Analytical estimation of tooth strength, restored by direct or indirect restorations

O. Bulbuk, A. Velychkovych, V. Mazurenko, L. Ropyak and T. Pryhorovska*

Ivano-Frankivsk National Medical University, Ukraine
Ivano-Frankivsk National Technical University of Oil and Gas, Ukraine
Vasyl Stefanyk Precarpathian National University, Ukraine

1. Introduction

Thin-walled structures are widespread distributed in nature and widely used in engineering because these structures successfully combine low material consumption and required strength. However, the thin-walled structure like any perfect product does not have a very high safety margin, and therefore is quite sensitive to the violation of nominal load modes and to various kinds of imperfections. Stress concentrators or other defects develop during operation and jeopardize these structures, thus they can significantly reduce structure’s life (Gross & Seelig, 2018). Frequent fractures of anatomical co-root of a tooth with a defect under full load are the clear example of this phenomenon (Jin et al., 2016; Listl et al., 2015). Also, residual stresses and contact loads appearing after restoration can be dangerous when the thin-walled structure contacts with an elastic body (Shatskyi et al., 2018; Velichkovich et al., 2018).

Today, the qualitative restoration of the destroyed anatomical crown of the tooth remains an urgent problem of modern biomechanics and dentistry (Listl et al., 2015). A correct selection of restorative materials and a crown restoring method are the keys to a successful solution to this problem. First, this approach should provide the durability of all elements of the composition (stored tooth and restoration material) and their adhesion. Tooth functional properties significantly depend on the chewing stresses of...
hard tissues of the tooth. Therefore, careful calculation of the stress-strain state indicators and the restored tooth crown strength assessment are necessary in order to choose an adequate restoration method. The inadequate study of this issue and the inconvenience for the practical application of already existing results have become the motivation for this research works.

The technology of tooth restoration includes direct and indirect methods of restoration, in particular filling, pin constructions, dental onlays and crowns. The crowns are obtained by the centrifugal casting method (Ropyak et al., 2016). The first attempts to analyze the destruction causes of chewing tooth restored crowns as a biomechanical problem were initiated in the work of Milikevich et al., (1981) and subsequently developed in the works of Mamoun & Napoletano, (2015); Srirekha & Bashetty, (2013); Nishta et al., (2014).

The method, proposed in Milikevich et al., (1981), also determines the tooth occlusive surface destruction index (TOSDI) and recommends using one of the four main types of filling (seal, dental onlay, artificial crown, pin construction) for tooth crown clinical restoration. The type of used filling depends on the diagnostic scale (0 – 0.55 – 0.6 – 0.8 – 1) of the TOSDI possible values. The TOSDI index is defined as the ratio of the 2nd class (by G. V. Black) carious cavity area and the tooth occlusive surface area. Practical recommendations for a tooth restoration method usage are based on the developed a mechanic-mathematical model of the tooth with the defect and relation between the tooth crown stresses and the TOSDI index. However, this index does not take into account destruction depth of the hard tooth tissues. Therefore, an improved index was proposed in Mamoun & Napoletano (2015), which consider, additionally, tooth crown fracture depth index (TCFDI) and gives recommendations for tooth restoration method depending on a two-dimensional diagnostic scale. The works published by Srirekha & Bashetty, (2013); Nishta et al., (2014) propose a finite-element model derived from the graph approach or three-dimensional modeling for the stress-strain state of the “tooth-filling” system.

Contact problems for elastic systems constitute the essential part of the mechanics of deformable solids. Statements and methods for contact problem solving by continuum models of a continuous medium are presented in the works of different researchers such as (Goryacheva & Martynyak, 2014; Kravchuk & Neitaitanaki, 2007; Dolgov et al., 1996). Pryhorovska & Chaplinskyy, (2014); Pryhorovska, (2017); Larson et al. (2013) studied the problems of contact interaction of the tooth and solids with loading. These works shows the general patterns of contact interaction and describe the state of the problem. However, these approaches cannot fully meet the needs of engineering practice because of their mathematical complexity. The works, which describes the numerical methods for contact problem solving and experimental approaches (for example, (Velichkovich & Dalyak, 2015; Velichkovich et al., 2011; Vlasiy et al., 2017)) deserve attention too. Attention is paid to the simulation of strength, stiffness and stability of shell and planar structures, especially of their complex forms (Panevnik & Velichkovich, 2017) and for cases of stress concentrators (Kolesov et al., 1992) or inclusions. In particular, the effects of stress concentrations in nonhomogeneous solids near cavities and inclusions were studied in (Kolesov et al., 1993; Shats’kyi, 2015), and a model was proposed for assessing the efficiency of rectifying crack-like defects with non-contrast material in (Striletskyy & Rovinskyy, 2017). The physical (Saakiyan et al., 1987), electrochemical (Saakiyan et al., 1989; Dolgov, 2016) and analytical (Shatskii, 1989) methods are used to study the behavior of materials, including layered structures with loading. However, models and methods of the theory of rods, shells and plates are more suitable for analytic studies of body contact interactions. According to these approaches, the main difficulties in contact tasks arise at the stage of research object simulations and the adequate choice of theory. But the contact tasks, in fact, for thin-walled elements have their own specificity associated with the emergence of irregular contact tensions, additional zones of detachment, etc. The analytical methods for solving such problems for plates and shells with defect-kits taking into account the effect of closing cracks under multiparameter loading were first proposed in the papers (Shatskii, 1989; Shats’kyi & Makoviichuk, 2005).
In our opinion, the analytical mechanical and mathematical models of the system "tooth-filling", studied in the previous papers, are somewhat simplified and do not consider a number of important factors. First of all, we mean neglecting of the numerical calculations of filling compressibility and not taking into account the effects of Poisson for the tooth tissues, which distorts the parameters of the stress-strain state and does not allow to estimate the possibility of loading influence of the system "tooth-filling" by stretching stresses. Such simplifications can significantly affect the proper choice of the tooth crown restoration method. Therefore, our work presents the results of research aimed at improving the methods of analytical evaluation of the stress-strain state of the system "tooth-filling" or "tooth-onlay". The authors developed the model, based on the idea the model is not universal, but directed to solving a specific problem; however it will make possible to receive analytical results, suitable for convenient use in dentistry. The model is, first, valuable in terms of obtaining a qualitative picture of the phenomena and studied processes. The model may be enriched by the methods (Shatskyi et al., 2016; Ropyak et al., 2017; Shatskii & Perepichka, 2013; Shats'kyi et al., 2016; Velichkovich, 2005) and it will make possible taking into account the layering of the tooth tissues and consider the research with local, arbitrarily oriented and dynamic loads, as well as the fatigue damage. The final quantitative data will be refined by means of point (single) virtual experiments.

The purpose of the study was to improve the analytical evaluation method of tooth crown restoration for the “tooth – filling” or “tooth–onlay” stress-strain state and to develop recommendations for destroyed crown recovery.

The purpose is detailed in the following tasks:
- to develop a mechanical - mathematical model of the “tooth – filling” contact as a system of equations, describing the static equilibrium of the system, physical relations of the tooth and filling solid tissues with external compressive loading, boundary conditions and contact conditions;
- to study the stress-strain state of the system and calculate all its indicators (in particular axial, contact and circumferential tensions);
- to develop and draw relations between the obtained results and the Tooth Linear Damage Index (TLDI), elastic-mechanical properties of the tooth and feeling materials used for the tooth crown restoration;
- to analyse the relations, in particular to evaluate theory, that takes into account maximum equivalent Guber-Mises stress magnitude;
- to develop practical recommendations on tooth restoration rational method choice.

2. Materials and methods of tooth tissue and monolithic filling or dental onlay strength study

The objective of the study is the “tooth – filling (onlay)” stress-strain state definition with consideration contact interaction of the tooth hard tissues with the filling (onlay). The problem of contact interaction of a cylindrical shell (a tooth crown model) and elastic filler (a onlay model) with hard action of a hard stamp (a chewing load model) was formulated and solved. The developed models of the shell and the filler made possible to formulate the boundary problem in a form, suitable for analytical solving. The method of stress-strain state determination and system strength evaluation was developed. Obtained result reliability is confirmed by validity of the geometric-linear problem formulation, the rigorous implementation of mathematical methods tested in the literature for analytical studies, the individual partial (marginal) case result convergence with literature known results.
2.1 Materials, used for experimental study

The study was conducted for a piecewise-homogeneous elastic composition with hard stamp loading, which simulates chewing of the tooth, restored by direct or indirect restorations. The cross-section of the model is a dual-binding region with elasticity and strength ranging from layer to layer, i.e. piecewise-constant functions related the radial coordinate. The stress-strain expressions for all material layers were developed based on the same static and kinematic hypotheses of the elasticity linear theory.

The following statements were assumed:
- material properties of the homogeneous regions are determined by the Young modules, Poisson's coefficients, and the boundaries of yield or strength (Chung & Park 2016).
- properties of tooth hard tissues are considered constant, and the properties of the restoration material are considered discrete-variable. The following materials were used to simulate tooth restoration:
  - a common light hardening filling material (a Gradia Direct type composite) with options: modulus of elasticity $E=1.2 \text{ GPa}$, Poisson coefficient $\mu=0.43$.
  - a ceramic material for inserts (type Vitablocs) with the following options: modulus of elasticity $E=6.0 \text{ GPa}$; Poisson coefficient $\mu=0.4$.

The averaged elastic constants for the hard tissues of the tooth were: $E_0=95.0 \text{ GPa}$, $\mu_0=0.32$.

2.2 Mechanical-mathematical model of the system “tooth-filling” or “tooth-onlay”

Due to complexity of the masticatory tooth occlusive surface and its layered structure, the authors assumed some statements. The restored tooth is simulated as a cylindrical body 1 with a non-through hole (Fig. 1), smoothly stubbed in a rigid obstacle. The model options are:
- vertical wall thickness $h$;
- outer diameter $D$;
- hole depth is the same order value with the outer diameter $D$;

The hole cavity is densely filled with an elastic deformable cylinder 2 (radii $R$ and height $a$). Tooth tissues are considered as a continuous isotropic medium with averaged elasticity constants. A flat smooth area represents the chewing surface. Chewing effort $Q$ is applied to the tooth surface by the hard stamp with average pressure $p = Q / (F + F_0)$, where $F$—filling (or onlay) cross-sectional area, $F_0$ — cross-sectional area of the tooth with a defect. The edge effect in the cylindrical body bottom is neglected at the initial research stage. The stress-strain state of the system is described in the cylindrical coordinates.

Fig. 1. Schematic diagram of the restored tooth: 1 — hard tooth tissues; 2 — filling (onlay)
The elastic equilibrium of the cylindrical body (the model of the tooth with the defect) is described by the momentless shell theory. The tooth static equilibrium is described the relations:

$$\frac{dh\sigma_{z_0}}{dz} - \tau = 0,$$

$$h\sigma_{\beta_0} = -R\sigma,$$  \hspace{1cm} (1) \hspace{1cm} (2)

The reaction of the loaded tooth hard tissue is described by the physical relationships (generalized Hooke's law):

$$\varepsilon_{z_0} = \frac{1}{E_0}\left(\sigma_{z_0} - \mu_0\sigma_{\beta_0}\right),$$

$$\varepsilon_{\beta_0} = \frac{1}{E_0}\left(\sigma_{\beta_0} - \mu_0\sigma_{z_0}\right)$$ \hspace{1cm} (3) \hspace{1cm} (4)

and the Cauchy relations: $$\varepsilon_{z_0} = \frac{du_0}{dz}, \varepsilon_{\beta_0} = \frac{w_0}{R},$$ where $$\sigma_{z_0}, \sigma_{\beta_0}$$ – axial and circular tooth strains, $$\varepsilon_{z_0}, \varepsilon_{\beta_0}$$ – axial and circular tooth deformations, $$\sigma, \tau$$ – normal and tangential contact stresses, $$u_0, w_0$$ – axial and radial displacements, $$E_0, \mu_0$$ – average Young's modulus and Poisson's coefficient of tooth hard tissues.

As the initial model relations for the internal cylinder (filling or onlay), we use the following equations of equilibrium (Popadyuk, 2016):

$$\frac{d\sigma_z}{dz} + 2\tau = 0,$$

$$\sigma_r = \sigma_\beta = \sigma,$$  \hspace{1cm} (5) \hspace{1cm} (6)

Hooke's law

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \mu(\sigma_r + \sigma_\beta)\right),$$

$$\varepsilon_\beta = \frac{1}{E}\left(\sigma_\beta - \mu(\sigma_r + \sigma_z)\right)$$ \hspace{1cm} (7) \hspace{1cm} (8)

and the Cauchy relations: $$\varepsilon_z = \frac{du}{dz}, \varepsilon_\beta = \frac{w}{R},$$ where $$\sigma_r, \sigma_\beta, \sigma_z$$ – radial, circumferential and axial stresses in the filling (or onlay); $$\varepsilon_z, \varepsilon_\beta$$ – radial, circumferential and axial deformations of the filling; $$u, w$$ – radial and axial displacement of the filling, $$E, \mu$$ – the elastic modulus and Poisson's ratio of the material of the filling (onlay). The ideal contact relations describe the tooth and filling (onlay) contact conditions: $$u_0 = u, \quad w_0 = w.$$ The axial and radial displacement equality conditions $$u_0 = u$$ and $$w_0 = w$$ are replaced by the compatibility conditions of the tooth and filling (onlay) deformations. The boundary conditions of the tooth free face end are:

$$\sigma_z(0)F + \sigma_{z_0}(0)F_0 + Q = 0.$$  \hspace{1cm} (9)
2.3 “Tooth-filling” and “tooth-onlay” stress-strain state indicator definition

The aforementioned stresses $\sigma_{z0}$, $\sigma_{\rho0}$, $\sigma_r$, $\sigma_{\rho}$, $\sigma_z$, are the main indicators of the “tooth-filling” and “tooth-onlay” stress-strain state. In order to find them, the relations (3–7) were transformed:

$$\begin{cases}
h\sigma_{\rho0} = -R\sigma,
\sigma_r = \sigma, \sigma_{\rho} = \sigma, \tau = 0,
\frac{1}{E}\sigma_z - \frac{1}{E_0}\sigma_{z0} - \left(\frac{2\mu}{E} \frac{\mu_0}{E_0} h\right)\sigma = 0, \\
-\frac{\mu}{E}\sigma_z + \frac{\mu_0}{E_0}\sigma_{z0} + \left(\frac{1}{E_0} h + \frac{1-\mu}{E}\right)\sigma = 0.
\end{cases}$$

(10)

All stress state components were defined by the system (10) solving with the boundary condition (9). The main analytical results are presented below.

Axial stresses in the filling are:

$$\sigma_z = -\frac{Q}{F_0} \cdot \zeta,$$

(11)

where $\zeta = \frac{1}{E_0} \left(\frac{1}{\lambda - \mu_0}\right)$, $\lambda = \frac{2E\frac{F}{F_0} + (1-\mu)E_0}{\mu E_0 + \mu_0 E \frac{F}{F_0}}$.

Axial stresses in the tooth hard tissues are:

$$\sigma_{z0} = -\frac{Q}{F_0} \cdot \left(1 - \frac{F}{F_0} \zeta\right).$$

(12)

The normal stresses on contact surfaces of fillings and tooth (contact pressure) are:

$$\sigma = \frac{Q}{2 \frac{F}{E_0} + (1-\mu)\frac{F_0}{E}} \left(\frac{\mu_0}{E_0} - \zeta \left(\frac{\mu}{E} \frac{\mu_0}{E_0} \frac{F}{F_0}\right)\right).$$

(13)

The circumferential stresses in the tooth hard tissues are:

$$\sigma_{\rho0} = -2\sigma \frac{F}{F_0}. $$

(14)
2.4 “Tooth-filling” and “tooth-onlay” stress-strain state indicator calculation results and their analysis

The obtained results made possible to specify the following cases. The Tooth Linear Damage Index (TLDI) \( k = h / D \) was used to describe the obtained numerical data. This index is convenient for practical usage, because of its easiness for calculations (Fig. 1). Fig. 2 depicts tooth hard tissue axial stresses related the TLDI. The more rigid material takes more share of external axial loading. Limit transitions by the expression (12) show the following:

- for the tooth with the defect (without restoration) \( E \to 0 \) and \( \mu \to 0 \), was obtained \( \sigma_{z0} = -Q / F_0 \);  
- for the healthy tooth \( E = E_0 \) r a \( \mu = \mu_0 \), was obtained \( \sigma_{z0} / p = -1 \);  
- if \( k = h / D = 0,5 \), then \( \sigma_{z0} / p = -1 \), which also corresponds to the healthy tooth.

Fig. 3 depicts the contact stresses of the filling (onlay) external surface and the tooth internal surface related the TLDI index. It worth mentioning, contact stresses were negative (compressing) for both a photopolymer filling and a ceramic onlay. An additional analysis of the formula (13) shows the following:

- if the filling with \( \mu < \mu_0 \) was used for restoration, the tensile stresses will change their sign - they will become stretching, and it can lead to a deterioration of adhesion or even the separation (removal) of the filling from the tooth;  
- for the healthy tooth \( E = E_0 \) r a \( \mu = \mu_0 \), it was obtained \( \sigma = 0 \);  
- contact stresses were, averagely, two orders lower, then the axial ones; so they can be neglected in strength evaluation (one of the main stresses of the tooth material is taken to be zero);  
- the simplified tooth model without Poisson's effect consideration \( \mu_0 = 0 \) gives significantly overpriced results regarding to the contact stresses.

![Fig. 2. Axial stresses in the crown of the tooth: 1 – restored by a light-cured composite; 2 – restored by a ceramic onlay; 3 – not restored (no defect).](image1)

![Fig. 3. Contact stresses between the filling and the tooth: 1 – restored by a light-cured composite; 2 – restored by a ceramic onlay.](image2)

Fig. 4 depicts the circumferential stresses of the restored crown tooth tissues related the TLDI index. These tensions were positive, that is, stretching. In both cases, the following was observed: the smaller thickness of the tooth crown wall \( h \), the higher the ring stresses; the intensity of circumferential tension increasing related to the TLDI index is accompanied by increasing of the Poisson coefficient of the restoration material. The stressed state of the loaded hard tissues of the restored tooth is complex. Therefore, one of the theories of strength was used to assess the strength, and the actual estimate is to be...
made according to the magnitude of the maximal equivalent stresses $\sigma_{eq0}$. We used the Guber-Mises criterion to find them: $\sigma_{eq0} = \sqrt{\sigma_{\beta0}^2 + \sigma_{\zeta0}^2 - \sigma_{\beta0} \sigma_{\zeta0}}$.

Fig. 4. Circumferential stresses in the crown of the tooth: 
1 – restored by the light-cured composite; 2 – restored by the ceramic onlay.

Fig. 5. Equivalent stresses in the crown of the tooth: 
1 – restored by the light-cured composite; 2 – restored by the ceramic onlay.

Equivalent stresses in the filling or onlay can be calculated by the same way. The Fig. 5 depicts the restored crown tooth tissues related to the TLDI index. Tooth tissue strength is provided in case of the equivalent stresses and do not exceed the ultimate values (determined in the experiment). However, filling material changes its properties – degrades because of cyclic load and temperature stresses, in particular, it may gradually lose its compressibility. For the ultimate estimation of this phenomenon, it was assumed that the material of the seal became insoluble (deformation occurs only by molding), i.e. $\mu=0.5$. This way, tooth equivalent stresses significantly increase. The Fig. 5 depicts this phenomenon for the curve 1, where the dotted line depicts the equivalent stresses for the incompressible filling. This phenomenon has much less influence and can be neglected for the ceramic onlay. The Fig. 6 depicts two plots on the same coordinate field $f_0(k) = [\sigma]_{eq0} / \sigma_{eq0}(k)$ and $f(k) = [\sigma]/\sigma_{eq}(k)$ for final strength assessment of the compositions "tooth-filling" or "tooth-onlay" for a particular case. The presented
functions $f_0(k), f(k)$ are the tooth and the filling (or onlay) strength factor distributions, and $\left[\sigma\right]_0$ and $\left[\sigma\right]$ – admissible normal stresses for the tissues of the tooth and the filling. If such plots have a point of mutual intersection, it will point to the zone of equal structure "tooth-filling". The area of rapid structure destruction risk is defined by the $k$ values, for which $f_0(k) < 1$ or $f(k) < 1$ (or the value of the two functions at once is less than 1). Fig. 6 depicts the strength of the tooth restored with a photopolymer sealing material. The composition "tooth - photopolymer filling" remains strong if $k \geq 0.175$. This means the following: if one assumes that the outer diameter $D = 10$ mm, the strength of the wall thickness should be no less then $h \geq 0.175D = 1.75$ mm; if wall thickness is less - the filling is not effective and the ceramic onlay should be used to restore the tooth.

![Fig. 6. Assessment of tooth strength restored by the light hardening composite.](image)

### 3. Discussion of the tooth strength assessment results with consideration “tooth-filling” contact interaction

The main results of our study are obtained by contact interaction mechanics analytical application. The TLDI index was proposed to for evaluation of defects in hard tooth tissues. The stress-stain study results make possible the following statements. The axial stresses in the hard tissues of the recovered tooth depend on the ratio of elasticity constants (Young modules and Poisson's coefficients) to the restoration material and tooth tissues. In particular, the stiffer compression of the restoration material is accompanied by the less axial stresses of the tooth tissues. It worth mentioning, the mechanical characteristics of composite sealing materials range depend on the filler size (in particular, the Young modulus - from 4 GPa to 16 GPa). For these reasons, when reducing the TLDI index, one should strive to increase the Young module of the restoration. The contact stress between the restoration and tooth tissues can be both compression and stretching, depending on the ratio of their Poisson coefficients. But as far as the contact stresses were two orders less than the axial ones, it can be assumed that the hard tooth tissues are in a flat stressed state (under the action of axial and circumferential stresses). The circumferential stresses in the hard tissues of the restored tooth are stretching. For more damaged tooth (with a smaller index $k$), the circumferential stresses are larger, and the intensity of their growth increases with an increase in the Poisson's coefficient of the restoration material.

Tooth strength assessment is in interest in terms of practical recommendations for choosing the tooth restoration method. To calculate it, the most used energy criterion of Guber-Mises has been chosen. The study made it possible to obtain an expression to determine the minimum thickness of the tooth wall with
a defect, in which filling restoration is still safe, and showed that neglecting the Poisson effect can significantly affect the values of equivalent stresses. In general, the results obtained in this article will be useful in applying to dentistry, because the success of restoration of a defective tooth depends, first of all, on the correct choice of restoration. The developed mechanical and mathematical model of “tooth-filling” contact interaction allowed obtaining clear qualitative and quantitative estimates of the behavior of the restored tooth with the masticatory load. Of course, in further research, we will try to enrich this model, taking into account the layering of the tooth tissues and considering it under the influence of local, randomly oriented and dynamic loads, and also to evaluate the possibility of fatal injuries, in order to bring the model closer to the real object of research.

4. Conclusions

1. The mechanical and mathematical model of the system "tooth-filling (onlay)" was developed, which allowed taking into account the influence of essential parameters of elasticity and strength and formulating the boundary-contact problem in a form suitable for analytical solving on the object of study.

2. The stress-strain state parameters were calculated, in particular the axial, contact and circumferential stresses (analytical expressions (11) – (14), which take into account the elastic-mechanical properties of the hard fabrics of the tooth and the material of the filling or onlay). Based on these data, plots of these indicators related the TLDI index was obtained. This index is convenient for practical application, because it is easy to obtain by direct measurements.

3. Analysis of showed that increasing the stiffness of the material of the filling (onlays) reduces the axial stresses in the tissues of the tooth. Therefore, the restoration material with the larger Young module should be used in case of the TLDI index decreasing. In addition, the contact stress between the filling (onlay) and the tissues of the tooth can be both compression and stretching, depending on the ratio of their Poisson coefficients. For this reason, it is impractical to use filling materials with \( \mu < \mu_0 \) for tooth restoration, because contact stresses will become stretched, which may lead to deterioration of adhesion or even to tear off the filling from the tooth. In addition, the contact stresses are, averagely, two orders less than the axial ones, so they can be neglected when evaluating the tooth strength.

4. The restored tooth strength assessment was made on the basis of the Guber-Mises energy criterion by the magnitude of the maximum equivalent stresses. The strength of the wall thickness should be no less then \( h \geq 0,175D = 1,75 \text{ mm} \); if wall thickness is less the filling is not effective and the ceramic onlay should be used to restore the tooth. In addition, the neglect of the Poisson effect can significantly affect the values of equivalent stresses.

5. Based on the analysis, practical recommendations for damaged tooth crown reconstruction was developed.

References


© 2019 by the authors; licensee Growing Science, Canada. This is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (http://creativecommons.org/licenses/by/4.0/).