence speed of BWO. The hybrid approach utilizes the strengths of both algorithms, enabling faster convergence in the optimization process.

Furthermore, a modified version of BWO called SDABWO is introduced by Hu et al. (2022) to address the challenges of the feature selection problem. SDABWO aims to overcome issues such as poor precision, low convergence speed, and susceptibility to getting trapped in local optima. The algorithm proposes three modifications to the original BWO. Firstly, a novel partner selection method based on the weight of female widows and the distance between them is introduced to improve precision and convergence speed. Secondly, the differential evolution mutation operator is incorporated during the mutation process to help the algorithm escape local optima. Lastly, three important parameters are adaptively adjusted as the number of iterations increases, enhancing the algorithm's performance over time.

In addition, a modified version of BWO called SDABWO is proposed to solve the feature selection problem. To address the drawbacks of poor precision, low convergence speed, and being prone to trap in local optima, the SDABWO algorithm proposed three modifications to the original BWO algorithm. First, a novel method of choosing partners is put forth, based on measuring the weight of female widows and the distance separating them. The method can be used to improve the BWO algorithm's precision and convergence speed. The second innovation uses the differential evolution mutation operator during the original algorithm mutation process to aid the algorithm's exit from local optima. Three important parameters are then specified to adapt as the number of iterations increases.

Moreover, an improved version of the BWO algorithm called LDBWO is proposed by Hu et al. (2023)to achieve optimal approximation of Q-Bézier surfaces. This modified algorithm addresses the limitations of the classic BWO method, including accuracy, slow convergence, and susceptibility to local optima. LDBWO introduces the golden sine learning technique and diffusion process to enhance the searchability of the BWO algorithm. The golden sine learning technique helps improve the accuracy of the algorithm by guiding the search process toward more promising regions. The diffusion process aids in avoiding local optima and ensures a more thorough exploration of the search space.

In this paper, a different perspective to improve the searchability of the BWO algorithm. The study proposes a novel approach that focuses on tracking and utilizing effective research areas during the search iteration. The goal of this approach is to enhance the exploration capability of the BWO algorithm. By identifying and remembering areas of the search space that have shown promising results in previous iterations, the algorithm gains valuable knowledge to guide its subsequent research process. This targeted exploration allows the algorithm to concentrate on the most promising regions, thereby increasing the chances of finding optimal or near-optimal solutions. Furthermore, it prevents premature convergence and ensures a more thorough exploration of the search space.

## 3. Black Widow Optimization algorithm

The black widow spider is a moderate-sized member of the Orygiidae species that is primarily found in European nations. Most of the female spider's activities, such as feeding, breeding, and egg hatching, take place in spider webs. A specific pheromone is produced by the female every time she desires to mate to draw the male. The very first spider male to join the net reduces the attractiveness of the female net to competitors. Throughout or after mating, the female will devour the husband before transferring her offspring to the egg sock for hatching. Upon hatching, the young start eating their siblings. When children are caught in their mother's net for a while, the powerful ones may devour the frail, and they may eventually devour their mother.

By using the black widow's way of existence as a mathematical model, Hayyolalam and Kazem introduced the BWO algorithm in 2020 Black widow optimization algorithm: a novel meta-heuristic approach for solving engineering optimization (Hayyolalam & Pourhaji Kazem, 2020). To find the optimal answer to the optimization challenges, the method models, both macroscopic and microscopic rules, govern spider population breeding and development. On a larger scale, the black widow's habit of reproducing and cannibalism reflects the idea of the Darwinian Theory of evolution, which is the survival of the strongest and dominance of the strongest. On the other hand, the alteration in the spider generation is an example of a tiny genetic transformation. By using this type of breeding and development method, the spider population can expand and become more efficient.

The population initialization, procreation, cannibalism and mutation processes of the BWO method are discussed below.

*3.1 population initialization*

The initial population of black widow spider is represented as$W = [X^1, X^2, \ldots, X^N]$, where N denotes the number of widows (i.e. solutions). Each solution in W is a vector of variables $X^i = [x_1, x_2, \ldots, x_D]$, where $i \in (1, N)$ and D is the number of decision variables. The initial population is initialized as given by Eq (1).

$$x_j^i = LB_j + U(0,1) \times \left(UB_j - LB_j\right), \tag{1}$$

Where$j \in (1, D)$,$U(0,1)$is a uniform distribution between 0 and 1.$UB$ and $LB$ are the upper and lower bounds, respectively.

After building the initial population, the fitness value of each solution is calculated using the objective function $f(X)$.

*3.2 Procreate*

The current breed is formed by black widows' distinctive mating habits. When mating begins, a pair of spiders designated as the mother and father are chosen arbitrarily from the population and paired up depending on their procreation rate (Pp). Eq (2)is used to make the offspring.

$$\begin{cases} Y_i = \alpha X_i + (1 - \alpha) X_j \\ Y_j = \alpha X_j + (1 - \alpha) X_i' \end{cases} \tag{2}$$

where $X_i$ is the mother and $X_j$ is the father. The results of the mating are $Y_i$ and $Y_j$. And $\alpha$ contains random numbers in a D-dimensional array.

*3.3 Cannibalism*

Three types of cannibalism are present in this period, including cannibalism between the female and the male, cannibalism between siblings, and cannibalism between offspring and mothers. Superior spiders are kept alive by removing the weak ones.

a. Cannibalism between the female and the male
Black widow females devour their partners either while or right after mating. The female spiders that survive are kept for the following generation.

b. Cannibalism between siblings
The powerful spiders devour weak ones since there aren't enough food supplies or they have natural adversaries. The toughness of the spider is regarded as its fitness value. The cannibalism rating (CR) is used in this method to control the amount of survivors.

c. Cannibalism between offspring and mothers
Offspring spiders can consume their mother if they are powerful enough. That is, if parents create a solution with a better fitness score, the solution will remove its mother and move on to the next iteration.

*3.4 Mutation*

Population to mutate (Pm) is used as a mutation rate in this stage to calculate the population to mutate number, which is a known constant. A random swap of two elements from the array $x_{im}$ and $x_{in}$, $(1 \leq m, n \leq D)$ is applied to the chosen member $X_i$, $(1 \leq i \leq N)$. Fig. 1 depicts the mutation execution.

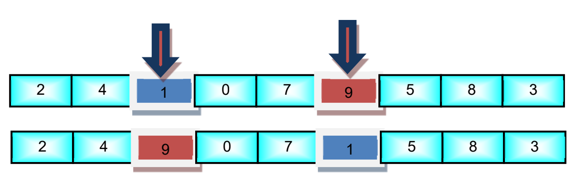


**Fig. 1.** Mutation process

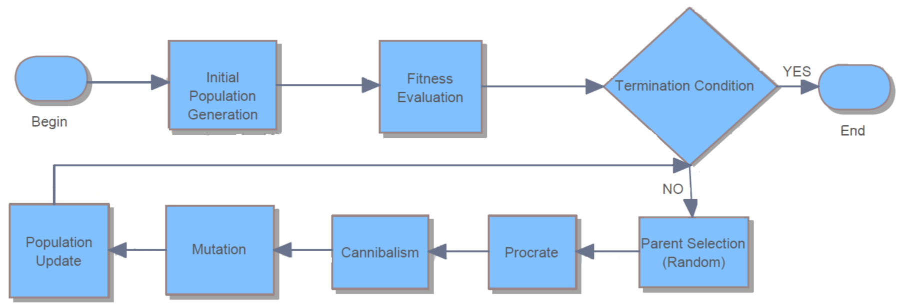


**Fig. 2.** Depicts the BWO algorithm's overall workflow

## 4. Improved Black Widow Optimization (IBWO) algorithm Methodology

The exploration search strategy plays a significant part in the evolution of the search process. In this paper, the mutation component of BOW is modified to explore the promised region of the search space by utilizing the algorithm's prior search experience. A novel exploration technique is added to the mutation operator of BWO to enhance the performance of the exploration search and hence the search efficiency of the algorithm. The basic steps of the proposed algorithm are explained as follows:

### 1) Initializing a dynamic pool (DP):

The proposed technique begins by initializing a DP of size $D \times D$ , where D is the problem dimension (i.e. number of decision variables) as shown in Fig.3. Each column in the DP corresponds to a certain decision variable (DV) of the optimization problem.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| NULL | NULL | NULL | NULL | ...................... | NULL | NULL | 1 |
| NULL | NULL | NULL | NULL | ...................... | NULL | NULL | 2 |
| ⋮ | ⋮ | ⋮ | ⋮ | ..........⋮.......... | ⋮ | ⋮ | |
| NULL | NULL | NULL | NULL | ...................... | NULL | NULL | D-1 |
| NULL | NULL | NULL | NULL | ...................... | NULL | NULL | D |
| $DV_1$ | $DV_2$ | $DV_3$ | $DV_4$ | | $DV_{D-1}$ | $DV_D$ | |

**Fig. 3.** An Initialized DP

### 2) Updating the DP:

During the BWO search process, the contents of the DP are particularly updated with the values of DVs that were assigned using the mutation operator of acceptable solutions, where values of DVs are recorded in their corresponding columns of DP, row by row, as explained in Fig.4. In case updating all rows of a certain DP's column. The next updating value will overwrite the content of a top row, and so on.

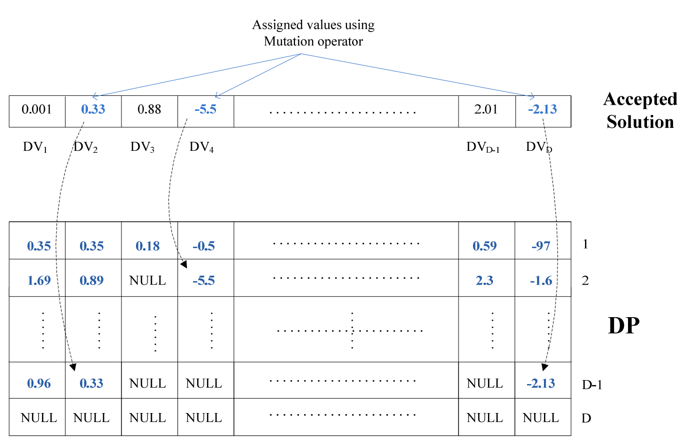

**Fig. 4.** Updating the operation of DP

### 3) Applying a modified mutation strategy:

The new proposed mutation strategy is based on randomly selecting two values from DP within the same block structure and generating a new mutation value using a simple mutation operation as given in Eq. (3).

$$x^{DV} = DP_{r_1}^{DV} + U(0,1) \times \left(DP_{r_2}^{DV} - DP_{r_1}^{DV}\right), \qquad r_1 \neq r_2, r_1 and\ r_2 \in [1, D] \tag{3}$$

The algorithm will randomly select one of the two mutation strategies to apply, either the traditional mutation strategy or the proposed mutation strategy, with equal probability.

## 5. Experiments and Results

This section evaluates the proposed IBWO algorithm to the original BWO algorithm and other optimization algorithms in the literature in terms of convergence speed and solution quality on various kinds of numerical benchmark functions.

### *5.1 Benchmark Functions*

A set of 39 benchmark functions that were extensively utilized in the literature was used to verify the effectiveness of the proposed IBWO method. Table 1 illustrates the test optimization functions along with their function formulation, domain range, and classified attributes which are denoted by the letters M, U, C, S, and N, respectively, for Multi-modal, Uni-modal, Composition, Separable, and Non-separable.

**Table 1**
Numerical benchmark functions

| Function | Equation | Range | Characteristics |
|---|---|---|---|
| **Powell Sum** | $the\ f_1(x) = \sum_{i=1}^{n} \lvert x_i \rvert^{i+1}$ | -5.12≤ $x_i$≤5.12 | U |
| **Cigar** | $f_2(x) = {x_1}^2 + 10^6 \sum_{i=2}^{n} {x_i}^2$ | -5.12≤ $x_i$≤5.12 | U, N |
| **Discus** | $f_3(x) = 10^6 {x_1}^2 + \sum_{i=2}^{n} {x_i}^2$ | -5.12≤ $x_i$≤5.12 | U |
| **Rosenbrock** | $f_4(x) = \sum_{i=2}^{n-1} 100({x_i}^2 - x_{i+1})^2 + (x_i - 1)^2$ | -30≤ $x_i$≤ 30 | U |
| **Ackley** | $f_5(x) = 20\,exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2}) - exp(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)) + 20 + e$ | -35≤ $x_i$≤ 35 | M, N |
| **Griewank** | $f_6(x) = \frac{1}{4000}\sum_{i=1}^{n} {x_i}^2 - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}}) + 1$ | −100 ≤ $x_i$≤ 100 | M, N |
| **Rastrigin** | $f_7(x) = \sum_{i=1}^{n}({x_i}^2 - 10\,cos(2\pi x_i) + 10)$ | -5.12≤ $x_i$≤5.12 | M |
| **HappyCat** | $f_8(x) = \left\lvert \sum_{i=1}^{n} {x_i}^2 - n \right\rvert^{\frac{1}{4}} + \frac{(\frac{1}{2}\sum_{i=1}^{n} {x_i}^2 + \sum_{i=1}^{n} x_i)}{n + 0.5}$ | -5.12≤ $x_i$≤5.12 | M, N |
| **HGBat** | $f_9(x) = \left\lvert (\sum_{i=1}^{n} {x_i}^2)^2 - (\sum_{i=1}^{n} x_i)^2 \right\rvert^{\frac{1}{2}} + \frac{(\frac{1}{2}\sum_{i=1}^{n} {x_i}^2 + \sum_{i=1}^{n} x_i)}{n + 0.5}$ | -5.12≤ $x_i$≤5.12 | M |
| **Expanded Griwank's plus Rosenbrock** | $f_{10}(x) = f_6(f_4(x_1, x_2)) + f_6(f_4(x_2, x_3)) + \cdots + f_6(f_4(x_{n-1}, x_n)) + f_6(f_4(x_n, x_1))$ | -5.12≤ $x_i$≤5.12 | C |
| **Expanded Scaffer's** | $g(x, y) = 0.5 + \frac{(\sin^2(\sqrt{x^2 + y^2} - 0.5)}{(1 + 0.001(x^2 + y^2))^2}$ <br> $f_{11}(x) = g(x_1, x_2) + g(x_2, x_3) + \cdots + g(x_{n-1}, x_n) + g(x_n, x_1)$ | -5.12≤ $x_i$≤5.12 | C |
| **Some of different powers** | $f_{12}(x) = 1 - \frac{1}{n}\sum_{i=1}^{n} \cos(kx_i)e^{\frac{-{x_i}^2}{2}}$ | - $\pi \le x_i \le \pi$ | M, S |
| **Sphere** | $f_{13}(x) = \sum_{i=1}^{n} {x_i}^2$ | -5.12≤ $x_i$≤5.12 | U, S |
| **Penalized** | $f_{14}(x) = \frac{\pi}{n}\{10\,sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\,sin^2(\pi y_{i+1})] + (y_n - 1)^2\} +$ <br> $+\sum_{i=1}^{n} u_i(x_i, 10, 100, 4), y_i = 1 + \frac{x_i + 1}{4}, u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, x_i > a \\ 0, -a < x_i < a \\ k(-x_i - a)^m, x_i < -a \end{cases}$ | −50 ≤ $x_i$≤ 50 | M, N |

**Table 2**
Numerical benchmark functions (Continued)

| Function | Equation | Range | Characteristics |
|---|---|---|---|
| **penalized2** | $f_{15}(x) = \sum_{i=1}^{n} u(x_i, 5,100,4) + 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i-1)^2[1+sin^2(3\pi x_{1+1})] + (x_n-1)^2 [1+sin^2(2\pi x_n)]\right\}$ | -50≤ $x_i$≤ 50 | M, N |
| **Quartic** | $f_{16}(x) = \sum_{i=1}^{n} i\, x_i^4 + random[0,1]$ | -1.28≤ $x_i$≤1.28 | U, S |
| **Schwefel 1.2** | $f_{17}(x) = \sum_{i=1}^{n}(\sum_{j=1}^{i} x_i)^2$ | -100≤ $x_i$≤ 100 | U, N |
| **Schwefel 2.21** | $f_{18}(x) = max_i\{\lvert x_i\rvert, 1 \le i \le n\}$ | -100≤ $x_i$≤ 100 | U, N |
| **Schwefel 2.22** | $f_{19}(x) = \sum_{i=1}^{n} \lvert x\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$ | -10≤ $x_i$≤ 10 | U, N |
| **Step 2** | $f_{20}(x) = \sum_{i=1}^{n}(\lvert x_i + 0.5\rvert)^2$ | -200≤ $x_i$≤ 200 | U |
| **Alpine1** | $f_{21}(x) = \sum_{i=1}^{n} \lvert x_i \sin(x_i) + 0.1x_i\rvert$ | -10≤ $x_i$≤ 10 | M |
| **Csendes** | $f_{22}(x) = \sum_{i=1}^{n} x_i^6(2 + sin\frac{1}{x_i})$ | -1≤ $x_i$≤ 1 | M |
| **Rotated Ellipse** | $f_{23}(x) = 7x_1^2 - 6\sqrt{3}\, x_1x_2 + 13x_2^2$ | -500≤ $x_i$≤ 500 | U |
| **Rotated Ellipse2** | $f_{24}(x) = x_1^2 - x_1x_2 + x_2^2$ | -500≤ $x_i$≤ 500 | U |
| **Sum Squares** | $f_{25}(x) = \sum_{i=1}^{n} ix_i^2$ | -10≤ $x_i$≤ 10 | U |
| **Step** | $f_{26}(x) = \sum_{i=1}^{n}(\lfloor\lvert x_i\rvert\rfloor)$ | -100≤ $x_i$≤ 100 | U, S |
| **Schewefel** | $f_{27}(x) = \sum_{i=1}^{n} 418.9829 - x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)$ | -500≤ $x_i$≤ 500 | M |
| **Xin-She Yang1** | $f_{28}(x) = \sum_{i=1}^{n} \varepsilon_i \lvert x_i\rvert^i$ | -5≤ $x_i$≤ 5 | S |
| **Schaffer** | $f_{29}(x) = 0.5 + \frac{\sin^2(x_1^2 + x_2^2)^2 - 0.5}{1 + 0.001(x_1^2 + x_2^2)^2}$ | -100≤ $x_i$≤ 100 | U, N |
| **Adjiman** | $f_{30}(x) = \cos(x_1)\sin(x_2) - \frac{x_1}{x_2^2 + 1}$ | -1≤ $x_i$≤ 2,<br>-1≤ $x_i$≤ 1 | M |
| **Bartels Conn** | $f_{31}(x) = \lvert x_1^2 + x_2^2 + x_1x_2\rvert + \lvert sin(x_1)\rvert + \lvert cos(x_2)\rvert$ | -500≤ $x_i$≤ 500 | M |
| **Ackley 2** | $f_{32}(x) = -200e^{-0.02\sqrt{x_1^2+x_2^2}}$ | -500≤ $x_i$≤ 500 | U |
| **Eggcrate** | $f_{33}(x,y) = x^2 + y^2 + 25(\sin^2 x + \cos^2 y)$ | $(x,y) \in [-2\pi, 2\pi] \times [-2\pi, 2\pi]$ | M |
| **F34** | $f_{34}(x,y) = x\sin(4x) + 1.1y\, sin(2y)$ | 0≤ x, y≤ 10 | |
| **Powell Singular 2** | $f_{35}(x) = \sum_{i=1}^{n-2}(x_{i-1} + 10\, x_i)^2 + (x_{i+1} - x_{i+2})^2 + (x_i - 2\, x_{i+1})^4 + 10(x_{i-1} - x_{i+2})^4$ | -4≤ $x_i$≤ 5 | U, N |
| **Quintic** | $f_{36}(x) = \sum_{i=1}^{n} \lvert x_i^5 - 3x_i^4 + 4x_i^3 + 2x_i^2 - 10x_i - 4\rvert$ | -10≤ $x_i$≤ 10 | M, S |
| **Qing** | $f_{37}(x) = \sum_{i=1}^{n}(x_i^2 - i)^2$ | -500≤ $x_i$≤ 500 | M, S |
| **Salomon** | $f_{38}(x) = 1 - cos(2\pi\sqrt{\sum_{i=1}^{n} x_i^2}) + 0.1\sqrt{\sum_{i=1}^{n} x_i^2}$ | −100 ≤ $x_i$≤ 100 | M, N |
| **Dixon & Price** | $f_{39}(x) = (x_1 - 1)^2 + \sum_{i=1}^{n} i(2x_i^2 - x_{i-1})^2$ | -10≤ $x_i$≤ 10 | U, N |

*5.2 Comparing IBWO with BWO algorithm*

The performance of IBWO is compared to the original BWO algorithm in terms of solution quality and convergence speed using six benchmark functions selected from Table 1 that span all category types (i.e. U, M, S, NS, and C). These benchmark functions are F1, F10, F16, F18, F27, and F37. The parameter settings for both algorithms are as follows: Pp= 0.6; CR= 0.44;

and Pm= 0.4 (Hayyolalam & Pourhaji Kazem, 2020). The experiments are performed 30 times each with a maximum of 500 iterations and varied population sizes (nPop) set to 20, 50, and 100 on 30 problem-dimensions (D). The results of both algorithms are summarized in Table 2, which also includes statistical measurements of the mean, median, best, and worst fitness values. The better mean values are indicated with bold fonts (lower values). Results show that the proposed IBWO offers better mean results in most carried-out experiments on 89% of all trials. The t-test results also reveal significant improvements in 44% of the overall best mean results attained by IBWO (marked with × symbol). Furthermore, two instances of the BWO algorithm outperforming the IBWO have been observed in experiments on the F16 (Quartic function) when the nPop numbers are set to 50 and 100. The other statistical results (median, best, and worst measures) demonstrate superiority for IBWO over BWO as producing better results in most of the performed experiments.

**Table 3**
Fitness evaluation results of IBWO and BWO algorithms with different population sizes

| | Function type | | IBWO | | | BWO | | |
|---|---|---|---|---|---|---|---|---|
| | | | nPop=20 | nPop=50 | nPop=100 | nPop=20 | nPop=50 | nPop=100 |
| **F1** | U | Mean | **2.09E+00** | **7.91E-10** | **1.70E-12** | 6.57E+00 | 3.63E-02 | 7.19E-02 |
| | | Med. | 6.61E-11 | 3.09E-16 | 1.28E-29 | 1.02E-02 | 3.34E-16 | 1.51E-18 |
| | | Best | 1.11E-22 | 7.99E-32 | 8.48E-52 | 3.23E-19 | 4.41E-24 | 8.87E-27 |
| | | Worst | 1.80E+01 | 2.20E-08 | 5.09E-11 | 5.70E+01 | 1.09E+00 | 2.16E+00 |
| **F10** | C | Mean | **1.09E+02** | **4.44E+00*** | **2.09E+00*** | 2.98E+02 | 5.94E+00 | 3.74E+00 |
| | | Med. | 1.96E+01 | 4.40E+00 | 2.12E+00 | 1.94E+01 | 5.62E+00 | 3.53E+00 |
| | | Best | 4.83E+00 | 2.39E+00 | 1.18E+00 | 6.34E+00 | 2.93E+00 | 2.28E+00 |
| | | Worst | 2.07E+03 | 7.71E+00 | 2.95E+00 | 4.60E+03 | 1.06E+01 | 5.56E+00 |
| **F16** | U, S | Mean | **2.88E-02** | 1.79E-03 | 6.18E-04 | 4.59E-02 | **1.58E-03** | **4.60E-04** |
| | | Med. | 1.17E-02 | 1.56E-03 | 5.31E-04 | 1.36E-02 | 1.53E-03 | 4.54E-04 |
| | | Best | 2.73E-03 | 7.19E-04 | 2.13E-04 | 3.76E-03 | 5.73E-04 | 2.05E-04 |
| | | Worst | 2.05E-01 | 4.11E-03 | 1.79E-03 | 2.65E-01 | 3.20E-03 | 1.03E-03 |
| **F18** | U, N | Mean | **2.57E+01*** | **8.28E-01*** | **4.01E-01** | 3.73E+01 | 6.72E+00 | 4.43E-01 |
| | | Med. | 2.62E+01 | 4.38E-01 | 2.83E-01 | 3.54E+01 | 5.87E+00 | 3.29E-01 |
| | | Best | 8.30E+00 | 7.28E-03 | 1.02E-03 | 2.33E+01 | 1.65E-01 | 2.32E-02 |
| | | Worst | 6.27E+01 | 3.14E+00 | 1.99E+00 | 5.69E+01 | 1.97E+01 | 1.18E+00 |
| **F27** | M | Mean | **1.43E+03** | **7.81E+02*** | **4.65E+02*** | 1.77E+03 | 1.56E+03 | 1.27E+03 |
| | | Med. | 1.05E+03 | 5.27E+02 | 4.17E+02 | 1.48E+03 | 1.59E+03 | 1.19E+03 |
| | | Best | 2.38E+01 | 2.92E+01 | 9.44E+01 | 4.94E+02 | 3.74E+01 | 7.66E+01 |
| | | Worst | 4.50E+03 | 3.85E+03 | 1.24E+03 | 5.03E+03 | 4.01E+03 | 2.84E+03 |
| **F37** | M, S | Mean | **5.39E+02** | **3.97E+01** | **1.69E+00*** | 2.04E+06 | 3.22E+02 | 2.40E+01 |
| | | Med. | 5.72E+01 | 6.03E+00 | 7.03E-01 | 1.04E+02 | 2.28E+01 | 1.16E+01 |
| | | Best | 1.56E+01 | 2.33E-01 | 3.52E-02 | 1.01E+01 | 3.89E+00 | 1.54E+00 |
| | | Worst | 1.39E+04 | 9.83E+02 | 1.40E+01 | 5.98E+07 | 5.98E+03 | 2.45E+02 |

** indicates a significant improvement at a = 0.05 by the two-tailed t-test estimator.*

Fig. 5 displays the convergence curves of the fitness function values for the best results achieved by IBWO and BWO algorithms in the different optimization functions. Both algorithms exhibit rapid convergence during the initial stages of the search iterations; however, the IBWO exhibits better convergence toward the optimal solution in several experiment trials, demonstrating that the ability of IBWO to gain knowledge about promising regions of search space from prior search experience makes it more effective at identifying and utilizing promising search space regions.

### *5.3 Comparing IBWO to the other optimization algorithms*

Several simulations are performed on various kinds of benchmark functions to assess the effectiveness of the proposed IBWO algorithm. The performance results of IBWO are compared to the performance results of five algorithms: BWO, GA, PSO, artificial bee colony (ABC), and biogeography-based optimization (BBO) (Simon, 2008). Table 3 shows the results of the fitness evaluation for the compared algorithms with 10 problem dimensions (D), 100 populations (nPop), and 500 iterations. The top scores among 30 runs generated by algorithms are highlighted in bold font for the best, mean, and median results. Note that the results of the comparison algorithms are taken from (Hayyolalam & Pourhaji Kazem, 2020).

The results of Table 3 show that IBWO provides excellent performance in achieving the best fitness values among other algorithms in 87.2% of the total experiments performed. Furthermore, the recorded mean and median results show better performance for IBWO compared to other algorithms in 66.7% and 71.8% of total experiments performed on all benchmark functions. The results also show that the optimality was triggered 14 times by IBWO, 9 times by BWO, 7 times by PSO, and 6 times by each of the other algorithms. Additionally, the BOW outperforms IBWO by obtaining the best results on just four instances (F8, F10, F13, and F34). This shows that IBWO has great performance when it comes to solving different kinds of issues that have small problem dimensions. Table 4 displays the fitness evaluation results for the comparison algorithms for 20 D, 150 nPop, and 1000 iterations. The results demonstrate that IBWO continues to provide consistent results and high-quality solutions. The IBWO outperforms other algorithms in terms of best, mean, and median fitness values at 74.4%, 69.2%, and 74.4%, respectively. Furthermore, the findings also demonstrate that IBWO triggered the optimality 13 times, followed by BWO with 11, GA with 8, and PSO, BBO, and ABC each with 7, 7, and 4 times, respectively.

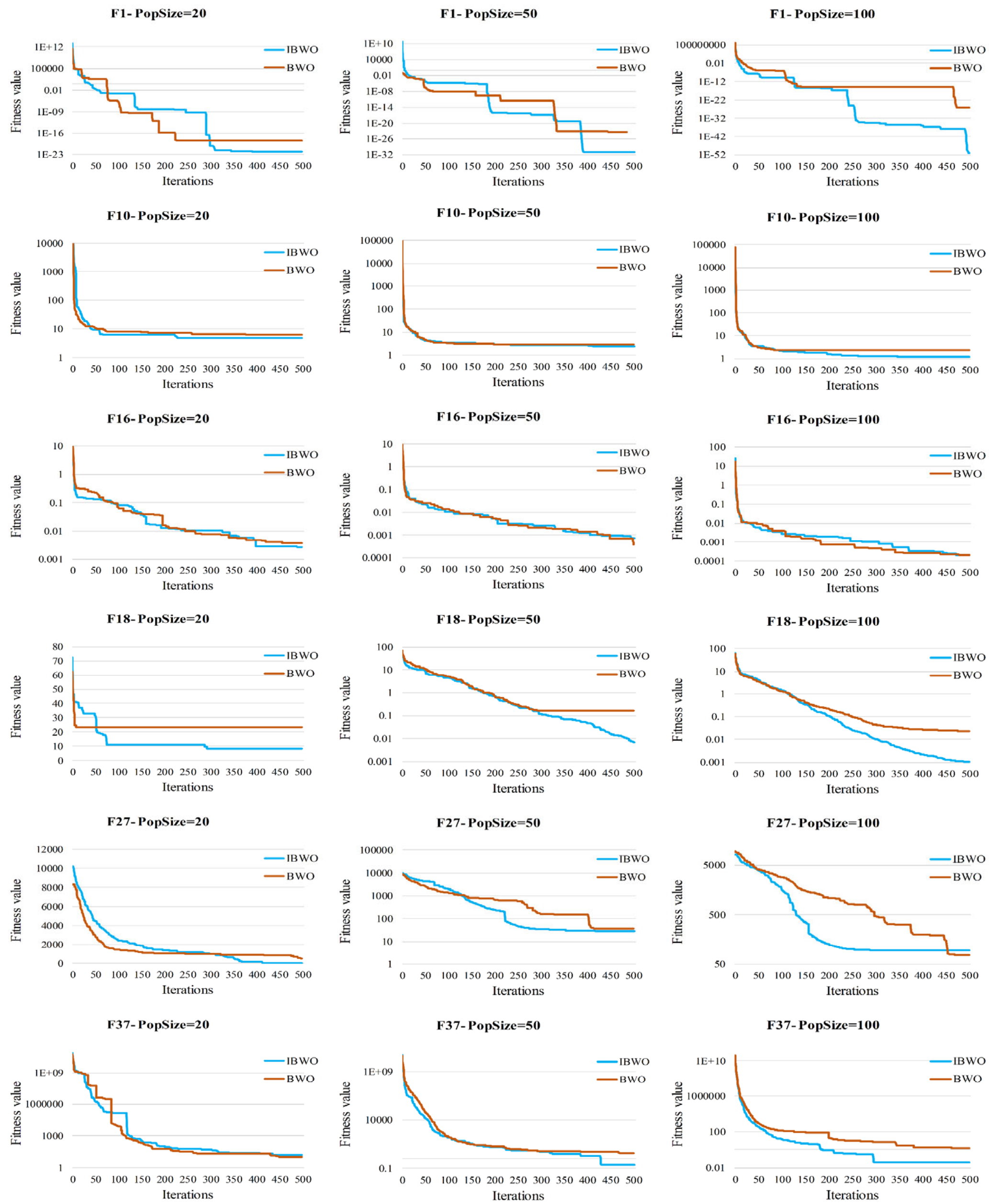


**Fig. 5.** Convergence fitness curves of IBWO and BWO algorithms on different benchmark functions

**Table 3**
Fitness evaluation results of IBWO and other algorithms with 10 D, 100 nPop, and 500 Iterations

| | F1 | | | F2 | | | F3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Best | Mean | Median | Best | Mean | Median | Best | Mean | Median |
| **IBWO** | **7.17E-52** | **1.47E-14** | **2.97E-33** | **1.62E-35** | **2.63E-02** | **1.19E-11** | **8.23E-42** | 6.61E-05 | **9.55E-16** |
| **BWO** | 1.58E-28 | 1.03E-12 | 2.28E-15 | 7.60E-03 | **1.81E-01** | 9.85E-02 | 1.13E-08 | 6.90E-04 | 9.16E-05 |
| **GA** | 4.09E-24 | 1.68E-09 | 2.60E-11 | 1.11E-01 | 8.24E-01 | 6.17E-01 | 1.50E-08 | 4.36E-03 | 1.19E-03 |
| **PSO** | 1.12E-18 | 1.21E-12 | 4.13E-14 | 9.07E-04 | 1.66E+01 | 2.62E+01 | 1.10E-04 | 1.79E-02 | 6.18E-03 |
| **ABC** | 3.35E-12 | 4.55E-09 | 9.34E-10 | 2.29E-02 | 6.67E-01 | 5.97E-01 | 3.63E-06 | 1.08E-03 | 3.97E-04 |
| **BBO** | 3.12E-22 | 1.81E-16 | 2.44E-17 | 1.84E-04 | 2.98E-01 | 4.79E-02 | 1.26E-07 | **1.31E-06** | 6.69E-07 |
| | F4 | | | F5 | | | F6 | | |
| **IBWO** | **3.51E-02** | 8.24E+00 | 8.19E+00 | **8.88E-16** | **1.50E-03** | **1.08E-05** | **0** | **4.00E-04** | **1.41E-10** |
| **BWO** | 3.54E-01 | 7.90E+00 | 7.22E+00 | 2.78E-13 | 3.07E+00 | 4.53E-05 | **0** | 6.99E-03 | 1.95E-05 |
| **GA** | 4.45E-01 | 1.02E+01 | 7.40E+00 | 4.82E-05 | 4.97E-02 | 1.22E-02 | 7.33E-08 | 4.29E-02 | 3.43E-02 |
| **PSO** | 9.12E-01 | 2.38E+02 | 1.61E+01 | 8.44E-05 | 4.43E-03 | 2.68E-03 | 1.22E-01 | 5.48E-01 | 5.61E-01 |
| **ABC** | 4.52E+00 | 1.38E+01 | 1.14E+01 | 1.17E-01 | 3.23E-01 | 2.72E-01 | 5.19E-02 | 1.51E-01 | 1.61E-01 |
| **BBO** | 6.52E-01 | **5.39E+00** | **4.68E+00** | 1.45E-03 | 1.89E-01 | 1.95E-03 | 7.40E-03 | 7.12E-02 | 6.02E-02 |

**Table 3**
Fitness evaluation results of IBWO and other algorithms with 10 D, 100 nPop, and 500 Iterations (Continued)

| | F7 | | | F8 | | | F9 | | |
|---|---|---|---|---|---|---|---|---|---|
| **IBWO** | **0** | **2.10E-02** | **2.15E-06** | 3.54E-02 | 1.40E-01 | 1.51E-01 | **6.23E-03** | **2.45E-01** | **2.25E-01** |
| **BWO** | **0** | 2.27E-02 | 1.93E-04 | **1.21E-02** | **3.25E-02** | **3.36E-02** | 1.47E-01 | 3.72E-01 | 4.09E-01 |
| **GA** | 4.81E-10 | 5.73E-01 | 7.90E-03 | 3.85E-02 | 1.18E-01 | 9.99E-02 | 1.51E-01 | 3.74E-01 | 4.07E-01 |
| **PSO** | 2.03E-01 | 7.90E+00 | 6.09E+00 | 1.39E-01 | 2.61E-01 | 2.55E-01 | 1.64E-01 | 3.00E-01 | 2.78E-01 |
| **ABC** | 1.30E+01 | 2.90E+01 | 2.92E+01 | 3.35E-01 | 6.78E-01 | 7.18E-01 | 2.34E-01 | 6.53E-01 | 5.74E-01 |
| **BBO** | 1.99E+00 | 5.87E+00 | 5.47E+00 | 4.16E-02 | 1.06E-01 | 9.85E-02 | 1.70E-01 | 4.41E-01 | 4.59E-01 |
| | **F10** | | | **F11** | | | **F12** | | |
| **IBWO** | 2.81E-01 | 5.39E-01 | 4.90E-01 | **1.58E-09** | **1.03E-01** | 9.76E-02 | **0** | 3.30E-03 | **2.50E-06** |
| **BWO** | **5.93E-02** | **2.35E-01** | **2.07E-01** | 5.83E-02 | **1.03E-01** | **4.72E-02** | 2.66E-15 | **1.79E-04** | 4.99E-06 |
| **GA** | 1.16E-01 | 2.48E-01 | 2.41E-01 | 7.79E-02 | 1.93E-01 | 9.76E-02 | 5.62E-13 | 6.45E-03 | 4.18E-04 |
| **PSO** | 9.10E-02 | 4.16E-01 | 3.72E-01 | 9.65E-02 | 3.71E-01 | 3.72E-01 | 1.19E-01 | 2.32E-01 | 2.32E-01 |
| **ABC** | 7.33E-01 | 1.05E+00 | 1.07E+00 | 2.40E-01 | 5.06E-01 | 4.91E-01 | 5.94E-02 | 1.04E-01 | 1.06E-01 |
| **BBO** | 1.60E-01 | 2.87E-01 | 2.75E-01 | 7.78E-02 | 1.31E-01 | 9.72E-02 | 5.33E-02 | 1.22E-01 | 1.08E-01 |
| | **F13** | | | **F14** | | | **F15** | | |
| **IBWO** | 5.10E-27 | **4.27E-08** | 1.45E-11 | **4.71E-32** | 5.42E-07 | **8.37E-11** | **1.35E-32** | 1.91E-04 | **1.99E-09** |
| **BWO** | **2.35E-30** | 2.45E-07 | **6.10E-12** | 5.28E-17 | 3.13E-06 | 2.20E-09 | 1.81E-07 | **1.70E-05** | 6.71E-06 |
| **GA** | 1.30E-11 | 6.15E-04 | 7.90E-06 | 1.14E-14 | 3.02E-03 | 1.41E-04 | 6.38E-07 | 1.72E-05 | 1.48E-05 |
| **PSO** | 8.33E-05 | 8.60E-03 | 3.72E-03 | 1.06E-07 | 1.33E-02 | 8.27E-06 | 2.00E-06 | 3.29E-03 | 1.26E-03 |
| **ABC** | 4.82E-05 | 6.54E-04 | 4.38E-04 | 1.96E-04 | 2.14E-03 | 1.89E-03 | 3.83E-04 | 1.91E-03 | 1.62E-03 |
| **BBO** | 5.70E-08 | 1.69E-07 | 1.71E-07 | 1.13E-07 | **2.12E-07** | 2.17E-07 | 2.25E-07 | 7.34E-04 | 9.14E-07 |
| | **F16** | | | **F17** | | | **F18** | | |
| **IBWO** | **1.04E-06** | **2.98E-05** | **2.48E-05** | **8.75E-33** | **3.63E-23** | **1.15E-27** | **5.69E-11** | 2.77E-02 | 9.13E-03 |
| **BWO** | 2.30E+00 | 1.33E+00 | 1.34E+00 | 8.34E-11 | 1.34E-02 | 2.91E-04 | 2.75E-02 | 7.74E-02 | 8.63E-02 |
| **GA** | 1.24E+00 | 1.54E+00 | 1.52E+00 | 3.04E-09 | 6.43E+00 | 1.07E-01 | 5.65E-02 | 1.38E-01 | 1.35E-01 |
| **PSO** | 1.38E+00 | 2.28E+00 | 2.30E+00 | 3.10E-05 | 7.61E-03 | 3.23E-03 | 7.04E-02 | 6.39E-01 | 4.97E-01 |
| **ABC** | 2.02E+00 | 2.77E+00 | 2.80E+00 | 5.34E-03 | 2.23E-01 | 1.95E-01 | 1.41E+01 | 2.44E+01 | 2.48E+01 |
| **BBO** | 1.11E+00 | 1.47E+00 | 1.50E+00 | 3.50E-05 | 1.04E-04 | 1.00E-04 | 1.69E-03 | **2.91E-03** | **3.02E-03** |
| | **F19** | | | **F20** | | | **F21** | | |
| **IBWO** | **4.26E-15** | 4.68E-04 | **4.59E-06** | **0** | **0** | **0** | **1.22E-11** | 1.22E-04 | **4.77E-05** |
| **BWO** | 3.22E-04 | **1.54E-04** | 1.37E-04 | **0** | **0** | **0** | 1.62E-05 | **5.85E-05** | 5.69E-05 |
| **GA** | 7.11E-04 | 2.22E-03 | 2.22E-03 | **0** | **0** | **0** | 4.68E-05 | 3.91E-04 | 1.30E-04 |
| **PSO** | 4.72E-05 | 2.18E-03 | 9.24E-03 | **0** | 2.33E-01 | **0** | 1.00E-04 | 4.77E-02 | 3.40E-03 |
| **ABC** | 2.31E-02 | 8.13E-02 | 7.49E-02 | **0** | 1.93E+00 | 2.00E+00 | 9.88E-03 | 7.99E-02 | 6.34E-02 |
| **BBO** | 7.34E-04 | 9.50E-04 | 9.46E-04 | **0** | **0** | **0** | 6.76E-05 | 1.17E-04 | 1.08E-04 |
| | **F22** | | | **F23** | | | **F24** | | |
| | **Best** | **Mean** | **Median** | **Best** | **Mean** | **Median** | **Best** | **Mean** | **Median** |
| **IBWO** | **1.31E-132** | **7.45E-16** | **8.41E-24** | **0** | **8.05E-41** | 1.62E-141 | **0** | **3.56E-44** | 3.95E-83 |
| **BWO** | 1.03E-24 | 2.02E-22 | 2.88E-23 | 1.56E-246 | 9.38E-24 | **2.53E-224** | 7.78E-250 | 5.33E-28 | **8.27E-238** |
| **GA** | 8.93E-23 | 1.03E-20 | 2.79E-21 | 7.26E-196 | 1.14E-23 | 1.33E-188 | 4.38E-201 | 5.55E-25 | 4.35E-194 |
| **PSO** | 2.13E-20 | 3.23E-14 | 9.45E-15 | 6.36E-05 | 4.58E-04 | 2.74E-05 | 4.99E-45 | 1.49E-43 | 2.73E-44 |
| **ABC** | 1.77E-19 | 8.66E-17 | 3.37E-17 | 8.64E-04 | 2.19E-02 | 1.29E-02 | 7.18E-06 | 1.42E-03 | 5.78E-04 |
| **BBO** | 1.11E-27 | 7.32E-27 | 4.07E-27 | 1.06E-22 | 2.13E-11 | 1.23E-12 | 3.29E-84 | 1.49E-10 | 2.13E-26 |
| | **F25** | | | **F26** | | | **F27** | | |
| **IBWO** | **3.36E-53** | **1.22E-06** | **2.81E-10** | **0** | **0** | **0** | **1.27E-04** | **1.08E+02** | **4.28E+01** |
| **BWO** | 1.18E-18 | 9.21E-04 | 4.65E-07 | **0** | **0** | **0** | 1.06E+00 | 1.81E+02 | 1.27E+02 |
| **GA** | 9.05E-10 | 1.10E-01 | 5.20E-03 | **0** | **0** | **0** | 6.64E+00 | 4.01E+02 | 3.61E+02 |
| **PSO** | 1.51E-07 | 1.76E-04 | 2.20E-05 | **0** | **0** | **0** | 1.18E+02 | 9.92E+02 | 9.54E+02 |
| **ABC** | 1.03E-03 | 3.33E-03 | 3.01E-03 | **0** | 6.67E-01 | 1 | 1.56E+02 | 5.13E+02 | 5.22E+02 |
| **BBO** | 3.50E-07 | 1.04E-06 | 9.67E-07 | **0** | **0** | **0** | 5.13E+02 | 1.17E+03 | 1.19E+03 |
| | **F28** | | | **F29** | | | **F30** | | |
| **IBWO** | **1.11E-16** | **8.28E-11** | **1.07E-12** | **0** | **0** | **0** | -2.0212 | -1.9567 | -1.9730 |
| **BWO** | 2.59E-15 | 7.34E-09 | 1.86E-11 | **0** | 1.88E-07 | **0** | -2.0202 | -1.9722 | -1.9798 |
| **GA** | 1.09E-14 | 2.99E-08 | 9.76E-11 | **0** | 5.91E-03 | **0** | -2.0164 | -1.9698 | -1.9809 |
| **PSO** | 2.41E-04 | 2.48E-02 | 4.74E-03 | **0** | 8.47E-07 | **0** | **-2.0218** | -2.0204 | -2.0216 |
| **ABC** | 3.09E-02 | 5.05E-01 | 4.36E-01 | 1.35E-10 | 1.91E-06 | 3.01E-07 | **-2.0218** | **-2.0218** | **-2.0218** |
| **BBO** | 2.57E-09 | 7.45E-06 | 1.30E-07 | **0** | 4.88E-03 | **0** | -2.8831 | -2.8185 | -2.8220 |
| | **F31** | | | **F32** | | | **F33** | | |
| **IBWO** | **1** | **1** | **1** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0** | **2.22E-49** | **3.84E-161** |
| **BWO** | **1** | **1** | **1** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | 1.48E-187 | 7.75E-48 | 2.59E-107 |
| **GA** | **1** | **1** | **1** | - | - | - | 1.04E-160 | 1.42E-33 | 5.15E-105 |
| **PSO** | **1** | **1** | **1** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | 4.87E-178 | 3.00E-02 | 1.19E-02 |
| **ABC** | **1** | **1** | **1** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | 4.37E-15 | 4.04E-09 | 1.37E-10 |
| **BBO** | **1** | **1** | **1** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | 3.34E-174 | 4.69E-15 | 3.51E-44 |
| | **F34** | | | **F35** | | | **F36** | | |
| **IBWO** | -18.5535 | -14.9915 | -14.3422 | **1.17E-11** | **6.96E-04** | **2.43E-07** | **0** | 7.60E-02 | 3.05E-03 |
| **BWO** | **-18.5547** | -18.5547 | -18.5547 | 3.80E-08 | 7.32E-03 | 1.29E-04 | 1.82E-08 | **2.19E-03** | **2.34E-04** |
| **GA** | **-18.5547** | -17.7399 | -18.5531 | 1.41E-05 | 4.71E-02 | 7.47E-03 | 2.19E-07 | 1.17E+00 | 2.75E-01 |
| **PSO** | -18.4961 | -15.1205 | -14.8086 | 3.07E+00 | 1.74E+01 | 1.62E+01 | 1.93E+01 | 7.67E+01 | 5.02E+01 |
| **ABC** | **-18.5547** | **-18.5547** | **-18.5547** | 8.95E-03 | 6.48E-02 | 5.76E-02 | 6.38E-01 | 1.02E+00 | 1.02E+00 |
| **BBO** | **-18.5547** | -17.0587 | -16.9847 | 1.35E-06 | 2.09E-02 | 9.96E-03 | 7.01E-03 | 1.26E-02 | 1.17E-02 |
| | **F37** | | | **F38** | | | **F39** | | |
| **IBWO** | **2.32E-03** | **5.71E-01** | **1.25E-01** | **9.99E-02** | **1.03E-01** | **9.99E-02** | **1.32E-01** | 6.34E-01 | 6.67E-01 |
| **BWO** | 1.99E-01 | 2.83E+00 | 2.18E+00 | 9.99E-02 | 1.03E-01 | 9.99E-02 | 4.37.E-01 | **3.26.E-01** | **4.77.E-01** |
| **GA** | 1.18E+00 | 2.00E+02 | 5.33E+00 | 9.99E-02 | 1.53E-01 | 9.99E-02 | 4.40.E-01 | 7.23.E-01 | 6.95.E-01 |
| **PSO** | 1.04E+01 | 1.38E+02 | 7.04E+01 | 2.11E+00 | 3.66E+00 | 3.77E+00 | 4.97.E-03 | 9.46.E+00 | 9.46.E+00 |
| **ABC** | 5.44E-01 | 5.85E+00 | 4.30E+00 | 1.41E+00 | 2.30E+00 | 2.30E+00 | 2.39.E-01 | 5.72.E-01 | 5.55.E-01 |
| **BBO** | 2.44E-02 | 4.85E-02 | 4.78E-02 | 9.99E-02 | 2.53E-01 | 2.00E-01 | 6.23.E-06 | 5.78.E-01 | 6.67.E-01 |

**Table 4**
Fitness evaluation results of IBWO and other algorithms with 20 D, 150 nPop, and 1000 Iterations

| | F1 | | | F2 | | | F3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Best | Mean | Median | Best | Mean | Median | Best | Mean | Median |
| **IBWO** | **4.78E-49** | **1.93E-21** | **1.03E-35** | **7.38E-38** | **5.75E-05** | **1.14E-10** | **3.92E-31** | **1.41E-06** | **1.02E-14** |
| **BWO** | 2.00E-36 | 5.93E-17 | 1.43E-18 | 3.44E-02 | 2.60E-01 | 2.50E-01 | 3.12E-07 | 6.77E-04 | 2.44E-04 |
| **GA** | 3.32E-33 | 3.05E-13 | 1.39E-18 | 3.81E-01 | 1.39E+00 | 1.34E+00 | 1.02E-06 | 4.42E-03 | 6.50E-04 |
| **PSO** | 7.47E-15 | 1.78E-09 | 3.83E-11 | 5.88E+00 | 2.12E+02 | 1.01E+02 | 1.40E-06 | 2.18E+01 | 2.62E+01 |
| **ABC** | 9.34E-10 | 2.14E-07 | 9.09E-08 | 4.26E+00 | 1.44E+01 | 1.14E+01 | 7.58E-05 | 3.09E-03 | 2.44E-03 |
| **BBO** | 5.70E-21 | 5.98E-18 | 7.98E-19 | 4.24E+02 | 2.71E+03 | 2.01E+03 | 5.57E-02 | 2.62E-01 | 1.88E-01 |
| | **F4** | | | **F5** | | | **F6** | | |
| **IBWO** | **1.64E+01** | **2.27E+01** | 1.86E+01 | **4.44E-15** | 2.57E-04 | 2.53E-07 | **0** | **7.18E-08** | **0** |
| **BWO** | 2.64E+00 | 2.47E+01 | **1.71E+01** | 7.99E-15 | **8.84E-05** | **6.33E-11** | **0** | 1.33E-03 | 3.85E-16 |
| **GA** | 4.07E+00 | 3.87E+01 | 1.82E+01 | 3.40E-07 | 1.05E-01 | 2.05E-02 | 2.44E-10 | 4.96E-02 | 1.41E-02 |
| **PSO** | 2.18E+01 | 9.70E+03 | 2.49E+02 | 9.86E-01 | 2.50E+00 | 2.52E+00 | 3.68E-01 | 8.43E-01 | 8.83E-01 |
| **ABC** | 2.68E+01 | 5.10E+01 | 5.09E+01 | 1.41E+00 | 2.27E+00 | 2.31E+00 | 2.31E-01 | 4.15E-01 | 4.31E-01 |
| **BBO** | 1.42E+01 | 8.03E+01 | 5.68E+01 | 3.05E-03 | 2.89E-01 | 4.55E-03 | 1.02E-05 | 2.56E-03 | 1.60E-05 |
| | **F7** | | | **F8** | | | **F9** | | |
| **IBWO** | **0** | **1.68E-03** | **3.86E-10** | 7.66E-02 | 1.64E-01 | 1.60E-01 | 2.91E-01 | **4.15E-01** | 4.23E-01 |
| **BWO** | **0** | 2.89E-03 | 3.13E-06 | **4.18E-02** | **7.37E-02** | **7.07E-02** | **2.10E-01** | 4.31E-01 | **4.21E-01** |
| **GA** | 3.29E-08 | 4.94E-02 | 1.75E-03 | 1.28E-01 | 2.17E-01 | 2.16E-01 | 3.41E-01 | 4.39E-01 | 4.39E-01 |
| **PSO** | 1.75E+01 | 5.11E+01 | 5.80E+01 | 3.65E-01 | 5.90E-01 | 5.94E-01 | 2.30E-01 | 6.04E-01 | 4.25E-01 |
| **ABC** | 8.68E+01 | 1.14E+02 | 1.17E+02 | 9.84E-01 | 1.79E+00 | 1.74E+00 | 2.08E+00 | 1.28E+01 | 1.35E+01 |
| **BBO** | 9.85E+00 | 2.26E+01 | 2.20E+01 | 1.10E-01 | 1.90E-01 | 1.87E-01 | 3.18E-01 | 4.32E-01 | 4.22E-01 |
| | **F10** | | | **F11** | | | **F12** | | |
| **IBWO** | 7.22E-01 | 1.26E+00 | 1.26E+00 | **5.02E-12** | **2.62E-01** | **2.11E-01** | **0** | **1.78E-05** | **2.08E-08** |
| **BWO** | **3.37E-01** | **5.45E-01** | **5.62E-01** | 1.57E-01 | 3.04E-01 | 3.16E-01 | **0** | 1.89E-04 | 4.76E-08 |
| **GA** | 4.19E-01 | 5.63E-01 | 5.67E-01 | 1.58E-01 | 4.11E-01 | 3.25E-01 | 3.98E-10 | 1.89E-03 | 3.49E-04 |
| **PSO** | 7.06E-01 | 1.59E+00 | 1.20E+00 | 5.78E-01 | 1.33E+00 | 1.37E+00 | 3.66E-01 | 4.50E-01 | 4.53E-01 |
| **ABC** | 1.27E+00 | 3.91E+00 | 2.82E+00 | 1.30E+00 | 2.38E+00 | 2.48E+00 | 1.79E-01 | 2.14E-01 | 2.14E-01 |
| **BBO** | 3.72E-01 | 7.40E-01 | 7.29E-01 | 2.66E-01 | 8.41E-01 | 7.48E-01 | 6.21E-02 | 1.69E-01 | 1.57E-01 |
| | **F13** | | | **F14** | | | **F15** | | |
| **IBWO** | 2.04E-37 | **1.27E-08** | 9.06E-21 | **2.36E-32** | **2.04E-07** | **1.29E-12** | 9.66E-25 | **1.39E-03** | **6.47E-08** |
| **BWO** | **3.41E-75** | 8.90E-08 | **1.36E-46** | **2.36E-32** | 1.96E-06 | 1.35E-09 | **1.35E-32** | 1.45E-03 | 2.80E-05 |
| **GA** | 5.86E-15 | 6.82E-04 | 1.77E-05 | 1.22E-11 | 1.32E-03 | 9.58E-05 | 4.48E-05 | 5.43E-02 | 2.91E-02 |
| **PSO** | 2.75E-03 | 1.33E-02 | 7.55E-03 | 9.25E-01 | 6.72E+00 | 6.02E+00 | 4.07E-01 | 6.56E+00 | 2.47E+00 |
| **ABC** | 2.85E-03 | 8.89E-03 | 8.18E-03 | 1.06E-03 | 7.05E-03 | 6.43E-03 | 7.97E-04 | 1.70E-02 | 1.49E-02 |
| **BBO** | 2.63E-07 | 4.67E-07 | 4.86E-07 | 3.56E-07 | 5.18E-03 | 5.33E-07 | 3.79E-06 | 8.80E-03 | 1.10E-02 |
| | **F16** | | | **F17** | | | **F18** | | |
| **IBWO** | **6.63E-06** | **2.19E-05** | **1.90E-05** | 1.06E-16 | **1.03E-12** | **9.15E-14** | **3.61E-06** | **1.01E-01** | **4.96E-02** |
| **BWO** | 3.63E+00 | 4.17E+00 | 4.18E+00 | **4.38E-23** | 2.10E-10 | 1.15E-10 | 1.05E-01 | 1.58E-01 | 1.53E-01 |
| **GA** | 4.20E+00 | 4.59E+00 | 4.55E+00 | 2.91E-03 | 5.05E-03 | 4.00E-03 | 1.84E-01 | 2.66E-01 | 2.64E-01 |
| **PSO** | 5.99E+00 | 6.99E+00 | 6.94E+00 | 1.44E+00 | 7.25E+02 | 4.17E+01 | 7.83E+00 | 1.51E+01 | 1.55E+01 |
| **ABC** | 7.48E+00 | 9.02E+00 | 8.95E+00 | 1.89E+00 | 5.11E+00 | 4.30E+00 | 3.38E+01 | 5.05E+01 | 5.14E+01 |
| **BBO** | 3.68E+00 | 4.49E+00 | 4.53E+00 | 4.19E-04 | 7.56E-04 | 7.20E-04 | 8.03E-01 | 2.97E+00 | 2.95E+00 |
| | **F19** | | | **F20** | | | **F21** | | |
| **IBWO** | **8.83E-20** | **2.27E-05** | **4.54E-09** | **0** | **0** | **0** | **0** | **3.67E-05** | **3.23E-08** |
| **BWO** | 8.25E-04 | 1.92E-03 | 1.79E-03 | **0** | **0** | **0** | 4.72E-05 | 1.10E-04 | 1.04E-04 |
| **GA** | 1.34E-03 | 3.78E-03 | 3.60E-03 | **0** | **0** | **0** | 1.19E-04 | 4.27E-04 | 2.49E-04 |
| **PSO** | 5.73E-02 | 4.01E-01 | 3.67E-01 | 6.0E+00 | 4.39E+01 | 2.85E+01 | 2.31E-02 | 8.31E-01 | 4.19E-01 |
| **ABC** | 2.78E-01 | 5.75E-01 | 5.79E-01 | 2.0E+00 | 1.47E+01 | 1.45E+01 | 3.66E-01 | 8.28E-01 | 8.32E-01 |
| **BBO** | 4.63E-02 | 2.08E-01 | 2.00E-01 | **0** | 8.23E+00 | 5.00E+00 | 2.42E-03 | 2.06E-02 | 1.29E-02 |
| | **F22** | | | **F23** | | | **F24** | | |
| | Best | Mean | Median | Best | Mean | Median | Best | Mean | Median |
| **IBWO** | **2.99E-68** | 1.92E-15 | 1.59E-20 | **0** | **1.61E-157** | **0** | **0.0** | 1.12E-104 | **0.0** |
| **BWO** | 4.37E-24 | **6.02E-23** | **2.83E-23** | 2.47E-325 | 1.36E-155 | 2.47E-323 | **0.0** | **0.0** | **0.0** |
| **GA** | 2.19E-21 | 1.73E-20 | 1.62E-20 | 6.29E-312 | 4.29E-143 | 5.36E-308 | **0.0** | **0.0** | **0.0** |
| **PSO** | 3.04E-10 | 2.05E-07 | 4.72E-08 | 5.23E-05 | 2.83E-03 | 3.42E-05 | **0.0** | **0.0** | **0.0** |
| **ABC** | 1.14E-12 | 7.64E-11 | 4.15E-11 | 9.25E-09 | 4.36E-03 | 3.35E-03 | 1.47E-10 | 4.13E-09 | 2.59E-09 |
| **BBO** | 1.28E-12 | 1.01E-09 | 2.39E-10 | 1.20E-36 | 2.59E-13 | 3.22E-15 | 3.00E-303 | 7.39E-17 | 1.30E-109 |
| | **F25** | | | **F26** | | | **F27** | | |
| **IBWO** | **1.95E-35** | 1.39E-06 | **2.94E-14** | **0.0** | **0.0** | **0.0** | **4.01E+01** | **3.86E+02** | **3.44E+02** |
| **BWO** | 4.69E-17 | **5.74E-13** | 1.18E-13 | **0.0** | **0.0** | **0.0** | 2.35E+03 | 2.30E+03 | 2.06E+03 |
| **GA** | 3.84E-06 | 5.45E-05 | 5.17E-05 | **0.0** | **0.0** | **0.0** | 2.68E+03 | 3.40E+03 | 3.31E+03 |
| **PSO** | 2.64E-02 | 1.18E+01 | 5.20E-01 | **0.0** | 2.23E+00 | 1.00E+00 | 9.92E+02 | 2.33E+03 | 2.15E+03 |
| **ABC** | 1.19E-02 | 5.87E-02 | 4.94E-02 | 5.00E+00 | 1.05E+01 | 1.10E+01 | 2.10E+03 | 2.73E+03 | 2.78E+03 |
| **BBO** | 1.20E-02 | 2.01E-01 | 1.60E-01 | **0.0** | 6.67E-02 | **0.0** | 2.52E+03 | 3.24E+03 | 3.16E+03 |
| | **F28** | | | **F29** | | | **F30** | | |
| **IBWO** | 4.78E-15 | **2.94E-11** | **1.90E-12** | **0.0** | **0.0** | **0.0** | -2.0195 | -1.9855 | -1.9880 |
| **BWO** | **3.62E-15** | 9.31E-11 | 4.29E-11 | **0.0** | **0.0** | **0.0** | -2.0196 | -1.9973 | -2.0024 |
| **GA** | 1.13E-14 | 1.58E-08 | 4.65E-11 | **0.0** | 9.59E-04 | **0.0** | -2.0217 | -1.9917 | -2.0021 |
| **PSO** | 1.90E-02 | 6.91E+00 | 1.60E+00 | **0.0** | 2.52E-12 | **0.0** | **-2.0218** | -2.0209 | -2.0216 |
| **ABC** | 1.00E+01 | 4.85E+02 | 4.70E+02 | 2.04E-11 | 1.80E-07 | 9.96E-09 | **-2.0218** | **-2.0218** | **-2.0218** |
| **BBO** | 5.08E-10 | 1.36E-05 | 1.65E-07 | **0.0** | 5.18E-17 | **0.0** | -4.6072 | -4.5380 | -4.5425 |

**Table 4**
Fitness evaluation results of IBWO and other algorithms with 20 D, 150 nPop, and 1000 Iterations (Continued)

| | F31 | | | F32 | | | F33 | | |
|---|---|---|---|---|---|---|---|---|---|
| **IBWO** | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | 3.76E-119 | **0.0** |
| **BWO** | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| **GA** | **1.0** | **1.0** | **1.0** | - | - | - | **0.0** | **0.0** | **0.0** |
| **PSO** | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **2.00E+02** | **0.0** | **0.0** | **0.0** |
| **ABC** | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | 2.48E-12 | 1.43E-09 | 6.91E-10 |
| **BBO** | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| | **F34** | | | **F35** | | | **F36** | | |
| **IBWO** | **-18.5547** | -15.2737 | -15.0863 | **4.16E-12** | 1.25E-01 | **3.83E-04** | **9.24E-08** | **5.78E-01** | **1.54E-01** |
| **BWO** | **-18.5547** | **-18.5547** | **-18.5547** | 1.38E-05 | **4.32E-03** | 3.57E-03 | 1.14E-03 | 6.17E-01 | 3.15E-02 |
| **GA** | **-18.5547** | -18.4496 | **-18.5547** | 5.16E-04 | 1.15E+00 | 2.71E-01 | 4.28E-01 | 5.87E+00 | 5.05E+00 |
| **PSO** | -18.5133 | -16.8102 | -16.9989 | 9.22E+01 | 4.28E+02 | 3.91E+02 | 9.48E+01 | 2.56E+03 | 2.45E+03 |
| **ABC** | **-18.5547** | **-18.5547** | **-18.5547** | 7.65E-02 | 3.35E-01 | 3.42E-01 | 1.78E+00 | 3.03E+00 | 3.12E+00 |
| **BBO** | **-18.5547** | -17.8118 | **-18.5547** | 4.01E-05 | 5.36E-03 | 5.30E-03 | 2.55E-02 | 7.04E-01 | 8.52E-02 |
| | **F37** | | | **F38** | | | **F39** | | |
| **IBWO** | 2.85E-02 | **7.29E+00** | **1.48E+00** | **9.99E-02** | 1.33E-01 | **9.99E-02** | 6.67E-01 | 6.90E-01 | 6.68E-01 |
| **BWO** | **2.75E-02** | 1.49E+01 | 8.09E+00 | **9.99E-02** | **1.13E-01** | **9.99E-02** | 6.71.E-01 | 1.12.E+00 | **6.65E-01** |
| **GA** | 9.92E+00 | 7.71E+01 | 2.93E+01 | **9.99E-02** | 1.97E-01 | 2.00E-01 | 8.68.E-01 | 3.73.E+00 | 2.86.E+00 |
| **PSO** | 1.35E+05 | 2.08E+06 | 1.44E+06 | 4.96E+00 | 8.79E+00 | 8.80E+00 | 1.17.E+00 | 9.11.E+01 | 1.95.E+01 |
| **ABC** | 2.59E+01 | 1.17E+02 | 1.06E+02 | 5.37E+00 | 7.87E+00 | 8.28E+00 | 1.03.E+00 | 2.56.E+00 | 2.50.E+00 |
| **BBO** | 1.32E-01 | 2.16E-01 | 2.12E-01 | 2.00E-01 | 4.37E-01 | 4.50E-01 | **3.24.E-03** | **6.67.E-01** | 6.68.E-01 |

Table 5 presents the experimental results for the algorithms for 50 D, 200 nPop, and 1500 Iterations. The results demonstrate that the IBWO algorithm still has better performance when compared to other competitors. However, IBWO had performance lower than the previously achieved results, with the best, mean, and median results being 69.2%, 64.1%, and 69.2%, respectively. Further analysis of the results reveals that IBWO triggered the optimality 11 times, followed by BWO (10 times), GA (7 times), PSO (6 times), BBO (6 times), and ABC (3 times).

**Table 5**
Fitness evaluation results of IBWO and other algorithms with 50 D, 200 nPop, and 1500 Iterations

| | F1 | | | F2 | | | F3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Best | Mean | Median | Best | Mean | Median | Best | Mean | Median |
| **IBWO** | **6.96E-60** | 5.16E-14 | **2.71E-35** | **8.28E-06** | **5.50E-02** | **1.99E-02** | **3.02E-49** | **3.09E-12** | **1.03E-22** |
| **BWO** | 1.13E-32 | **3.09E-17** | 8.87E-21 | 4.22E-01 | 1.78E+00 | 1.61E+00 | 2.98E-07 | 3.43E-03 | 3.45E-04 |
| **GA** | 6.82E-17 | 3.73E-11 | 3.17E-13 | 1.87E+01 | 2.66E+01 | 2.63E+01 | 2.85E-05 | 1.10E-02 | 3.18E-03 |
| **PSO** | 3.79E-09 | 1.66E-04 | 2.67E-05 | 3.76E+05 | 1.65E+06 | 1.54E+06 | 9.04E+00 | 9.15E+01 | 8.53E+01 |
| **ABC** | 3.63E-06 | 2.40E-04 | 1.74E-04 | 7.78E+01 | 1.17E+03 | 8.62E+02 | 2.31E-03 | 7.42E-02 | 3.83E-02 |
| **BBO** | 1.64E-18 | 1.86E-13 | 1.02E-14 | 1.12E+05 | 2.11E+05 | 1.93E+05 | 1.19E+00 | 2.31E+00 | 2.22E+00 |
| | **F4** | | | **F5** | | | **F6** | | |
| **IBWO** | 3.67E+01 | **9.89E+01** | **1.01E+02** | **7.99E-15** | 1.16E-06 | 9.76E-12 | **0.0** | **6.10E-03** | **0.0** |
| **BWO** | **2.21E+01** | 1.13E+02 | **1.01E+02** | 2.93E-14 | **3.70E-14** | **3.29E-14** | **0.0** | 2.48E-02 | 2.78E-16 |
| **GA** | 3.89E+01 | 1.38E+02 | 1.51E+02 | 2.92E-12 | 4.62E-02 | 1.76E-02 | 2.59E-12 | 5.68E-02 | 2.05E-04 |
| **PSO** | 5.90E+04 | 2.08E+05 | 1.95E+05 | 7.10E+00 | 1.01E+01 | 1.02E+02 | 1.17E+00 | 1.67E+00 | 1.55E+00 |
| **ABC** | 1.00E+02 | 3.18E+02 | 3.09E+02 | 9.19E+00 | 1.15E+01 | 1.16E+01 | 8.98E-01 | 1.10E+00 | 1.09E+00 |
| **BBO** | 5.32E+02 | 2.19E+03 | 2.11E+03 | 8.20E-03 | 2.48E-01 | 9.75E-03 | 6.30E-05 | 1.32E-03 | 9.31E-05 |
| | **F7** | | | **F8** | | | **F9** | | |
| **IBWO** | **0.0** | 2.18E-07 | **0.0** | **1.67E-01** | **2.56E-01** | **2.31E-01** | 3.50E-01 | **4.61E-01** | **4.62E-01** |
| **BWO** | **0.0** | **0.0** | **0.0** | 1.71E-01 | 2.60E-01 | 2.60E-01 | 3.33E-01 | 4.80E-01 | 4.56E-01 |
| **GA** | 4.01E-06 | 1.12E-01 | 5.97E-02 | 3.34E-01 | 5.17E-01 | 5.07E-01 | 4.03E-01 | 5.30E-01 | 4.61E-01 |
| **PSO** | 1.56E+02 | 3.28E+02 | 3.43E+02 | 4.96E-01 | 8.02E-01 | 7.99E-01 | **3.15E-01** | 7.39E-01 | 6.72E-01 |
| **ABC** | 1.19E+02 | 1.51E+02 | 1.53E+02 | 4.31E+00 | 5.09E+00 | 5.08E+00 | 1.09E+02 | 1.54E+02 | 1.56E+02 |
| **BBO** | 6.70E+01 | 1.02E+02 | 1.03E+02 | 2.82E-01 | 5.47E-01 | 5.48E-01 | 3.75E-01 | 4.83E-01 | 4.72E-01 |
| | **F10** | | | **F11** | | | **F12** | | |
| **IBWO** | 3.20E+00 | 3.98E+00 | 3.87E+00 | 4.20E-01 | **7.78E-01** | **7.68E-01** | **0** | 2.76E-06 | 2.88E-13 |
| **BWO** | **6.60E-01** | **7.46E-01** | **7.56E-01** | **4.18E-01** | 9.31E-01 | 8.48E-01 | **0** | **2.30E-06** | **2.22E-16** |
| **GA** | 6.91E-01 | 7.59E-01 | 7.70E-01 | 6.53E-01 | 1.10E+00 | 1.01E+00 | 5.87E-10 | 1.20E-03 | 2.56E-04 |
| **PSO** | 2.24E+01 | 3.76E+02 | 1.47E+02 | 3.23E+00 | 6.38E+00 | 6.45E+00 | 5.56E-01 | 6.79E-01 | 6.88E-01 |
| **ABC** | 7.05E+02 | 1.27E+05 | 1.19E+05 | 3.03E+00 | 4.77E+00 | 4.86E+00 | 3.45E-01 | 3.87E-01 | 3.86E-01 |
| **BBO** | 1.00E+00 | 1.09E+00 | 1.08E+00 | 3.67E+00 | 5.50E+00 | 5.21E+00 | 1.77E-01 | 2.90E-01 | 2.90E-01 |
| | **F13** | | | **F14** | | | **F15** | | |
| **IBWO** | **7.29E-48** | 3.50E-12 | 9.24E-26 | **9.42E-33** | **7.83E-08** | **4.26E-14** | 9.73E-22 | 4.71E-05 | 6.09E-08 |
| **BWO** | 2.57E-44 | **4.25E-39** | **2.03E-43** | 1.64E-07 | 1.86E-06 | 7.03E-07 | **1.35E-32** | **1.98E-07** | **6.02E-28** |
| **GA** | 1.39E-15 | 6.46E-04 | 2.23E-05 | 1.02E-05 | 1.53E-05 | 1.36E-05 | 5.45E-10 | 2.21E-01 | 3.28E-02 |
| **PSO** | 9.66E-01 | 3.52E+00 | 3.39E+00 | 1.50E+01 | 1.94E+04 | 4.58E+03 | 8.23E+02 | 1.36E+05 | 4.98E+04 |
| **ABC** | 4.76E-02 | 2.17E-01 | 1.89E-01 | 3.67E-02 | 1.63E-01 | 1.51E-01 | 1.32E-01 | 4.78E-01 | 4.23E-01 |
| **BBO** | 3.57E-06 | 4.94E-06 | 4.99E-06 | 1.86E-06 | 2.00E-02 | 2.93E-06 | 4.48E-05 | 2.57E-02 | 1.10E-02 |
| | **F16** | | | **F17** | | | **F18** | | |
| **IBWO** | **1.10E-04** | **2.43E-04** | **2.32E-04** | 3.66E+00 | 4.38E+01 | 4.10E+01 | **2.04E-04** | **2.94E-01** | **2.67E-01** |
| **BWO** | 1.22E+01 | 1.38E+01 | 1.39E+01 | **3.55E-10** | **3.28E-08** | **5.16E-08** | 3.88E-01 | 4.74E-01 | 4.77E-01 |
| **GA** | 1.37E+01 | 1.59E+01 | 1.61E+01 | 1.14E-04 | 2.62E+02 | 1.42E+02 | 6.81E-01 | 9.25E-01 | 9.17E-01 |
| **PSO** | 2.50E+01 | 3.01E+01 | 3.05E+01 | 1.46E+04 | 8.87E+04 | 7.88E+04 | 4.45E+01 | 5.31E+01 | 5.37E+01 |
| **ABC** | 3.74E+01 | 5.29E+01 | 5.29E+01 | 6.73E+01 | 1.14E+03 | 9.07E+02 | 7.16E+01 | 7.72E+01 | 7.73E+01 |
| **BBO** | 1.38E+01 | 1.58E+01 | 1.60E+01 | 1.88E-02 | 9.67E-02 | 3.44E-02 | 4.48E+00 | 7.42E+00 | 6.69E+00 |
| | **F19** | | | **F20** | | | **F21** | | |
| **IBWO** | **2.76E-25** | **5.23E-09** | **1.89E-11** | **0.0** | **0.0** | **0.0** | **3.41E-26** | **1.83E-10** | **5.18E-12** |
| **BWO** | 3.41E-03 | 6.05E-03 | 5.74E-03 | **0.0** | **0.0** | **0.0** | 1.69E-04 | 3.29E-04 | 3.41E-04 |
| **GA** | 1.90E-02 | 2.89E-02 | 3.04E-02 | **0.0** | **0.0** | **0.0** | 1.26E-03 | 1.95E-03 | 1.72E-03 |
| **PSO** | 4.50E+00 | 1.65E+01 | 1.60E+01 | 2.89E+03 | 6.49E+03 | 5.48E+03 | 2.27E+00 | 2.09E+01 | 1.71E+01 |
| **ABC** | 4.45E+00 | 7.20E+00 | 7.16E+00 | 1.63E+02 | 1.05E+03 | 8.33E+02 | 6.00E+00 | 9.82E+00 | 9.92E+00 |
| **BBO** | 2.40E+00 | 3.47E+00 | 3.43E+00 | 1.72E+02 | 3.29E+02 | 3.20E+02 | 1.71E+00 | 3.46E+00 | 3.46E+00 |

**Table 5**
Fitness evaluation results of IBWO and other algorithms with 50 D, 200 nPop, and 1500 Iterations (Continued)

| | F22 | | | F23 | | | F24 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Best | Mean | Median | Best | Mean | Median | Best | Mean | Median |
| IBWO | **1.65E-43** | 1.80E-18 | **1.21E-26** | **0.0** | **0.0** | **0.0** | **0.0** | **0.0** | **0.0** |
| BWO | 7.56E-21 | **2.06E-20** | 2.06E-20 | 2.47E-323 | 2.47E-123 | 2.47E-172 | **0.0** | **0.0** | **0.0** |
| GA | 2.85E-17 | 7.01E-17 | 6.52E-17 | 4.96E-312 | 2.47E-96 | 2.47E-126 | **0.0** | **0.0** | **0.0** |
| PSO | 2.73E-04 | 1.07E-03 | 8.00E-04 | 3.26E-08 | 4.08E-06 | 2.76E-06 | **0.0** | 4.31E-04 | 4.78E-05 |
| ABC | 3.12E-07 | 1.14E-05 | 5.43E-06 | 1.27E-05 | 3.27E-04 | 1.77E-04 | 7.88E-07 | 3.71E-05 | 2.72E-05 |
| BBO | 3.44E-08 | 2.01E-07 | 1.25E-07 | 3.25E-316 | 1.97E-20 | 8.92E-167 | **0.0** | 1.51E-108 | **0.0** |
| | F25 | | | F26 | | | F27 | | |
| IBWO | **1.07E-11** | **2.73E-03** | **4.91E-06** | **0.0** | **0.0** | **0.0** | **6.32E+01** | **1.33E+03** | **1.22E+03** |
| BWO | 1.71E-10 | 2.73E-01 | 1.53E-01 | **0.0** | 3.33E-02 | 0.00E+00 | 8.57E+03 | 1.02E+04 | 1.00E+04 |
| GA | 7.04E-06 | 1.47E+00 | 8.70E-01 | **0.0** | 3.36E-02 | 1.00E-02 | 7.88E+03 | 9.45E+03 | 9.29E+03 |
| PSO | 9.22E+01 | 7.76E+02 | 5.88E+02 | 6.40E+01 | 1.62E+02 | 1.62E+02 | 6.55E+03 | 8.23E+03 | 7.98E+03 |
| ABC | 1.18E+00 | 1.60E+01 | 1.02E+01 | 7.70E+01 | 1.31E+02 | 1.36E+02 | 6.36E+03 | 7.21E+03 | 7.29E+03 |
| BBO | 1.46E-04 | 1.04E-03 | 4.20E-04 | 2.00E+00 | 9.77E+00 | 9.00E+00 | 8.33E+03 | 1.09E+04 | 1.11E+04 |
| | F28 | | | F29 | | | F30 | | |
| IBWO | 5.52E-11 | 1.03E-07 | 9.44E-09 | **0.0** | **0** | **0.0** | -2.0192 | -1.9923 | -2.0000 |
| BWO | **1.17E-12** | **5.14E-09** | **8.99E-10** | **0.0** | **0** | **0.0** | -2.0217 | -2.0058 | -2.0083 |
| GA | 9.33E-11 | 2.77E-05 | 3.03E-08 | **0.0** | 2.90E-04 | **0.0** | -2.0210 | -2.0092 | -2.0106 |
| PSO | 1.30E+07 | 3.27E+11 | 4.87E+09 | **0.0** | 0.00E+00 | **0.0** | **-2.0218** | **-2.0218** | **-2.0218** |
| ABC | 1.68E+14 | 7.89E+16 | 4.48E+16 | 5.57E-13 | 1.57E-09 | 3.67E-10 | **-2.0218** | **-2.0218** | **-2.0218** |
| BBO | 1.05E-08 | 5.99E-04 | 3.71E-06 | **0.0** | **0.0** | **0.0** | -5.6480 | -5.5841 | -5.5818 |
| | F31 | | | F32 | | | F33 | | |
| IBWO | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| BWO | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| GA | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| PSO | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| ABC | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | 9.49E-12 | 5.75E-09 | 3.27E-09 |
| BBO | **1.0** | **1.0** | **1.0** | **-2.00E+02** | **-2.00E+02** | **-2.00E+02** | **0.0** | **0.0** | **0.0** |
| | F34 | | | F35 | | | F36 | | |
| IBWO | **-18.5547** | -14.4970 | -13.5379 | 1.27E-02 | 8.26E-01 | 3.75E-01 | 1.03E-03 | **6.19E-01** | **3.57E-01** |
| BWO | **-18.5547** | **-18.5547** | **-18.5547** | **6.09E-03** | 4.65E-01 | 2.84E-01 | **8.93E-05** | 7.26E-01 | 3.76E-01 |
| GA | **-18.5547** | **-18.5547** | **-18.5547** | 7.13E-02 | 4.72E+00 | 3.10E+00 | 4.80E+00 | 2.27E+01 | 2.11E+01 |
| PSO | -18.4897 | -17.7351 | -17.8868 | 2.62E+03 | 6.62E+03 | 6.54E+03 | 3.28E+04 | 7.26E+04 | 7.45E+04 |
| ABC | **-18.5547** | **-18.5547** | **-18.5547** | 5.50E+00 | 1.46E+01 | 1.12E+01 | 1.16E+01 | 1.83E+01 | 1.83E+01 |
| BBO | **-18.5547** | -18.4500 | **-18.5547** | 7.72E-03 | **3.93E-02** | **3.17E-02** | 6.52E+00 | 1.95E+01 | 1.85E+01 |
| | F37 | | | F38 | | | F39 | | |
| IBWO | **2.71E-01** | **6.24E+00** | **3.72E+00** | 9.99E-02 | 2.07E-01 | 2.00E-01 | 6.86E-01 | 3.09E+00 | 2.04E+00 |
| BWO | 8.54E-01 | 9.84E+02 | 1.00E+01 | **9.98E-02** | **1.76E-01** | **1.99E-01** | **6.03E-01** | **3.27E-01** | **3.56E-01** |
| GA | 4.98E+01 | 1.77E+02 | 1.09E+02 | 2.00E-01 | 3.53E-01 | 3.00E-01 | 2.51E+00 | 1.78E+01 | 1.93E+01 |
| PSO | 1.26E+08 | 7.91E+08 | 6.83E+08 | 1.47E+01 | 2.22E+01 | 2.27E+01 | 1.23E+03 | 1.84E+04 | 5.13E+03 |
| ABC | 2.30E+03 | 4.13E+04 | 2.92E+04 | 2.21E+01 | 2.61E+01 | 2.64E+01 | 1.91E+01 | 4.50E+01 | 4.67E+01 |
| BBO | 4.53E+00 | 6.94E+00 | 6.87E+00 | 1.10E+00 | 1.56E+00 | 1.50E+00 | 6.67E-01 | 1.36E+00 | 7.06.E-01 |

The analysis of previous results has shown that the IBWO algorithm outperforms those of the other competitors in several experiments on 39 benchmark functions and parameter settings for the number of populations, the dimensions of the problem, and the number of iterations. This indicates the significant ability of the proposed method to examine and recognize potential search space regions during search iterations, which was attained through utilizing the preceding experiences of the exploration search strategy. However, IBWO's performance declined while solving several of the problems belonging to the family of unimodal and separable test functions, such as Expanded Griwank's plus Rosenbrock (F10), Schwefel 1.2 (F17), Powell Singular 2 (F35), and Dixon & Price(F39). The findings also demonstrate that either IBWO or BWO achieves the best score in all experiments conducted on all benchmark functions, except for Adjiman (F30), where neither has been achieved under various parameter settings. Additionally, only the ABC algorithm received the highest scores for the best, mean, and median results across all parameter settings for F30 and F34.

## 6. Conclusion

This study has introduced an enhanced version of the Black Widow Optimization (BWO) algorithm called Improved BWO (IBWO). IBWO incorporates a mechanism that enhances the global search performance of the algorithm by tracking and remembering good search regions during the search iteration. This mechanism has enabled the optimization process to focus on promising regions of the search space, leading to improved convergence towards the global optimum and the generation of high-quality solutions. The effectiveness of IBWO has been demonstrated through experiments comparing its performance with the original BWO algorithm and four additional optimization methods (GA, PSO, ABC, and BBO) across a set of benchmark functions. The results have shown that IBWO outperforms the alternatives in terms of solution quality and convergence speed. Statistical analysis has confirmed the significant improvement offered by IBWO over the BWO algorithm. Furthermore, IBWO has been evaluated on 39 benchmark functions, covering various categories, to assess its effectiveness across a broader range of optimization problems. The results have indicated that IBWO consistently outperforms the alternatives and maintains stable performance accuracy across different parameter settings. However, it is noted that IBWO's performance was slightly lower when dealing with certain types of unimodal and separable test functions. Future work in this area will focus on addressing the weaknesses of IBWO to enhance its effectiveness across various forms of benchmark (Al-Wesabi et al., 2022)functions. Additionally, the proposed method will be evaluated on real-world optimization problems in different industries such as transportation (Basalamah et al., 2023; Khan & Shambour, 2023b), energy (Alrajhi, 2020; Al-Wesabi et al., 2022), robotics (Loganathan & Ahmad, 2023), smart environment (Malibari et al., 2022), scheduling and timetabling (Khan & Shambour, 2023a; Shambour & Khan, 2019). This expansion into real-world challenges will provide insights into the practical applicability and performance of IBWO in solving complex optimization problems outside the realm of benchmark functions (Shambour & Abu-Hashem, 2023). For instance, it can contribute solutions to improve real-world challenges such as effective management of pilgrim crowds and transportation during the Hajj season, streamline supply chain logistics, and enhance energy distribution efficiency.

**Competing interests**

The authors declare no conflicts of interest that are relevant to the content of this article.

**Data availability**

The data used to support the findings of this study are available upon request.

**Ethical approval**

We declare that this work is original and not considered to be published in any other publication media.

**Acknowledgements**

The authors extend their appreciation to the Deputyship for Research and Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project number: IFP22UQU4361183DSR060.

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