Decision Science Letters 14 (2025) ***–***

Contents lists available at GrowingScience

Decision Science Letters

homepage: www.GrowingScience.com/dsl

Evaluating women’s happiness levels with ARASsort: The case of Türkiye

Semiha Bal[a], Gül Tekin Temur[b] and Sait Gül[b*]

[a]*Independent Researcher, Turkey*
[b]*Bahçeşehir University, Turkey*

C H R O N I C L E

Article history:
Received: February 11, 2025
Received in revised format: March 15, 2025
Accepted: March 19 2025
Available online: March 19, 2025

Keywords:
ARASsort
Happiness
Life satisfaction
Multiple attribute sorting
Women’s happiness

A B S T R A C T

The happiness levels of women exhibit variations attributable to a myriad of factors, encompassing economic, social, cultural, and demographic variables. Numerous governments incorporate the measurement of happiness levels as part of life-satisfaction analyses; nonetheless, these analyses lack a comprehensive framework for predicting happiness levels over specific periods. Notably, in developing countries, women confront the adverse consequences of economic, social, cultural, and demographic determinants to a greater extent than men. Paradoxically, they remain significantly underrepresented in both academic and industrial domains. In light of this, the primary objective of this study is to conduct an in-depth analysis of happiness levels and their underlying determinants from a gender-oriented perspective. Therefore, the pertinent literature has not dedicated a systematic approach to classify and forecast the happiness of women. The present paper initiates by elucidating the factors influencing women's perceptions of happiness through a comprehensive review of the existing literature. Then, a multiple attribute decision-making algorithm-based sorting methodology, ARASsort, is utilized to evaluate how women’s happiness levels are affected by life satisfaction components in a developing country, Türkiye. The selection of ARASsort is based on its performance over other traditional sorting approaches in terms of time and effort attachment. Various factors affecting the happiness levels of women in different cities in the country sample were discussed and analyzed in detail in accordance with the main findings of the OECD Better Life Index (2020), through representative data selected from TÜİK's life satisfaction dataset.

1. Introduction

Well-being which is accepted as an indicator of societal improvement is a significant value for human lives. In the relevant area, researchers who are interested in well-being evaluation have offered two ways; (1) objective and (2) subjective well-being (Voukelatou *et al.* 2020). Subjective well-being (SWB) is a generic set of circumstances that encompass an individual’s emotional acts, domain satisfactions, and judgments of life satisfaction (Diener *et al*., 1999) and it addresses the assessments made by people about their lives (Shmotkin, 2005), where SWB is seen as a hypernymic structure, which includes both cognitive and emotional elements (Diener, 1984). SWB is defined as a common field of scientific interest, whose divisions include pleasant and unpleasant effects, as well as life and domain satisfactions (Diener *et al*., 1999). Previously, the referential term used was *life satisfaction*, but that of *happiness* has now come to predominate (Shmotkin, 2005). “Happiness” and “well-being” are used synonymously in many studies; however, “happiness” is a concept that indicates the superiority of pleasure over negative feelings (Nunes and Proença, 2023).

* Corresponding author.
E-mail address sait.gul@bau.edu.tr (S. Gül)

ISSN 1929-5812 (Online) - ISSN 1929-5804 (Print)

doi: 10.5267/j.dsl.2025.3.009

Life satisfaction is one of the most important facets of SWB and it is measured by several components including happiness. Wilson (1967) defines a happy person as one, who is "young, healthy, well-educated, well-paid, extroverted, optimistic, worry-free, religious, married person with high self-esteem, job morale, modest aspiration, of either sex and of a wide range of intelligence" (Diener *et al.*, 1999). Over the years, this definition evolved as a result of research undertaken in the fields of SWB psychology and economics. While Bolonkin (2012) states that "happiness is a state of mind or feeling characterized by contentment, love, satisfaction, pleasure or joy"; attempts have also been made to define the term from various other perspectives, including those falling within the domains of biology, psychology, religion, and philosophy. Easterlin (2006) also contends that the happiness life cycle and its sources can vary depending on psychological, economic, and demographic status. In the study of Shin and Kim (2021), the definition of happiness is given as "physical comfort and emotional pleasure related to satisfying personal needs in a specific social and cultural environment" and it is mentioned that happiness belongs to characteristics of people and cognitive measurement of the external conditions.

The emotions of women and men are affected in different ways by individual or social circumstances. This means that levels of happiness are contingent on gender. The previous studies prove that dimensions of measuring "happiness" and "life satisfaction" include handling objective and subjective criteria of well-being, and there are high differences between happiness levels of women and men (Ngamaba *et al.*, 2023). In developing countries, gender statistics indicate that women are emotionally damaged and obliged to struggle with the negative effects of economic, social, cultural, and demographic factors more than men. However, they are severely underrepresented in academia and industry. As a result, this study will focus on women. One case, in point from the last century, is that in relation to Turkish women whose happiness levels have changed apparently as a result of important advances in many areas of their lives. One of the most evident and important improvements is labour force participation with a resultant increase in their educational status, as given in Table 1.

Table 1
Labor force participation status by sex and educational level (2020 and 2021) (Turkish Statistical Institute (TÜİK) Gender Statistics, 2022)

	Labor force participation rate					
	2020			2021		
Educational level	Total	Male	Female	Total	Male	Female
Total	49.3	68.2	30.9	51.4	70.3	32.8
Illiterate	14.7	26.4	12.4	15.6	29.6	12.8
Less than high school	44.0	63.4	24.1	45.5	65.3	25.3
High school	49.5	66.4	29.9	52.0	68.9	32.5
Vocational high school	61.6	77.9	37.0	63.9	80.1	38.5
Higher education	75.0	83.3	65.6	76.5	84.6	67.6

The historical background to the development of gender equality in Türkiye can be summarized as follows (TÜİK Gender Statistics, 2013): with the proclamation of the Turkish Republic on the 29th of October 1923, legal and structural reforms have accelerated leading to an increase in women's participation and influence in the public sphere. Women acquired equal rights with men under the Education Law Association Act, which entered into force on the 3rd of March 1924. 17 women deputies entered parliament - the Grand National Assembly of Türkiye - for the first time in the 5th term elections on the 8th of February 1935. On the 8th of June 1936, the new Labor Law came into force and women's working lives were arranged with some regulations. In 1975, the first World Women's Conference was held by the United Nations in Mexico City, where the period between the years 1975-85 was declared to be the "Women Decade". Türkiye signed the United Nations Convention on the Elimination of All Forms of Discrimination against Women (CEDAW) and the agreement came into force in 1986. In 1989, the first Women's Issues Research and Application Centre was established at Istanbul University and today the number of these centres has reached 62 within universities across the country. In accordance with the commitments made by the World Women's Conference in 1996 and the participation of voluntary women's organizations, deploy commissions were created in four areas: education, health, law, and employment, with a focus on women's problems under the auspices of the Department of Women Status and Problems. In order to evaluate the results and ensure full implementation of the Beijing Declaration and Platform for Action, and focus on new actions and initiatives, Türkiye participated in the Special Session of the General Assembly from the 5th to the 9^{th} of June 2000 in New York – the conference was entitled "Women 2000: Gender Equality, Development and Peace for 21st Century" and was arranged by the United Nations. The new Turkish Penal Code, containing regulations in relation to violence against women, gender equality, and contemporary was adopted on the 26th of September 2004. The Parliamentary Investigation Commission, which was established in order to conduct research into so-called custom and honour crimes, the causes of violence perpetrated against women and children, and to determine how to address these, began its work on the 18th of October 2005. Prime Ministry Circular no. 2006/17 on *Measures to be taken to Prevent Honor and Custom-Motivated Murders and Acts of Violence against Children and Women* entered into law on 4 July 2006. The Law on the Protection of the Family, enacted with the goal of preventing violence against women entered into law in 2007, with the statute being subsequently extended. The Law on the Protection of the Family and the Prevention of Violence against Women was enacted in 2012.

Legislation such as this has a direct impact on the life satisfaction levels, feelings, and emotions of Turkish women in the context of their daily lives. Statistical assessments on the happiness levels of Turkish women would indicate that there is substantial variability in subjective happiness ratings over the period 2003-2014 (as seen in Fig. 1). However, an increasing trend in happiness is clearly detected between 2009 and 2011. It reached its highest value in 2011. Turkish women appeared to become more dissatisfied after 2007 and following 2011. There is an inevitable increase between 2014 and 2016. However, after 2017, it has been dramatically decreased up to 2020.

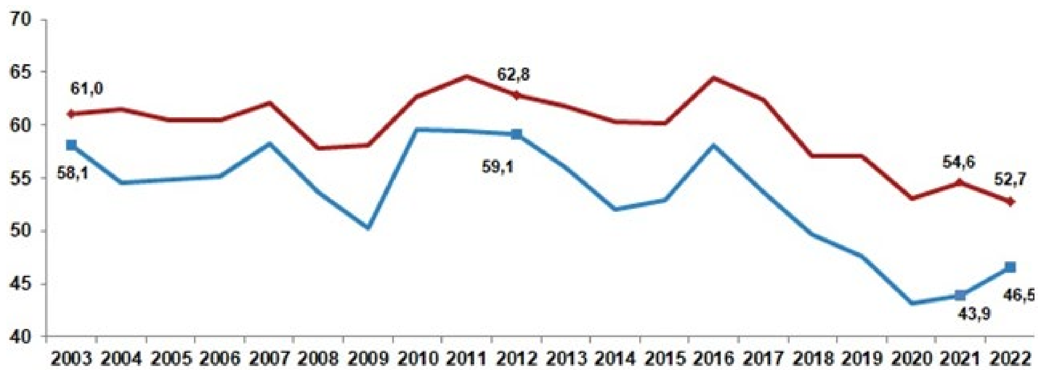

Fig. 1. Turkish citizens' happiness levels between 2003 and 2022 (red line refers to women, blue line refers to men) (TÜİK Life Satisfaction Index, 2022)

A steady trend in the happiness levels of Turkish women across time is not in evidence. As a result of the volatility associated with Türkiye's political and economic situation, time-series-based forecasting and sorting the happiness levels is difficult and forces governments and women's institutions to investigate these trend levels by utilizing effective support tools. Under these circumstances, the contributions of this study can be categorized into two main parts: first, this study clarifies the main macroeconomic, social and demographic indicators that have an impact on happiness levels, secondly it proposes a supportive tool to sort the status of happiness levels of different regions. The proposed model is flexible and generalizable, which means it can also be utilized in forecasting different countries' and different demographic groups' happiness or other SWB levels.

Within this scope, this study is structured as follows. First, SWB and happiness literature are briefly reviewed, and the main problem is stated by clarifying the motivation of the study. Then, the research method is specified with descriptions of relevant data and their sources; the sorting model is examined through the provision of detailed information about working principles and the specifications of ARASsort method. Thirdly, the proposed model is developed through a consideration of the historical data on the happiness levels of Turkish women. This study ends with a discussion of the results and further study suggestions.

2. Literature Review and Problem Statement

Many studies claim to reveal correlations between components of life satisfaction in the context of SWB. However, it is unfortunate that there is an absence of research focusing on tracking the status of happiness levels in society. Lane (2000) states that judicial offenses (such as aggression and crime), disruptions in family life, and loneliness cause depression that makes a society less happy. According to Diener and Diener (2002), and Myers (2000), wealth has little influence on happiness under the supply of basic requirements. Headey and Wearing (1989) and Suh *et al*. (1996) point out that good and bad life events have a transitory impact on SWB. In addition, a number of studies have conducted analyses to highlight how people's age impacts their level of happiness (Diener & Suh, 1997; Charles *et al*., 2001; Mroczek, 2001). Besides life satisfaction components, Diener *et al*. (1999) also criticize some demographic components (such as marital status, age etc.), which have an impact on happiness. Oishi *et al*. (2003) examined judgments of life satisfaction by conducting five different studies. One of their main findings indicates that if life satisfaction is stable, it does not always equate to stability of life-satisfaction judgments. Furthermore, they note that life satisfaction cannot be associated with all positive emotions. Kahneman *et al*. (2009) consider the well-being levels of women in two cities by utilizing the day-reconstruction method (DRM), which enables the usage of on-time data. It is stated that although the structure of well-being can be the same, the content differs in different regions. In Sheldon *et al*. (2010), a sample, which included university-based community members, is considered and a longitudinal experiment was conducted in order to observe the change in their happiness level.

One of the main findings notes that "well-being is characterized by a set-range rather than a set-point, and that continued appropriate and successful activity can keep people in the upper end of their set-range". Extremera and Fernandez-Berrocal (2014) created a metric to determine the happiness level and validate the reliability of the method for different women groups. Alquwez *et al.* (2021) focus on Saudi women's subjective happiness and they offer validation research that was made surveying 300 Saudi working women. In the study, the factors that are affecting happiness are also figured as employment position, working hours, and monthly salary. In the study of Shin and Kim (2021), a methodology is proposed to assess the happiness of middle-aged women in Korea. In this methodology, happiness is handled under four factors which are self-value, positive thinking, self-management, and family relationship. Ehrlich (2022) utilized a multiple-intervention method titled Goal-Setting Training to measure happiness. In the study, four reasons are assumed as the pursuit of goals out of pleasure, altruism, fear of self-esteem loss, or necessity.

It is evident that most of the studies in the *happiness* literature (Brickman & Campbell, 1971; Lykken & Tellegen, 1996; Lucas, 2007; Lyubomirsky *et al.*, 2005; Alquwez *et al.*, 2021; Shin and Kim, 2021) propose methodologies to address the reasons for and the origins of the change in happiness by having regard to historical events or observable experiments.

Notably, none of these researchers has tried to develop a forecasting system to determine the future status of the level of happiness. Under these circumstances, the motivation and scope of this study are determined by the following two principal issues:

1. Several governments from all around the world conduct life satisfaction analyses and note happiness levels periodically. The content and components can vary according to the specific social and political status. Some governments employ more comprehensive analyses that include an evaluation of each state and/or city, as in some regions, the factors affecting life satisfaction levels can differ as a result of different demographic, economic, political, social, and psychological factors. Comprehensive assessments of this kind cannot be easily undertaken by governments, because they consume too much time given the number of variables to be considered. Additional complications and issues to be addressed include the need for more specific samples, including the need for a greater number of responders, the lack of motivation on the part of responders, and the difficulty of the evaluation process because of missing and incomplete data. It also results in insufficiency in developing periodical assessments. It thus becomes increasingly difficult to evaluate life satisfaction components with respect to time, and other entities (such as governmental-based foundations, municipalities, ministries etc.) that require on-time results in order to deal with unsatisfactory segments and take necessary actions against inconvenient status, become late to have information on status of life satisfaction components. For instance, in Türkiye, only one life satisfaction analysis has been conducted since 2003 and there is only one comprehensive statistical assessment (conducted in 2013) that includes separate assessments of each city. It is largely futile to hope that the government will conduct a comprehensive analysis of the life satisfaction levels of women on a city-by-city basis; a further problem also arises in that it would be some time before the results of any such surveys were published. Under such circumstances, the need for a supportive decision-making tool to help forecast trends in women's lives has inevitably become more pronounced.
2. The Ministry of Family and Social Services of Turkish Republic plans to increase the number of women's guesthouses from 149 to 159 in 2023, 164 in 2024, 169 in 2025, and 174 in 2026. In most cities, women's committees and centres work to protect women's rights (The Ministry of Family and Social Services, 2022). In most cities, women's committees and centres work to protect women's rights. Although they cannot fully meet the demand for their services or address all the issues, they currently exist. Türkiye tries to guarantee equality between women and men in its constitution and within the context of its institutions. The main aims of these institutions are not only to strengthen women's status in society and provide them with an appropriate quality of life but also to observe and gauge the status quo and take necessary action as and when required. Therefore, they need to identify changes in women's life satisfaction levels and have some foresight into women's requirements at an early stage. By increasing the importance attached to women's rights and increasing the number of women's institutions, especially in developing countries, where the political and economic climate may be unstable, it is essential to constantly track the status of women to address emerging issues or take preventative measures where possible.

3. Methodology

3.1. Assumptions

Our methodology begins with a determination of the main assumptions of the model. There are two main assumptions particular to this study:

1. As Bolonkin (2012) contends, happiness is affected by many components: social interactions, extraversion, marital status, employment status, health, freedom, democracy, optimism, physical exercise, eating habits, religious status, income, and closeness to other happy people. Some of these components and happiness levels are measured under life-satisfaction analyses conducted by governments. For instance, Fig. 2 addresses the components of a regular life-satisfaction analysis in Türkiye which was first handled by TÜİK in the year of 2013 (and it has not been repeated for city-based research until now). However, it should be noted that there is a degree of interaction between components measured under life-satisfaction analyses. For instance, some life-satisfaction components such as health systems, municipality services, income, etc. have a direct impact on levels of happiness and are correlated with them. Besides that, as mentioned before,

happiness and life satisfaction have been used synonymously in the literature. In this respect, our first assumption is that happiness levels can be forecasted by considering the rest of the life-satisfaction components.

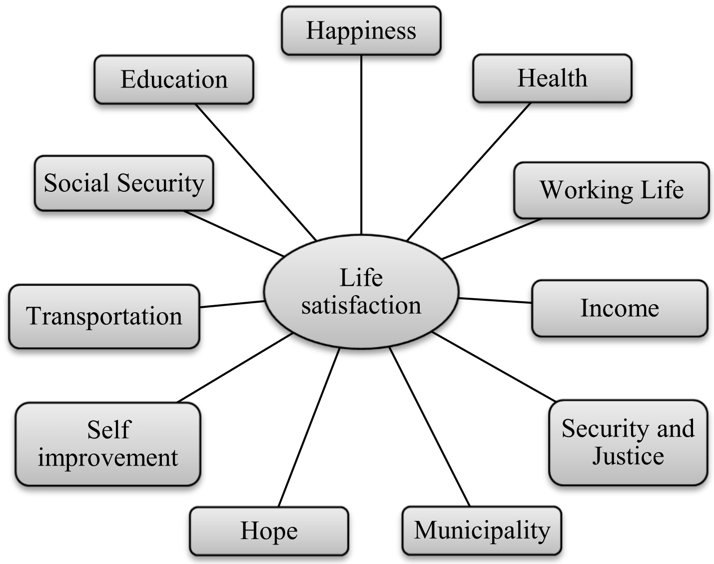

Fig. 2. The components of the regular life-satisfaction analysis (TÜİK, 2013)

2. However, the nature of the responses, which refer to the satisfaction level of responders with respect to these components includes changefulness, because life satisfaction analyses include questions such as “Are you satisfied with public services?”, “Are you satisfied with health services?”, “Are you satisfied with transportation services?”, etc., which are a measurement more of perceptions than real situations. Responses are inevitably subjective, and they can differ according to the psychological status of responders during the questionnaire. Within the scope of this study, an attempt is made to reduce the risk of changefulness through a consideration of exact values that represent these unobservable components affecting happiness. The main goal is to consider observable values that have an impact on happiness levels. These representative observable values (that can be called indicators) are obtained from macroeconomic, social, and demographic data that are periodically published by relevant government institutions. However, as stated in many studies (such as Diener *et al*. 1999; Bolonkin, 2012), happiness levels are not only affected by life satisfaction components but also by other gender-based demographic elements such as marital status, marriage age, and fertility rate. Therefore, the list of indicators affecting happiness levels is extended through a consideration of gender-based demographic features. It should also be stressed that, because there is a lack of data in some life-satisfaction components, the most relevant indicators, containing existing data are taken into consideration. Table 2 addresses all main indicators affecting happiness levels and the selected representative data for each of the main indicators.

Table 2
The Indicators of Happiness Level (TÜİK, 2013)

Main Indicator	Representative Data
Health	C1: Number of Health Institutions per 100,000 People
Working Life	C2: Number of Bully or Harassment Complaints C3: Number of Work Accidents
Income	C4: Women Labor Force Participation Rate
Security and Justice	C5: Number of Concerned Women
Municipality	C6: Existence of Women Institutions
Hope	C7: Number of Suicides in Women
Self-Improvement	C8: Library Utilization per 1000 People
Transportation	C9: Number of Traffic Accidents
Social Security	C10: Number of Insured People
Education	C11: University Educated Women Rate
Demographic	C12: Divorcement C13: Fertility Rate of Women

3.2. Multiple Attribute Decision-Making-based Sorting and ARASsort-cp

In the study, we aim to evaluate the women's happiness levels by considering the attributes listed in Table 2. For this purpose, we applied ARASsort MADM method to obtain the happiness levels of women living in the cities of Türkiye. In the end, we will be classifying the cities into some predefined classes of happiness. This chapter is dedicated to the introduction of the basics of MADM-sorting and ARASsort-cp as a version of it.

Multiple Attribute Decision-Making (MADM) can deal with sorting problems by including a new element named classes or categories in the decision model. Special assumptions of this model are listed as follows (de Lima Silva *et al.*, 2020; de Lima Silva and de Almeida Filho, 2020):

- The decision-maker defines the classes before analysis.
- There is an order between classes, that is, a class holding a greater index is worse than its follower: $C_1 \succ C_2 \succ C_3 \succ \cdots \succ C_q$. It means that C_1 collects better alternatives than C_2 can collect.
- The classes are arranged in accordance with one of the two perspectives:
    - Limits/boundaries can specify the classes. Each class is defined by an interval, i.e., C_k consists of alternatives ranging between $P_{k-1,j}$ and P_{kj}. Thus, the number of limiting profiles is equal to the "number of classes – 1" as the upper limit of C_k is the lower limit of C_{k-1}.
    - Central/characteristic profiles can specify the classes. The central profile is a reference or ideal point for the attribute. Each class is defined by a central profile, i.e., C_k consists of the alternatives showing the highest closeness to P_{kj}. So, the number of central profiles is equal to the "number of classes".
- For any consecutive profiles, P_k should be better than $P_{k+1,j}$: $P_k \succcurlyeq P_{k+1,j}$, where strict preference ($\succ$) holds for one attribute at least.

Fig. 3 and Fig. 4 demonstrate these assumptions in detail for limiting profiles and central profiles, respectively.

The main features of the current MADM-based sorting methods that can be found in the literature are as follows:

- Outranking-based methods as the most cited algorithms in the literature such as PROMSORT (Araz & Ozkarahan, 2005) and Flowsort (Nemery & Lamboray, 2008) need a decision-maker to determine an appropriate preference function for each one of the attributes, and also some thresholds.
- Distance-based algorithms as extensions of compensatory methods such as TOPSISsort (Sabokbar *et al.*, 2016; de Lima Silva and de Almeida Filho, 2020; de Lima Silva *et al.*, 2020; Ocampo *et al.*, 2021; Yamagishi and Ocampo, 2022) and VIKORsort (Demir et al., 2018; Ocampo and Yamagishi, 2021; Polat *et al.*, 2021; Sabbagh *et al.*, 2021) require a process for determining positive and negative ideal points and also calculate the distance between each alternative and these ideals.
- Pairwise comparison-based algorithms considering subjective assessments of the experts, construct a hierarchy representing the relations between elements of the decision model, and an excessive number of comparison questions needed to be answered, such as AHPsort (Ishizaka *et al.*, 2012; Toledo *et al.*, 2019), AHPsort II (Miccoli & Ishizaka, 2017; Xie *et al.*, 2019; Labella *et al.*, 2020), fuzzy AHP sorting methods (Ishizaka *et al.*, 2020; Krejci and Ishizaka, 2018; Xu *et al.*, 2019; Du *et al.*, 2022), and group AHPsort approaches (Lopez & Ishizaka, 2017; Assumma *et al.*, 2021; Labella *et al.*, 2021).

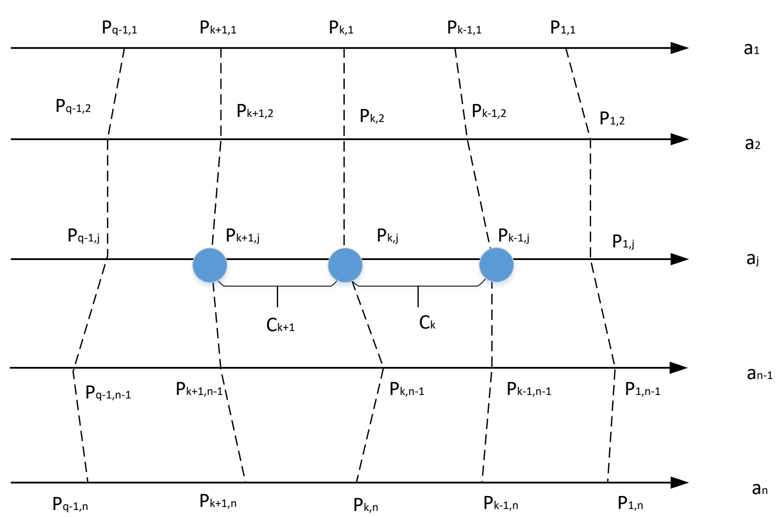

Fig. 3. Representation of limiting profiles

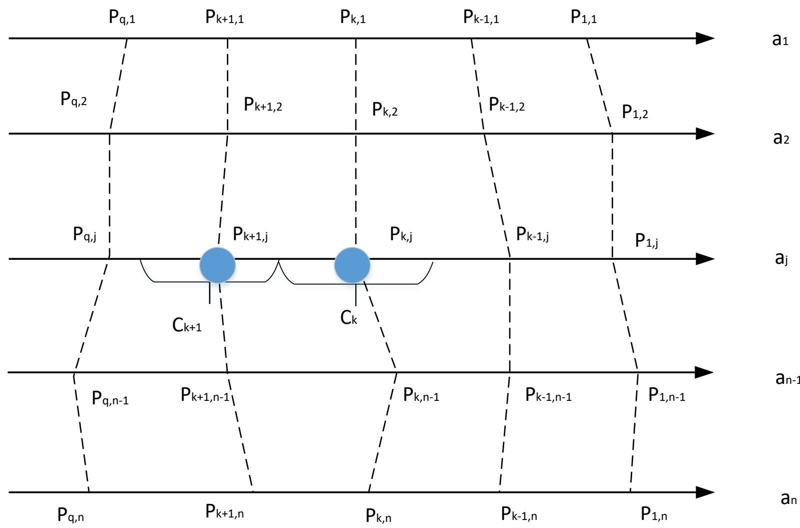

Fig. 4. Representation of central profiles

All these valuable methods possessing different levels of mathematical complexities, parameter determination processes, and several concepts such as distance measurement and pairwise comparisons, are found challenging by researchers having different business or academic background and practitioners having expertise in the real industry who knows nothing or few about MADM. As a result, they usually hesitate to use sorting-based MADM algorithms. Gül (2023) developed a more practical algorithm called ARASsort to ease the calculations of sorting-based algorithms.

Zavadskas and Turskis (2010) initiated the concept of ARAS. Its theory is not so hard to understand and implement. This advantageous feature has created a good publication potential for the studies that utilize the method. The practicality is the most important feature of the method, especially, for the researchers or practitioners who have little or no information about MADM techniques. ARAS guides decision-makers in handling complicated and challenging managerial decision problems with a broad domain: alternatives can be ordered or prioritized under a multiple-attribute and multiple-expert environment.

Conventional ARAS has 5 consecutive steps. The data of alternatives, attributes, and performance scores of alternatives are collected from proper resources in Step 1. Also, the attribute weights should be obtained by using another method as ARAS has no specific attribute weighting procedure. Additionally, the decision-maker should determine an optimal (ideal) alternative consisting of the best performance scores of the alternatives with respect to each attribute. The normalized decision matrix is structured in Step 2. ARAS normalized the matrix by using the Manhattan-type procedure. Before normalization, the cost type data must be switched to the benefit type by computing their reciprocals. Step 3 multiplies the attribute weights and the normalized performance scores to construct the weighted normalized decision matrix. Step 4 calculates the alternatives' optimality values by summing the weighted normalized performance scores of alternatives. Each optimality value is divided by the optimality value of the ideal alternative and the degree of alternative utility is generated. In Step 5, the alternatives are ranked in descending order of utility values.

It is obvious that ARAS eliminates many technical requirements such as the determination of the ideal points required by TOPSIS or VIKOR and the assignment of preference functions and threshold values needed by PROMETHEE or ELECTRE. This practicality may be the reason for being selected by many researchers having different expertise other than MADM while combining the qualitative and quantitative measures more intensively. The current literature on ARAS includes studies from energy management, supply chain management, performance assessment, maritime port management, material management, product design, IT technologies, tourism management, technology management, strategy management, credit rating, drug selection, food waste management, etc (Liu and Xu, 2021; Gül 2021a; Gül, 2021b; Gül, 2023).

To the best of our knowledge, there is only one sorting-based version of ARAS in the literature. It is called ARASsort (Gül, 2023). In this version, the decision matrix includes some additional information, and all the other steps are applied as usual. In the end, there is a new step that is dedicated to the assignment of the alternatives into classes. Gül (2023) developed two versions: while ARASsort-cp is the algorithm that can be used for the problem including class definitions based on central references, ARASsort-lp is appropriate for limiting profile-based classes. In this study, ARASsort-cp is preferred because the determination of the central profiles is easier.

Let $X = (x_1, \dots, x_m)$ be a set of *m* alternatives (cities of Türkiye in our case); $A = (a_1, \dots, a_n)$ be a set of *n* attributes (happiness factors); $W = (w_1, \dots, w_n)$ be a vector of attribute weights satisfying $\sum_j w_j = 1$ and $1 \geq w_j \geq 0$ where w_j is the weight of the attribute a_j; $C = (C_1, \dots, C_q)$ be a set of q pre-defined ordered classes: $C_1 \succ C_2 \succ C_3 \succ \cdots \succ C_q$ (five classes from the most happy to the most unhappiest). Assume A^* and A^- be the subsets of benefit and cost type attributes, respectively, and ξ_{ij} shows the performance of alternative x_i obtained with respect to the attribute a_j where $i = 1, \dots, m$ and $j = 1, \dots, n$. Let P_{kj} be the performance of the profile P_k obtained with respect to the attribute a_j where $k = 1, \dots, q$. It means P_k be the central profile representing the class C_k. The steps of ARASsort-cp are as follows:

Step 1. The decision matrix: ARASsort-cp has three sub-matrices in its aggregated decision matrix $\Im = \begin{bmatrix} \aleph \\ \sigma \\ \wp \end{bmatrix}$ where

- $\aleph = \begin{bmatrix} \xi_{11} & \cdots & \xi_{1n} \\ \vdots & \ddots & \vdots \\ \xi_{m1} & \cdots & \xi_{mn} \end{bmatrix}$ is the original data matrix consisting of alternatives in rows, attributes in columns, and performance scores in entries,
- $\sigma = [\xi_{01} \quad \xi_{02} \quad \xi_{03} \quad \dots \quad \xi_{0n}]$ is the optimal alternative's row vector,
- $\wp = \begin{bmatrix} P_{11} & \cdots & P_{1n} \\ \vdots & \ddots & \vdots \\ P_{q,1} & \cdots & P_{q,n} \end{bmatrix}$ is the matrix regarding the performance values of central profiles.

σ is the row vector comprising the optimal (best, ideal) value of each attribute. If the optimal solution cannot be easily obtained, the best value of each column of the matrix $\aleph$ can be chosen. For this purpose, Eq. (1) is applied.

$$\xi_{0j} = \begin{cases} \max_i \xi_{ij} & if\ a_j \in A^* \\ \min_i \xi_{ij} & if\ a_j \in A^- \end{cases} \quad (1)$$

Step 2. The normalized decision matrix: In order to let the alternatives and central profiles in the matrix $\Im$ be compared, the data should be nondimensionalized. Manhattan-type normalization is applied here to convert all values of $\Im$ into a decimal number ranging between 0 and 1. Eq. (2) and Eq. (3) are the operations for alternatives and central profiles. However, when the concerned attribute has a cost characteristic ($a_j \in A^-$), the included values are replaced by their reciprocals.

$$\bar{\xi}_{ij} = \xi_{ij} \Big/ \left(\sum_{i=0}^{m} \xi_{ij} + \sum_{k=1}^{q} P_{kj}\right) \quad (2)$$

$$\bar{P}_{kj} = P_{kj} \Big/ \left(\sum_{i=0}^{m} \xi_{ij} + \sum_{k=1}^{q} P_{kj}\right) \quad (3)$$

The normalized decision matrix is structured as $\bar{\Im} = \begin{bmatrix} \bar{\aleph} \\ \bar{\sigma} \\ \bar{\wp} \end{bmatrix}$ where $\bar{\aleph} = \begin{bmatrix} \bar{\xi}_{11} & \cdots & \bar{\xi}_{1n} \\ \vdots & \ddots & \vdots \\ \bar{\xi}_{m1} & \cdots & \bar{\xi}_{mn} \end{bmatrix}$, $\bar{\sigma} = [\bar{\xi}_{01} \quad \dots \quad \bar{\xi}_{0n}]$, and $\bar{\wp} = \begin{bmatrix} \bar{P}_{11} & \cdots & \bar{P}_{1n} \\ \vdots & \ddots & \vdots \\ \bar{P}_{q1} & \cdots & \bar{P}_{qn} \end{bmatrix}$.

Step 3. The weighted normalized decision matrix: When attributes have different importance for the problem at hand, we need to consider the attribute weights: w_j ($w_j \in [0,1]$ and $\sum_j w_j = 1$). There is no step serving for revealing the attribute weights in ARASsort. So, any weighting procedure may be utilized for this need. Since we don't have any weight value ready in hand or experts who can be consulted for this project, the Criteria Importance Through Intercriteria Correlation (CRITIC) method, an objective attribute weighting procedure, is selected for this purpose. CRITIC was developed by Diakoulaki *et al.* (1995) and it aims to determine the objective attribute weights in a MADM problem. CRITIC is useful for problems possessing complex and interrelated issues. It is assumed that the weights of two or more attributes should be lower when they are correlated because it means they measure similar levels of information and they do not discriminate alternatives effectively. The CRITIC approach for allocating objective weights to attributes is explained below (Diakoulaki *et al.*, 1995):

Step 3.1. A decision matrix is built utilizing information about the available alternatives: $\aleph = \begin{bmatrix} \xi_{11} & \cdots & \xi_{1n} \\ \vdots & \ddots & \vdots \\ \xi_{m1} & \cdots & \xi_{mn} \end{bmatrix}$ which is defined in Step 1.

Step 3.2. Standardization-based normalization is used as given in Eqs. (4-5). The first equation is utilized for benefit attributes while the second is performed for cost attributes.

$$x_{ij} = \frac{\xi_{ij} - \min_i \xi_{ij}}{\max_i \xi_{ij} - \min_i \xi_{ij}} \qquad i = 1,2,\dots,m\,;\; j = 1,2,\dots,n \tag{4}$$

$$x_{ij} = \frac{\max_i \xi_{ij} - \xi_{ij}}{\max_i \xi_{ij} - \min_i \xi_{ij}} \qquad i = 1,2,\dots,m\,;\; j = 1,2,\dots,n \tag{5}$$

Step 3.3. Calculate the correlation coefficients between attribute pairs via Eq. (6).

$$\rho_{ij} = \frac{\sum_{i=1}^{m}(x_{ij} - \underline{x_j})(x_{ik} - \underline{x_k})}{\sqrt{\sum_{i=1}^{m}\left(x_{ij} - \underline{x_j}\right)^2 \sum_{i=1}^{m}\left(x_{ik} - \underline{x_k}\right)^2}} \tag{6}$$

where $\underline{x_j}$ and $\underline{x_k}$ are the means of j^{th} and k^{th} attributes.

Step 3.4. The standard deviation of each attribute is computed via Eq. (7).

$$\sigma_j = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} \left(x_{ij} - \underline{x_j}\right)^2} \tag{7}$$

Step 3.5. The attribute weights are revealed by Eq. (8).

$$w_j = \frac{C_j}{\sum_{j=1}^{n} C_j} \text{ where } C_j = \sigma_j \sum_{k=1}^{n}(1 - \rho_{ik}) \tag{8}$$

The weighted normalized decision matrix is generated by operating Eq. (9).

$$\bar{\bar{\Im}} = \begin{bmatrix} \bar{\bar{\aleph}} \\ \bar{\bar{\sigma}} \\ \bar{\bar{\wp}} \end{bmatrix} \text{ where } \bar{\bar{\aleph}} = \left[\bar{\bar{\xi}}_{ij} = \bar{\xi}_{ij} * w_j\right]_{i*j},\ \bar{\bar{\sigma}} = \left[\bar{\bar{\xi}}_{0j} = \bar{\xi}_{0j} * w_j\right]_{0*j},\ \bar{\bar{\wp}} = \left[\bar{\bar{P}}_{kj} = \bar{P}_{kj} * w_j\right]_{q*j} \tag{9}$$

Step 4. Optimality value of each alternative and central profile: The row sums (Eqs. 10-11) of the matrix $\bar{\bar{\Im}}$ presents the optimality value of alternatives and central profiles (S_i and S_k where $i = 0,\dots,m$; $k = 1,\dots,q$):

$$S_i = \sum_{j=1}^{n} \bar{\bar{\xi}}_{ij} \ \ where\ i = 0,\dots,m \tag{10}$$

$$S_k = \sum_{j=1}^{n} \bar{\bar{P}}_{kj} \ \ where\ k = 1,\dots,q \tag{11}$$

Step 5. The degree of utility: Optimality values are compared to the optimality value of the ideal alternative (S_0). $K_i = {}^{S_i}/_{S_0}$ $where\ i = 1,\dots,m$ is called the alternative's degree of utility and $K_k = {}^{S_k}/_{S_0}$ $where\ k = 1,\dots,q$ is called the profile's degree of utility.

Step 6. Classification: The alternatives are classified by comparing the degree of utilities (K_i and K_k). For this purpose, ARASsort-cp checks three rules for each alternative:

- $x_i \in C_1 \ \ iff \ \ |K_i - K_{k=1}| \le |K_i - K_{k=2}|$
- $x_i \in C_k \ \ iff \ \ \begin{cases} |K_i - K_k| < |K_i - K_{k-1}| \\ |K_i - K_k| \le |K_i - K_{k+1}| \end{cases},\ \ k = 2,\dots,q-1$
- $x_i \in C_q \ \ iff \ \ |K_i - K_q| < |K_i - K_{q-1}|$

4. ARASsort-cp Implementation

The sample used in the study is drawn from the life-satisfaction analysis conducted by the Turkish Statistical Institute (TÜİK), covering an analysis of 81 cities in 2013. It is the first and only comprehensive analysis, which includes a life-satisfaction analysis of all cities in Türkiye. This database provides the results of the most detailed life-satisfaction questionnaire covering information for a wide range of subjective individual well-being outcomes, as well as demographic specifications. The sample consists of 196,203 people aged between 18 and 64. A face–to–face interview was held at 103,312 houses in order to gather data. The missing data in relation to the factors have been excluded. The dataset used in the methodology is given in Table 3.

Table 3

Decision Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
City 1	294.00	66.00	52.00	13.20	243.00	yes	34.00	187.00	4859.00	13275.00	8.17	34198.00	2.23
City 2	171.00	4.00	0.00	9.10	16.00	no	6.00	503.00	1026.00	1952.00	7.54	3310.00	2.72
City 3	278.00	14.00	3.00	5.60	67.00	no	11.00	225.00	1777.00	5205.00	8.81	7011.00	1.94
City 4	149.00	11.00	0.00	6.80	31.00	yes	13.00	314.00	685.00	547.00	5.13	914.00	3.90
City 5	183.00	16.00	3.00	5.80	36.00	no	5.00	590.00	1208.00	3175.00	7.69	5257.00	2.22
City 6	255.00	4.00	2.00	6.60	19.00	yes	3.00	298.00	950.00	3159.00	9.81	2868.00	1.76
City 7	357.00	422.00	77.00	10.20	297.00	yes	43.00	110.00	11883.00	38768.00	14.38	105039.00	1.69
City 8	234.00	77.00	53.00	7.90	302.00	yes	31.00	183.00	7078.00	25100.00	10.18	48219.00	1.84
City 9	243.00	2.00	0.00	5.80	8.00	no	2.00	577.00	153.00	385.00	12.85	361.00	2.07
City 10	210.00	7.00	1.00	7.10	6.00	no	1.00	678.00	335.00	1202.00	11.27	1149.00	1.72
City 11	267.00	24.00	25.00	6.90	149.00	yes	18.00	288.00	2831.00	12001.00	8.42	18759.00	1.76
City 12	254.00	54.00	32.00	6.00	159.00	yes	16.00	340.00	3458.00	15396.00	8.20	19143.00	1.62
City 13	223.00	13.00	3.00	6.20	19.00	yes	4.00	264.00	479.00	1306.00	13.22	2326.00	1.55
City 14	192.00	2.00	2.00	23.40	18.00	yes	7.00	285.00	655.00	605.00	5.14	1251.00	3.30
City 15	198.00	389.00	0.00	6.20	20.00	no	1.00	401.00	175.00	375.00	9.29	295.00	2.11
City 16	170.00	11.00	14.00	6.50	5.00	yes	1.00	522.00	547.00	1491.00	9.20	2125.00	1.74
City 17	248.00	10.00	0.00	7.00	4.00	no	7.00	501.00	423.00	328.00	7.39	818.00	2.50
City 18	236.00	5.00	0.00	10.60	4.00	yes	6.00	505.00	412.00	514.00	6.14	467.00	3.40
City 19	499.00	14.00	15.00	9.60	46.00	no	4.00	427.00	879.00	2907.00	11.45	3262.00	1.61
City 20	265.00	13.00	5.00	6.90	35.00	no	4.00	640.00	905.00	4898.00	11.07	3323.00	1.64
City 21	237.00	82.00	142.00	6.60	239.00	yes	22.00	111.00	5524.00	28009.00	8.77	39864.00	1.82
City 22	282.00	25.00	20.00	6.10	74.00	yes	6.00	400.00	1441.00	8610.00	12.92	6687.00	1.50
City 23	230.00	9.00	2.00	6.80	12.00	yes	1.00	1346.00	607.00	1153.00	8.63	1720.00	1.77
City 24	264.00	15.00	8.00	5.90	31.00	yes	6.00	904.00	1556.00	4700.00	7.91	5527.00	1.88
City 25	261.00	39.00	48.00	6.50	128.00	yes	17.00	506.00	2945.00	15612.00	8.07	16917.00	1.73
City 26	292.00	33.00	13.00	18.70	60.00	yes	31.00	374.00	2059.00	1347.00	4.80	5685.00	3.13
City 27	192.00	10.00	17.00	8.70	29.00	yes	2.00	165.00	1042.00	2731.00	9.56	4134.00	1.85
City 28	483.00	27.00	6.00	7.80	45.00	yes	2.00	545.00	756.00	4223.00	11.78	5069.00	1.46
City 29	512.00	14.00	3.00	7.80	49.00	no	16.00	584.00	1279.00	2029.00	8.98	4947.00	1.91
City 30	293.00	8.00	4.00	6.70	9.00	yes	5.00	604.00	652.00	1446.00	11.22	1938.00	1.87
City 31	468.00	36.00	3.00	6.60	25.00	no	6.00	492.00	1337.00	1955.00	6.15	3260.00	2.44
City 32	436.00	21.00	20.00	8.50	80.00	yes	12.00	388.00	1949.00	6070.00	11.63	14568.00	1.46
City 33	241.00	37.00	23.00	6.90	97.00	yes	16.00	234.00	3607.00	7152.00	5.12	16831.00	3.11
City 34	329.00	14.00	10.00	6.50	43.00	no	1.00	918.00	817.00	4563.00	10.41	4332.00	1.61
City 35	236.00	7.00	0.00	7.20	29.00	no	25.00	585.00	363.00	1116.00	10.82	816.00	1.80
City 36	146.00	7.00	1.00	11.70	6.00	yes	6.00	351.00	221.00	340.00	5.86	258.00	2.91
City 37	182.00	25.00	15.00	12.20	64.00	yes	8.00	170.00	3298.00	11488.00	6.27	15636.00	2.56
City 38	155.00	9.00	1.00	6.90	7.00	no	2.00	342.00	329.00	651.00	8.84	1273.00	3.10
City 39	470.00	15.00	3.00	8.70	64.00	yes	4.00	959.00	1472.00	4174.00	12.11	5070.00	1.66
City 40	234.00	437.00	417.00	11.20	572.00	yes	98.00	34.00	15224.00	133848.00	11.51	282414.00	1.78
City 41	280.00	184.00	201.00	15.40	495.00	yes	43.00	73.00	9687.00	38870.00	11.93	107609.00	1.62
City 42	203.00	15.00	13.00	11.60	93.00	yes	11.00	217.00	2187.00	5114.00	5.23	8931.00	2.73
City 43	290.00	10.00	3.00	8.00	16.00	no	13.00	363.00	476.00	1843.00	9.94	2638.00	1.56
City 44	253.00	15.00	9.00	4.20	13.00	no	5.00	602.00	790.00	2004.00	8.49	3090.00	2.06
City 45	257.00	9.00	0.00	6.60	12.00	yes	7.00	296.00	364.00	788.00	10.87	985.00	2.68
City 46	295.00	10.00	2.00	6.20	28.00	no	2.00	286.00	850.00	4184.00	9.97	3671.00	1.68
City 47	307.00	33.00	23.00	9.90	147.00	no	16.00	165.00	4146.00	9258.00	7.82	14353.00	2.15
City 48	296.00	17.00	2.00	8.00	16.00	no	1.00	717.00	898.00	1474.00	7.74	3703.00	1.61
City 49	221.00	14.00	15.00	8.00	22.00	no	6.00	407.00	763.00	3722.00	11.20	4537.00	1.43
City 50	223.00	16.00	0.00	7.30	8.00	yes	2.00	1744.00	612.00	2232.00	9.18	2845.00	1.75
City 51	151.00	6.00	0.00	7.70	6.00	no	3.00	644.00	441.00	662.00	6.61	975.00	2.92
City 52	242.00	64.00	58.00	10.10	111.00	yes	15.00	96.00	3276.00	9137.00	9.22	21608.00	1.89
City 53	322.00	53.00	13.00	4.70	150.00	yes	29.00	896.00	6450.00	14687.00	6.46	24976.00	2.15
City 54	303.00	26.00	6.00	6.00	43.00	no	7.00	306.00	1295.00	4821.00	8.90	6089.00	1.50
City 55	305.00	14.00	15.00	7.80	54.00	yes	12.00	282.00	1415.00	4604.00	8.43	6589.00	1.93
City 56	285.00	40.00	55.00	5.10	161.00	no	16.00	456.00	3961.00	12509.00	6.65	17731.00	1.88
City 57	124.00	19.00	3.00	20.60	14.00	yes	11.00	258.00	1122.00	1856.00	3.45	2174.00	3.22
City 58	197.00	46.00	15.00	12.40	159.00	yes	25.00	282.00	5394.00	12830.00	8.43	30228.00	2.09
City 59	224.00	28.00	10.00	7.30	88.00	yes	13.00	252.00	3806.00	11615.00	10.35	18632.00	1.68
City 60	183.00	5.00	0.00	10.40	5.00	no	10.00	691.00	387.00	487.00	6.53	659.00	3.51
City 61	203.00	11.00	2.00	6.00	33.00	yes	3.00	836.00	937.00	2808.00	9.74	3998.00	1.91
City 62	198.00	7.00	4.00	6.10	28.00	no	5.00	423.00	860.00	2345.00	10.89	3422.00	2.25
City 63	260.00	22.00	8.00	6.10	60.00	no	9.00	367.00	1368.00	5650.00	6.13	6171.00	1.81
City 64	189.00	2.00	1.00	14.00	50.00	no	5.00	372.00	1653.00	2938.00	8.41	4238.00	2.50
City 65	310.00	12.00	3.00	6.70	18.00	no	1.00	313.00	754.00	6241.00	10.10	2408.00	1.77
City 66	184.00	22.00	21.00	9.40	67.00	yes	10.00	181.00	2495.00	7641.00	7.12	10837.00	1.80
City 67	323.00	37.00	23.00	6.60	97.00	yes	8.00	390.00	2808.00	13621.00	7.85	14644.00	1.81
City 68	192.00	1.00	0.00	20.50	2.00	yes	4.00	536.00	473.00	243.00	5.61	553.00	3.66
City 69	249.00	6.00	1.00	6.20	24.00	no	3.00	797.00	464.00	2403.00	11.19	2158.00	1.74
City 70	392.00	24.00	5.00	10.00	36.00	yes	10.00	467.00	1350.00	4745.00	9.76	4844.00	1.91
City 71	150.00	33.00	7.00	16.30	39.00	yes	34.00	283.00	2212.00	4352.00	2.95	5647.00	4.31
City 72	144.00	13.00	0.00	20.10	10.00	yes	4.00	226.00	576.00	340.00	4.09	689.00	4.08
City 73	214.00	28.00	66.00	7.20	48.00	no	11.00	104.00	1720.00	7869.00	8.28	12117.00	1.80
City 74	319.00	14.00	8.00	6.60	42.00	no	3.00	436.00	1447.00	3425.00	8.93	4631.00	1.78
City 75	439.00	26.00	10.00	7.40	27.00	yes	14.00	301.00	1461.00	5792.00	8.68	6060.00	1.76
City 76	219.00	6.00	0.00	8.10	9.00	yes	2.00	1072.00	168.00	363.00	15.73	889.00	1.58
City 77	282.00	10.00	20.00	5.40	34.00	yes	3.00	150.00	1077.00	6315.00	9.58	5902.00	1.73
City 78	228.00	22.00	0.00	10.30	15.00	yes	18.00	112.00	1377.00	881.00	3.13	1667.00	3.61
City 79	174.00	16.00	8.00	11.00	21.00	no	2.00	94.00	606.00	2415.00	10.40	5091.00	1.67
City 80	229.00	9.00	0.00	8.60	32.00	no	2.00	301.00	1014.00	3614.00	7.36	3907.00	1.89
City 81	342.00	16.00	3.00	7.60	67.00	no	7.00	229.00	970.00	3887.00	10.76	8949.00	1.57

As seen in Table 3, there are 13 attributes in total. While five of them (C2: Number of Bully or Harassment Complaints; C3: Number of Work Accidents; C7: Number of Suicides in Women; C9: Number of Traffic Accidents; C13: Fertility Rate of Women) are cost type attributes, the other eight have benefit characteristics. ARASsort-cp algorithm is initiated with the purpose of classifying the cities of Türkiye into five happiness categories. Happiness categories are identified as follows: very happy, happy, average, unhappy, and very unhappy. Let's study the application step by step.

Table 4
Aggregated Decision Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
City 1	294.00	0.0152	0.0192	13.20	243.00	1.00	0.0294	187.00	0.0002	13275.00	8.17	34198.00	0.4484
City 2	171.00	0.2500	1.0000	9.10	16.00	0.00	0.1667	503.00	0.0010	1952.00	7.54	3310.00	0.3676
City 3	278.00	0.0714	0.3333	5.60	67.00	0.00	0.0909	225.00	0.0006	5205.00	8.81	7011.00	0.5155
City 4	149.00	0.0909	1.0000	6.80	31.00	1.00	0.0769	314.00	0.0015	547.00	5.13	914.00	0.2564
City 5	183.00	0.0625	0.3333	5.80	36.00	0.00	0.2000	590.00	0.0008	3175.00	7.69	5257.00	0.4505
City 6	255.00	0.2500	0.5000	6.60	19.00	1.00	0.3333	298.00	0.0011	3159.00	9.81	2868.00	0.5682
City 7	357.00	0.0024	0.0130	10.20	297.00	1.00	0.0233	110.00	0.0001	38768.00	14.38	105039.00	0.5917
City 8	234.00	0.0130	0.0189	7.90	302.00	1.00	0.0323	183.00	0.0001	25100.00	10.18	48219.00	0.5435
City 9	243.00	0.5000	1.0000	5.80	8.00	0.00	0.5000	577.00	0.0065	385.00	12.85	361.00	0.4831
City 10	210.00	0.1429	1.0000	7.10	6.00	0.00	1.0000	678.00	0.0030	1202.00	11.27	1149.00	0.5814
City 11	267.00	0.0417	0.0400	6.90	149.00	1.00	0.0556	288.00	0.0004	12001.00	8.42	18759.00	0.5682
City 12	254.00	0.0185	0.0313	6.00	159.00	1.00	0.0625	340.00	0.0003	15396.00	8.20	19143.00	0.6173
City 13	223.00	0.0769	0.3333	6.20	19.00	1.00	0.2500	264.00	0.0021	1306.00	13.22	2326.00	0.6452
City 14	192.00	0.5000	0.5000	23.40	18.00	1.00	0.1429	285.00	0.0015	605.00	5.14	1251.00	0.3030
City 15	198.00	0.0026	1.0000	6.20	20.00	0.00	1.0000	401.00	0.0057	375.00	9.29	295.00	0.4739
City 16	170.00	0.0909	0.0714	6.50	5.00	1.00	1.0000	522.00	0.0018	1491.00	9.20	2125.00	0.5747
City 17	248.00	0.1000	1.0000	7.00	4.00	0.00	0.1429	501.00	0.0024	328.00	7.39	818.00	0.4000
City 18	236.00	0.2000	1.0000	10.60	4.00	1.00	0.1667	505.00	0.0024	514.00	6.14	467.00	0.2941
City 19	499.00	0.0714	0.0667	9.60	46.00	0.00	0.2500	427.00	0.0011	2907.00	11.45	3262.00	0.6211
City 20	265.00	0.0769	0.2000	6.90	35.00	0.00	0.2500	640.00	0.0011	4898.00	11.07	3323.00	0.6098
City 21	237.00	0.0122	0.0070	6.60	239.00	1.00	0.0455	111.00	0.0002	28009.00	8.77	39864.00	0.5495
City 22	282.00	0.0400	0.0500	6.10	74.00	1.00	0.1667	400.00	0.0007	8610.00	12.92	6687.00	0.6667
City 23	230.00	0.1111	0.5000	6.80	12.00	1.00	1.0000	1346.00	0.0016	1153.00	8.63	1720.00	0.5650
City 24	264.00	0.0667	0.1250	5.90	31.00	1.00	0.1667	904.00	0.0006	4700.00	7.91	5527.00	0.5319
City 25	261.00	0.0256	0.0208	6.50	128.00	1.00	0.0588	506.00	0.0003	15612.00	8.07	16917.00	0.5780
City 26	292.00	0.0303	0.0769	18.70	60.00	1.00	0.0323	374.00	0.0005	1347.00	4.80	5685.00	0.3195
City 27	192.00	0.1000	0.0588	8.70	29.00	1.00	0.5000	165.00	0.0010	2731.00	9.56	4134.00	0.5405
City 28	483.00	0.0370	0.1667	7.80	45.00	1.00	0.5000	545.00	0.0013	4223.00	11.78	5069.00	0.6849
City 29	512.00	0.0714	0.3333	7.80	49.00	0.00	0.0625	584.00	0.0008	2029.00	8.98	4947.00	0.5236
City 30	293.00	0.1250	0.2500	6.70	9.00	1.00	0.2000	604.00	0.0015	1446.00	11.22	1938.00	0.5348
City 31	468.00	0.0278	0.3333	6.60	25.00	0.00	0.1667	492.00	0.0007	1955.00	6.15	3260.00	0.4098
City 32	436.00	0.0476	0.0500	8.50	80.00	1.00	0.0833	388.00	0.0005	6070.00	11.63	14568.00	0.6849
City 33	241.00	0.0270	0.0435	6.90	97.00	1.00	0.0625	234.00	0.0003	7152.00	5.12	16831.00	0.3215
City 34	329.00	0.0714	0.1000	6.50	43.00	0.00	1.0000	918.00	0.0012	4563.00	10.41	4332.00	0.6211
City 35	236.00	0.1429	1.0000	7.20	29.00	0.00	0.0400	585.00	0.0028	1116.00	10.82	816.00	0.5556
City 36	146.00	0.1429	1.0000	11.70	6.00	1.00	0.1667	351.00	0.0045	340.00	5.86	258.00	0.3436
City 37	182.00	0.0400	0.0667	12.20	64.00	1.00	0.1250	170.00	0.0003	11488.00	6.27	15636.00	0.3906
City 38	155.00	0.1111	1.0000	6.90	7.00	0.00	0.5000	342.00	0.0030	651.00	8.84	1273.00	0.3226
City 39	470.00	0.0667	0.3333	8.70	64.00	1.00	0.2500	959.00	0.0007	4174.00	12.11	5070.00	0.6024
City 40	234.00	0.0023	0.0024	11.20	572.00	1.00	0.0102	34.00	0.0001	133848.00	11.51	282414.00	0.5618
City 41	280.00	0.0054	0.0050	15.40	495.00	1.00	0.0233	73.00	0.0001	38870.00	11.93	107609.00	0.6173
City 42	203.00	0.0667	0.0769	11.60	93.00	1.00	0.0909	217.00	0.0005	5114.00	5.23	8931.00	0.3663
City 43	290.00	0.1000	0.3333	8.00	16.00	0.00	0.0769	363.00	0.0021	1843.00	9.94	2638.00	0.6410
City 44	253.00	0.0667	0.1111	4.20	13.00	0.00	0.2000	602.00	0.0013	2004.00	8.49	3090.00	0.4854
City 45	257.00	0.1111	1.0000	6.60	12.00	1.00	0.1429	296.00	0.0027	788.00	10.87	985.00	0.3731
City 46	295.00	0.1000	0.5000	6.20	28.00	0.00	0.5000	286.00	0.0012	4184.00	9.97	3671.00	0.5952
City 47	307.00	0.0303	0.0435	9.90	147.00	0.00	0.0625	165.00	0.0002	9258.00	7.82	14353.00	0.4651
City 48	296.00	0.0588	0.5000	8.00	16.00	0.00	1.0000	717.00	0.0011	1474.00	7.74	3703.00	0.6211
City 49	221.00	0.0714	0.0667	8.00	22.00	0.00	0.1667	407.00	0.0013	3722.00	11.20	4537.00	0.6993
City 50	223.00	0.0625	1.0000	7.30	8.00	1.00	0.5000	1744.00	0.0016	2232.00	9.18	2845.00	0.5714
City 51	151.00	0.1667	1.0000	7.70	6.00	0.00	0.3333	644.00	0.0023	662.00	6.61	975.00	0.3425
City 52	242.00	0.0156	0.0172	10.10	111.00	1.00	0.0667	96.00	0.0003	9137.00	9.22	21608.00	0.5291
City 53	322.00	0.0189	0.0769	4.70	150.00	1.00	0.0345	896.00	0.0002	14687.00	6.46	24976.00	0.4651
City 54	303.00	0.0385	0.1667	6.00	43.00	0.00	0.1429	306.00	0.0008	4821.00	8.90	6089.00	0.6667
City 55	305.00	0.0714	0.0667	7.80	54.00	1.00	0.0833	282.00	0.0007	4604.00	8.43	6589.00	0.5181
City 56	285.00	0.0250	0.0182	5.10	161.00	0.00	0.0625	456.00	0.0003	12509.00	6.65	17731.00	0.5319
City 57	124.00	0.0526	0.3333	20.60	14.00	1.00	0.0909	258.00	0.0009	1856.00	3.45	2174.00	0.3106
City 58	197.00	0.0217	0.0667	12.40	159.00	1.00	0.0400	282.00	0.0002	12830.00	8.43	30228.00	0.4785
City 59	224.00	0.0357	0.1000	7.30	88.00	1.00	0.0769	252.00	0.0003	11615.00	10.35	18632.00	0.5952
City 60	183.00	0.2000	1.0000	10.40	5.00	0.00	0.1000	691.00	0.0026	487.00	6.53	659.00	0.2849
City 61	203.00	0.0909	0.5000	6.00	33.00	1.00	0.3333	836.00	0.0011	2808.00	9.74	3998.00	0.5236
City 62	198.00	0.1429	0.2500	6.10	28.00	0.00	0.2000	423.00	0.0012	2345.00	10.89	3422.00	0.4444
City 63	260.00	0.0455	0.1250	6.10	60.00	0.00	0.1111	367.00	0.0007	5650.00	6.13	6171.00	0.5525
City 64	189.00	0.5000	1.0000	14.00	50.00	0.00	0.2000	372.00	0.0006	2938.00	8.41	4238.00	0.4000
City 65	310.00	0.0833	0.3333	6.70	18.00	0.00	1.0000	313.00	0.0013	6241.00	10.10	2408.00	0.5650
City 66	184.00	0.0455	0.0476	9.40	67.00	1.00	0.1000	181.00	0.0004	7641.00	7.12	10837.00	0.5556
City 67	323.00	0.0270	0.0435	6.60	97.00	1.00	0.1250	390.00	0.0004	13621.00	7.85	14644.00	0.5525
City 68	192.00	1.0000	1.0000	20.50	2.00	1.00	0.2500	536.00	0.0021	243.00	5.61	553.00	0.2732
City 69	249.00	0.1667	1.0000	6.20	24.00	0.00	0.3333	797.00	0.0022	2403.00	11.19	2158.00	0.5747
City 70	392.00	0.0417	0.2000	10.00	36.00	1.00	0.1000	467.00	0.0007	4745.00	9.76	4844.00	0.5236
City 71	150.00	0.0303	0.1429	16.30	39.00	1.00	0.0294	283.00	0.0005	4352.00	2.95	5647.00	0.2320
City 72	144.00	0.0769	1.0000	20.10	10.00	1.00	0.2500	226.00	0.0017	340.00	4.09	689.00	0.2451
City 73	214.00	0.0357	0.0152	7.20	48.00	0.00	0.0909	104.00	0.0006	7869.00	8.28	12117.00	0.5556
City 74	319.00	0.0714	0.1250	6.60	42.00	0.00	0.3333	436.00	0.0007	3425.00	8.93	4631.00	0.5618
City 75	439.00	0.0385	0.1000	7.40	27.00	1.00	0.0714	301.00	0.0007	5792.00	8.68	6060.00	0.5682
City 76	219.00	0.1667	1.0000	8.10	9.00	1.00	0.5000	1072.00	0.0060	363.00	15.73	889.00	0.6329
City 77	282.00	0.1000	0.0500	5.40	34.00	1.00	0.3333	150.00	0.0009	6315.00	9.58	5902.00	0.5780
City 78	228.00	0.0455	1.0000	10.30	15.00	1.00	0.0556	112.00	0.0007	881.00	3.13	1667.00	0.2770
City 79	174.00	0.0625	0.1250	11.00	21.00	0.00	0.5000	94.00	0.0017	2415.00	10.40	5091.00	0.5988
City 80	229.00	0.1111	1.0000	8.60	32.00	0.00	0.5000	301.00	0.0010	3614.00	7.36	3907.00	0.5291
City 81	342.00	0.0625	0.3333	7.60	67.00	0.00	0.1429	229.00	0.0010	3887.00	10.76	8949.00	0.6369
σ	512.00	1.0000	1.0000	23.40	572.00	1.00	1.0000	1744.00	0.0065	133848.00	15.73	282414.00	0.6993
P1	473.20	0.9002	0.9002	21.48	515.00	0.90	0.9010	1573.00	0.0059	120487.50	14.45	254198.40	0.6526
P2	395.60	0.7007	0.7007	17.64	401.00	0.70	0.7031	1231.00	0.0046	93766.50	11.90	197767.20	0.5591
P3	318.00	0.5011	0.5012	13.80	287.00	0.50	0.5051	889.00	0.0033	67045.50	9.34	141336.00	0.4657
P4	240.40	0.3016	0.3017	9.96	173.00	0.30	0.3071	547.00	0.0020	40324.50	6.78	84904.80	0.3722
P5	162.80	0.1021	0.1022	6.12	59.00	0.10	0.1092	205.00	0.0007	13603.50	4.23	28473.60	0.2787

ARASsort-cp needs a decision matrix including three sub-matrices: $\Im = \begin{bmatrix} \aleph \\ \sigma \\ \wp \end{bmatrix}$. $\aleph = [\xi_{ij}]$ (i=1,…,81; j=1,…,13) original decision matrix is given in Table 3.

Table 5

Normalized Decision Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
City 1	0.0127	0.0013	0.0006	0.0164	0.0322	0.0198	0.0012	0.0046	0.0016	0.0124	0.0106	0.0165	0.0102
City 2	0.0074	0.0215	0.0291	0.0113	0.0021	0.0000	0.0070	0.0122	0.0077	0.0018	0.0098	0.0016	0.0084
City 3	0.0120	0.0061	0.0097	0.0070	0.0089	0.0000	0.0038	0.0055	0.0045	0.0048	0.0114	0.0034	0.0118
City 4	0.0064	0.0078	0.0291	0.0084	0.0041	0.0198	0.0032	0.0076	0.0116	0.0005	0.0066	0.0004	0.0059
City 5	0.0079	0.0054	0.0097	0.0072	0.0048	0.0000	0.0084	0.0144	0.0066	0.0030	0.0099	0.0025	0.0103
City 6	0.0110	0.0215	0.0146	0.0082	0.0025	0.0198	0.0140	0.0073	0.0084	0.0029	0.0127	0.0014	0.0130
City 7	0.0154	0.0002	0.0004	0.0127	0.0393	0.0198	0.0010	0.0027	0.0007	0.0361	0.0186	0.0506	0.0135
City 8	0.0101	0.0011	0.0005	0.0098	0.0400	0.0198	0.0014	0.0045	0.0011	0.0234	0.0132	0.0232	0.0124
City 9	0.0105	0.0429	0.0291	0.0072	0.0011	0.0000	0.0210	0.0141	0.0519	0.0004	0.0166	0.0002	0.0110
City 10	0.0091	0.0123	0.0291	0.0088	0.0008	0.0000	0.0420	0.0165	0.0237	0.0011	0.0146	0.0006	0.0133
City 11	0.0115	0.0036	0.0012	0.0086	0.0197	0.0198	0.0023	0.0070	0.0028	0.0112	0.0109	0.0090	0.0130
City 12	0.0110	0.0016	0.0009	0.0075	0.0210	0.0198	0.0026	0.0083	0.0023	0.0143	0.0106	0.0092	0.0141
City 13	0.0096	0.0066	0.0097	0.0077	0.0025	0.0198	0.0105	0.0064	0.0166	0.0012	0.0171	0.0011	0.0147
City 14	0.0083	0.0429	0.0146	0.0291	0.0024	0.0198	0.0060	0.0069	0.0121	0.0006	0.0066	0.0006	0.0069
City 15	0.0085	0.0002	0.0291	0.0077	0.0026	0.0000	0.0420	0.0098	0.0453	0.0003	0.0120	0.0001	0.0108
City 16	0.0073	0.0078	0.0021	0.0081	0.0007	0.0198	0.0420	0.0127	0.0145	0.0014	0.0119	0.0010	0.0131
City 17	0.0107	0.0086	0.0291	0.0087	0.0005	0.0000	0.0060	0.0122	0.0188	0.0003	0.0096	0.0004	0.0091
City 18	0.0102	0.0172	0.0291	0.0132	0.0005	0.0198	0.0070	0.0123	0.0193	0.0005	0.0079	0.0002	0.0067
City 19	0.0215	0.0061	0.0019	0.0119	0.0061	0.0000	0.0105	0.0104	0.0090	0.0027	0.0148	0.0016	0.0142
City 20	0.0114	0.0066	0.0058	0.0086	0.0046	0.0000	0.0105	0.0156	0.0088	0.0046	0.0143	0.0016	0.0139
City 21	0.0102	0.0010	0.0002	0.0082	0.0316	0.0198	0.0019	0.0027	0.0014	0.0261	0.0113	0.0192	0.0125
City 22	0.0122	0.0034	0.0015	0.0076	0.0098	0.0198	0.0070	0.0097	0.0055	0.0080	0.0167	0.0032	0.0152
City 23	0.0099	0.0095	0.0146	0.0084	0.0016	0.0198	0.0420	0.0328	0.0131	0.0011	0.0112	0.0008	0.0129
City 24	0.0114	0.0057	0.0036	0.0073	0.0041	0.0198	0.0070	0.0220	0.0051	0.0044	0.0102	0.0027	0.0121
City 25	0.0113	0.0022	0.0006	0.0081	0.0169	0.0198	0.0025	0.0123	0.0027	0.0145	0.0104	0.0081	0.0132
City 26	0.0126	0.0026	0.0022	0.0232	0.0079	0.0198	0.0014	0.0091	0.0039	0.0013	0.0062	0.0027	0.0073
City 27	0.0083	0.0086	0.0017	0.0108	0.0038	0.0198	0.0210	0.0040	0.0076	0.0025	0.0124	0.0020	0.0123
City 28	0.0208	0.0032	0.0049	0.0097	0.0060	0.0198	0.0210	0.0133	0.0105	0.0039	0.0152	0.0024	0.0156
City 29	0.0221	0.0061	0.0097	0.0097	0.0065	0.0000	0.0026	0.0142	0.0062	0.0019	0.0116	0.0024	0.0120
City 30	0.0126	0.0107	0.0073	0.0083	0.0012	0.0198	0.0084	0.0147	0.0122	0.0013	0.0145	0.0009	0.0122
City 31	0.0202	0.0024	0.0097	0.0082	0.0033	0.0000	0.0070	0.0120	0.0059	0.0018	0.0080	0.0016	0.0094
City 32	0.0188	0.0041	0.0015	0.0106	0.0106	0.0198	0.0035	0.0094	0.0041	0.0057	0.0150	0.0070	0.0156
City 33	0.0104	0.0023	0.0013	0.0086	0.0128	0.0198	0.0026	0.0057	0.0022	0.0067	0.0066	0.0081	0.0073
City 34	0.0142	0.0061	0.0029	0.0081	0.0057	0.0000	0.0420	0.0224	0.0097	0.0043	0.0135	0.0021	0.0142
City 35	0.0102	0.0123	0.0291	0.0089	0.0038	0.0000	0.0017	0.0142	0.0219	0.0010	0.0140	0.0004	0.0127
City 36	0.0063	0.0123	0.0291	0.0145	0.0008	0.0198	0.0070	0.0085	0.0359	0.0003	0.0076	0.0001	0.0078
City 37	0.0079	0.0034	0.0019	0.0151	0.0085	0.0198	0.0053	0.0041	0.0024	0.0107	0.0081	0.0075	0.0089
City 38	0.0067	0.0095	0.0291	0.0086	0.0009	0.0000	0.0210	0.0083	0.0241	0.0006	0.0114	0.0006	0.0074
City 39	0.0203	0.0057	0.0097	0.0108	0.0085	0.0198	0.0105	0.0234	0.0054	0.0039	0.0157	0.0024	0.0138
City 40	0.0101	0.0002	0.0001	0.0139	0.0757	0.0198	0.0004	0.0008	0.0005	0.1247	0.0149	0.1360	0.0128
City 41	0.0121	0.0005	0.0001	0.0191	0.0655	0.0198	0.0010	0.0018	0.0008	0.0362	0.0154	0.0518	0.0141
City 42	0.0088	0.0057	0.0022	0.0144	0.0123	0.0198	0.0038	0.0053	0.0036	0.0048	0.0068	0.0043	0.0084
City 43	0.0125	0.0086	0.0097	0.0099	0.0021	0.0000	0.0032	0.0088	0.0167	0.0017	0.0129	0.0013	0.0146
City 44	0.0109	0.0057	0.0032	0.0052	0.0017	0.0000	0.0084	0.0147	0.0100	0.0019	0.0110	0.0015	0.0111
City 45	0.0111	0.0095	0.0291	0.0082	0.0016	0.0198	0.0060	0.0072	0.0218	0.0007	0.0141	0.0005	0.0085
City 46	0.0127	0.0086	0.0146	0.0077	0.0037	0.0000	0.0210	0.0070	0.0093	0.0039	0.0129	0.0018	0.0136
City 47	0.0132	0.0026	0.0013	0.0123	0.0195	0.0000	0.0026	0.0040	0.0019	0.0086	0.0101	0.0069	0.0106
City 48	0.0128	0.0051	0.0146	0.0099	0.0021	0.0000	0.0420	0.0175	0.0088	0.0014	0.0100	0.0018	0.0142
City 49	0.0095	0.0061	0.0019	0.0099	0.0029	0.0000	0.0070	0.0099	0.0104	0.0035	0.0145	0.0022	0.0160
City 50	0.0096	0.0054	0.0291	0.0091	0.0011	0.0198	0.0210	0.0425	0.0130	0.0021	0.0119	0.0014	0.0130
City 51	0.0065	0.0143	0.0291	0.0096	0.0008	0.0000	0.0140	0.0157	0.0180	0.0006	0.0085	0.0005	0.0078
City 52	0.0104	0.0013	0.0005	0.0125	0.0147	0.0198	0.0028	0.0023	0.0024	0.0085	0.0119	0.0104	0.0121
City 53	0.0139	0.0016	0.0022	0.0058	0.0199	0.0198	0.0014	0.0218	0.0012	0.0137	0.0084	0.0120	0.0106
City 54	0.0131	0.0033	0.0049	0.0075	0.0057	0.0000	0.0060	0.0075	0.0061	0.0045	0.0115	0.0029	0.0152
City 55	0.0132	0.0061	0.0019	0.0097	0.0071	0.0198	0.0035	0.0069	0.0056	0.0043	0.0109	0.0032	0.0118
City 56	0.0123	0.0021	0.0005	0.0063	0.0213	0.0000	0.0026	0.0111	0.0020	0.0117	0.0086	0.0085	0.0121
City 57	0.0054	0.0045	0.0097	0.0256	0.0019	0.0198	0.0038	0.0063	0.0071	0.0017	0.0045	0.0010	0.0071
City 58	0.0085	0.0019	0.0019	0.0154	0.0210	0.0198	0.0017	0.0069	0.0015	0.0120	0.0109	0.0146	0.0109
City 59	0.0097	0.0031	0.0029	0.0091	0.0116	0.0198	0.0032	0.0061	0.0021	0.0108	0.0134	0.0090	0.0136
City 60	0.0079	0.0172	0.0291	0.0129	0.0007	0.0000	0.0042	0.0168	0.0205	0.0005	0.0084	0.0003	0.0065
City 61	0.0088	0.0078	0.0146	0.0075	0.0044	0.0198	0.0140	0.0204	0.0085	0.0026	0.0126	0.0019	0.0120
City 62	0.0085	0.0123	0.0073	0.0076	0.0037	0.0000	0.0084	0.0103	0.0092	0.0022	0.0141	0.0016	0.0101
City 63	0.0112	0.0039	0.0036	0.0076	0.0079	0.0000	0.0047	0.0089	0.0058	0.0053	0.0079	0.0030	0.0126
City 64	0.0082	0.0429	0.0291	0.0174	0.0066	0.0000	0.0084	0.0091	0.0048	0.0027	0.0109	0.0020	0.0091
City 65	0.0134	0.0072	0.0097	0.0083	0.0024	0.0000	0.0420	0.0076	0.0105	0.0058	0.0131	0.0012	0.0129
City 66	0.0079	0.0039	0.0014	0.0117	0.0089	0.0198	0.0042	0.0044	0.0032	0.0071	0.0092	0.0052	0.0127
City 67	0.0139	0.0023	0.0013	0.0082	0.0128	0.0198	0.0053	0.0095	0.0028	0.0127	0.0102	0.0071	0.0126
City 68	0.0083	0.0859	0.0291	0.0255	0.0003	0.0198	0.0105	0.0131	0.0168	0.0002	0.0073	0.0003	0.0062
City 69	0.0107	0.0143	0.0291	0.0077	0.0032	0.0000	0.0140	0.0194	0.0171	0.0022	0.0145	0.0010	0.0131
City 70	0.0169	0.0036	0.0058	0.0124	0.0048	0.0198	0.0042	0.0114	0.0059	0.0044	0.0126	0.0023	0.0120
City 71	0.0065	0.0026	0.0042	0.0202	0.0052	0.0198	0.0012	0.0069	0.0036	0.0041	0.0038	0.0027	0.0053
City 72	0.0062	0.0066	0.0291	0.0250	0.0013	0.0198	0.0105	0.0055	0.0138	0.0003	0.0053	0.0003	0.0056
City 73	0.0092	0.0031	0.0004	0.0089	0.0064	0.0000	0.0038	0.0025	0.0046	0.0073	0.0107	0.0058	0.0127
City 74	0.0138	0.0061	0.0036	0.0082	0.0056	0.0000	0.0140	0.0106	0.0055	0.0032	0.0115	0.0022	0.0128
City 75	0.0189	0.0033	0.0029	0.0092	0.0036	0.0198	0.0030	0.0073	0.0054	0.0054	0.0112	0.0029	0.0130
City 76	0.0095	0.0143	0.0291	0.0101	0.0012	0.0198	0.0210	0.0261	0.0472	0.0003	0.0203	0.0004	0.0145
City 77	0.0122	0.0086	0.0015	0.0067	0.0045	0.0198	0.0140	0.0037	0.0074	0.0059	0.0124	0.0028	0.0132
City 78	0.0098	0.0039	0.0291	0.0128	0.0020	0.0198	0.0023	0.0027	0.0058	0.0008	0.0040	0.0008	0.0063
City 79	0.0075	0.0054	0.0036	0.0137	0.0028	0.0000	0.0210	0.0023	0.0131	0.0022	0.0135	0.0025	0.0137
City 80	0.0099	0.0095	0.0291	0.0107	0.0042	0.0000	0.0210	0.0073	0.0078	0.0034	0.0095	0.0019	0.0121
City 81	0.0148	0.0054	0.0097	0.0094	0.0089	0.0000	0.0060	0.0056	0.0082	0.0036	0.0139	0.0043	0.0145
σ	0.0221	0.0859	0.0291	0.0291	0.0757	0.0198	0.0420	0.0425	0.0519	0.1247	0.0203	0.1360	0.0160
P1	0.0204	0.0773	0.0262	0.0267	0.0682	0.0178	0.0379	0.0383	0.0467	0.1122	0.0187	0.1224	0.0149
P2	0.0171	0.0602	0.0204	0.0219	0.0531	0.0139	0.0296	0.0300	0.0365	0.0873	0.0154	0.0952	0.0128
P3	0.0137	0.0430	0.0146	0.0171	0.0380	0.0099	0.0212	0.0217	0.0262	0.0625	0.0121	0.0680	0.0106
P4	0.0104	0.0259	0.0088	0.0124	0.0229	0.0059	0.0129	0.0133	0.0159	0.0376	0.0088	0.0409	0.0085
P5	0.0070	0.0088	0.0030	0.0076	0.0078	0.0020	0.0046	0.0050	0.0057	0.0127	0.0055	0.0137	0.0064

The cost-type attributes are converted to benefit-type attributes by taking the reciprocals of the scores they include. The values listed in the column of C6 involving YES/NO scores are converted to numerical values by equalizing YES to 1 and NO to 0. $\sigma = [\xi_{0j}]$ (j=1,…,13) optimal alternative is formed by using Eq. (1) selecting the highest scores in each attribute. $\wp = [P_{k,13}]$ (k=1,…,5) representing the central profiles are obtained by implementing a rule: first, the maximum and minimum values are computed in each attribute; then, the interval is determined by taking their difference; last, the interval is divided into 5 equal parts. Each part is accepted as the central profile. Table 4 represents the aggregated decision matrix of $\mathfrak{J}$.

In Step 2, Eqs. (2-3) are performed to normalize the aggregated decision matrix $\bar{\mathfrak{J}}$ (Table 5).

Step 3 calculates the objective attribute weights via CRITIC. The method is applied to the data stored in the original decision matrix $\aleph$. Table 6 depicts the correlation matrix, standard deviation values of each attribute, and the weights in the latest column. Eqs. (4-8) are run consecutively to obtain the weights. Eq. (9) forms the weighted normalized decision matrix $\bar{\bar{\mathfrak{J}}}$ which is shown in Table 7.

In Step 4, Eqs. (10-11) calculates the optimality values of alternatives and central profiles by taking the sum of rows. Then, K_i and K_k values are calculated in Step 5. Table 7 presents S_i, S_k, K_i, and K_k values in the last two columns.

In Step 6, a classification operation is performed. According to K_i and K_k values, the algorithm assigns cities to happiness classes. Two examples are given as follow:

- $|K_{i=1} - K_{k=4}| = |0.2673 - 0.3269| = 0.0596 < |K_{i=1} - K_{k=3}| = |0.2673 - 0.5192| = 0.2519$

  $|K_{i=1} - K_{k=4}| = 0.0596 \leq |K_{i=1} - K_{k=5}| = |0.2673 - 0.1345| = 0.1327$

  Since both conditions are satisfied, $x_{i=1} \in C_{k=4}$ which means City 1 is assigned to the fourth happiness class.
- $|K_{i=2} - K_{k=q=5}| = |0.1751 - 0.1345| = 0.0406 < |K_{i=2} - K_{k=q-1=4}| = |0.1751 - 0.3269| = 0.1518$. Condition is satisfied and $x_{i=2} \in C_{k=5}$ which means City 2 is assigned to the fifth happiness category.

Table 8 presents all assignments. The total number of cities which are included in classes is as follows: 1 in the third class (City 40), 33 cities in the fourth class, and 47 cities in the fifth class representing the lowest level of women's happiness.

Table 6
Calculations of CRITIC

	ρ_{ij} *Matrix*																
	C1	*C2*	*C3*	**C4**	**C5**	**C6**	*C7*	**C8**	*C9*	**C10**	**C11**	**C12**	*C13*	$\sum_{k=1}^{n}(1-\rho_{ik})$	σ_j	C_j	w_j
C1	1.0000	-0.0482	-0.0107	-0.2591	0.1080	-0.0645	-0.0375	0.0985	-0.0886	0.0393	0.3505	0.0357	0.4656	11.4110	0.2188	2.4964	0.0797
C2	-0.0482	1.0000	0.6800	-0.0588	-0.6781	-0.1269	0.6675	0.2571	0.7347	-0.7285	-0.2282	-0.7680	-0.1277	12.4252	0.1797	2.2334	0.0713
C3	-0.0107	0.6800	1.0000	-0.0999	-0.8729	-0.2067	0.8312	0.3212	0.8260	-0.9557	-0.1839	-0.9506	-0.1734	12.7954	0.1284	1.6430	0.0525
C4	-0.2591	-0.0588	-0.0999	1.0000	0.1077	0.3086	-0.2368	-0.2604	-0.1206	0.0605	-0.4327	0.1172	-0.5850	13.4594	0.2007	2.7018	0.0863
C5	0.1080	-0.6781	-0.8729	0.1077	1.0000	0.2787	-0.8587	-0.3623	-0.9482	0.8658	0.1890	0.8808	0.2160	13.0742	0.1717	2.2447	0.0717
C6	-0.0645	-0.1269	-0.2067	0.3086	0.2787	1.0000	-0.2982	-0.0780	-0.3152	0.2212	-0.1040	0.2195	-0.1630	12.3285	0.4935	6.0843	0.1943
C7	-0.0375	0.6675	0.8312	-0.2368	-0.8587	-0.2982	1.0000	0.3565	0.8823	-0.8639	0.0063	-0.8815	0.0417	12.3912	0.1411	1.7484	0.0558
C8	0.0985	0.2571	0.3212	-0.2604	-0.3623	-0.0780	0.3565	1.0000	0.3540	-0.2887	0.1547	-0.2920	0.1365	11.6029	0.1685	1.9546	0.0624
C9	-0.0886	0.7347	0.8260	-0.1206	-0.9482	-0.3152	0.8823	0.3540	1.0000	-0.8739	-0.1316	-0.8925	-0.1517	12.7252	0.1677	2.1338	0.0682
C10	0.0393	-0.7285	-0.9557	0.0605	0.8658	0.2212	-0.8639	-0.2887	-0.8739	1.0000	0.1996	0.9855	0.1858	13.1530	0.1195	1.5718	0.0502
C11	0.3505	-0.2282	-0.1839	-0.4327	0.1890	-0.1040	0.0063	0.1547	-0.1316	0.1996	1.0000	0.2177	0.7502	11.2124	0.1976	2.2152	0.0708
C12	0.0357	-0.7680	-0.9506	0.1172	0.8808	0.2195	-0.8815	-0.2920	-0.8925	0.9855	0.2177	1.0000	0.1543	13.1739	0.1239	1.6316	0.0521
C13	0.4656	-0.1277	-0.1734	-0.5850	0.2160	-0.1630	0.0417	0.1365	-0.1517	0.1858	0.7502	0.1543	1.0000	11.2507	0.2355	2.6493	0.0846

Table 7
Weighted Normalized Decision Matrix

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**	**C11**	**C12**	**C13**	S_i and S_k	K_i and K_k
City 1	0.00101	0.00009	0.00003	0.00141	0.00231	0.00385	0.00007	0.00028	0.00011	0.00062	0.00075	0.00086	0.00087	0.0123	0.2673
City 2	0.00059	0.00153	0.00153	0.00098	0.00015	0.00000	0.00039	0.00076	0.00053	0.00009	0.00069	0.00008	0.00071	0.0080	0.1751
City 3	0.00096	0.00044	0.00051	0.00060	0.00064	0.00000	0.00021	0.00034	0.00030	0.00024	0.00081	0.00018	0.00100	0.0062	0.1356
City 4	0.00051	0.00056	0.00153	0.00073	0.00029	0.00385	0.00018	0.00048	0.00079	0.00003	0.00047	0.00002	0.00050	0.0099	0.2165
City 5	0.00063	0.00038	0.00051	0.00062	0.00034	0.00000	0.00047	0.00090	0.00045	0.00015	0.00070	0.00013	0.00087	0.0062	0.1342
City 6	0.00088	0.00153	0.00076	0.00071	0.00018	0.00385	0.00078	0.00045	0.00057	0.00015	0.00090	0.00007	0.00110	0.0119	0.2601
City 7	0.00123	0.00001	0.00002	0.00109	0.00282	0.00385	0.00005	0.00017	0.00005	0.00181	0.00132	0.00264	0.00114	0.0162	0.3531
City 8	0.00081	0.00008	0.00003	0.00085	0.00287	0.00385	0.00008	0.00028	0.00008	0.00117	0.00093	0.00121	0.00105	0.0133	0.2893
City 9	0.00084	0.00306	0.00153	0.00062	0.00008	0.00000	0.00117	0.00088	0.00353	0.00002	0.00118	0.00001	0.00093	0.0138	0.3018
City 10	0.00072	0.00088	0.00153	0.00076	0.00006	0.00000	0.00235	0.00103	0.00161	0.00006	0.00103	0.00003	0.00112	0.0112	0.2436
City 11	0.00092	0.00026	0.00006	0.00074	0.00141	0.00385	0.00013	0.00044	0.00019	0.00056	0.00077	0.00047	0.00110	0.0109	0.2376
City 12	0.00087	0.00011	0.00005	0.00064	0.00151	0.00385	0.00015	0.00052	0.00016	0.00072	0.00075	0.00048	0.00119	0.0110	0.2398
City 13	0.00077	0.00047	0.00051	0.00066	0.00018	0.00385	0.00059	0.00040	0.00113	0.00006	0.00121	0.00006	0.00125	0.0111	0.2427
City 14	0.00066	0.00306	0.00076	0.00251	0.00017	0.00385	0.00034	0.00043	0.00083	0.00003	0.00047	0.00003	0.00059	0.0137	0.2992
City 15	0.00068	0.00002	0.00153	0.00066	0.00019	0.00000	0.00235	0.00061	0.00309	0.00002	0.00085	0.00001	0.00092	0.0109	0.2380
City 16	0.00058	0.00056	0.00011	0.00070	0.00005	0.00385	0.00235	0.00079	0.00099	0.00007	0.00084	0.00005	0.00111	0.0120	0.2627
City 17	0.00085	0.00061	0.00153	0.00075	0.00004	0.00000	0.00034	0.00076	0.00128	0.00002	0.00068	0.00002	0.00077	0.0076	0.1666
City 18	0.00081	0.00123	0.00153	0.00114	0.00004	0.00385	0.00039	0.00077	0.00131	0.00002	0.00056	0.00001	0.00057	0.0122	0.2665
City 19	0.00172	0.00044	0.00010	0.00103	0.00044	0.00000	0.00059	0.00065	0.00062	0.00014	0.00105	0.00008	0.00120	0.0080	0.1753
City 20	0.00091	0.00047	0.00031	0.00074	0.00033	0.00000	0.00059	0.00097	0.00060	0.00023	0.00101	0.00008	0.00118	0.0074	0.1618
City 21	0.00082	0.00007	0.00001	0.00071	0.00227	0.00385	0.00011	0.00017	0.00010	0.00131	0.00080	0.00100	0.00106	0.0123	0.2676
City 22	0.00097	0.00025	0.00008	0.00065	0.00070	0.00385	0.00039	0.00061	0.00038	0.00040	0.00118	0.00017	0.00129	0.0109	0.2379

Table 7
Weighted Normalized Decision Matrix (Continued)

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	S_i and S_k	K_i and K_k
City 23	0.00079	0.00068	0.00076	0.00073	0.00011	0.00385	0.00235	0.00205	0.00089	0.00005	0.00079	0.00004	0.00109	0.0142	0.3094
City 24	0.00091	0.00041	0.00019	0.00063	0.00029	0.00385	0.00039	0.00137	0.00035	0.00022	0.00072	0.00014	0.00103	0.0105	0.2290
City 25	0.00090	0.00016	0.00003	0.00070	0.00121	0.00385	0.00014	0.00077	0.00018	0.00073	0.00074	0.00042	0.00112	0.0109	0.2387
City 26	0.00100	0.00019	0.00012	0.00200	0.00057	0.00385	0.00008	0.00057	0.00026	0.00006	0.00044	0.00014	0.00062	0.0099	0.2158
City 27	0.00066	0.00061	0.00009	0.00093	0.00028	0.00385	0.00117	0.00025	0.00052	0.00013	0.00087	0.00010	0.00104	0.0105	0.2292
City 28	0.00166	0.00023	0.00025	0.00084	0.00043	0.00385	0.00117	0.00083	0.00072	0.00020	0.00108	0.00013	0.00132	0.0127	0.2768
City 29	0.00176	0.00044	0.00051	0.00084	0.00047	0.00000	0.00015	0.00089	0.00042	0.00009	0.00082	0.00012	0.00101	0.0075	0.1639
City 30	0.00101	0.00077	0.00038	0.00072	0.00009	0.00385	0.00047	0.00092	0.00083	0.00007	0.00103	0.00005	0.00103	0.0112	0.2442
City 31	0.00161	0.00017	0.00051	0.00071	0.00024	0.00000	0.00039	0.00075	0.00040	0.00009	0.00056	0.00008	0.00079	0.0063	0.1375
City 32	0.00150	0.00029	0.00008	0.00091	0.00076	0.00385	0.00020	0.00059	0.00028	0.00028	0.00106	0.00037	0.00132	0.0115	0.2504
City 33	0.00083	0.00017	0.00007	0.00074	0.00092	0.00385	0.00015	0.00036	0.00015	0.00033	0.00047	0.00042	0.00062	0.0091	0.1977
City 34	0.00113	0.00044	0.00015	0.00070	0.00041	0.00000	0.00235	0.00140	0.00066	0.00021	0.00095	0.00011	0.00120	0.0097	0.2116
City 35	0.00081	0.00088	0.00153	0.00077	0.00028	0.00000	0.00009	0.00089	0.00149	0.00005	0.00099	0.00002	0.00107	0.0089	0.1934
City 36	0.00050	0.00088	0.00153	0.00125	0.00006	0.00385	0.00039	0.00053	0.00245	0.00002	0.00054	0.00001	0.00066	0.0127	0.2760
City 37	0.00063	0.00025	0.00010	0.00131	0.00061	0.00385	0.00029	0.00026	0.00016	0.00054	0.00057	0.00039	0.00075	0.0097	0.2117
City 38	0.00053	0.00068	0.00153	0.00074	0.00007	0.00000	0.00117	0.00052	0.00164	0.00003	0.00081	0.00003	0.00062	0.0084	0.1827
City 39	0.00162	0.00041	0.00051	0.00093	0.00061	0.00385	0.00059	0.00146	0.00037	0.00020	0.00111	0.00013	0.00116	0.0129	0.2819
City 40	0.00081	0.00001	0.00000	0.00120	0.00543	0.00385	0.00002	0.00005	0.00004	0.00626	0.00105	0.00709	0.00109	0.0269	0.5864
City 41	0.00096	0.00003	0.00001	0.00165	0.00470	0.00385	0.00005	0.00011	0.00006	0.00182	0.00109	0.00270	0.00119	0.0182	0.3973
City 42	0.00070	0.00041	0.00012	0.00124	0.00088	0.00385	0.00021	0.00033	0.00025	0.00024	0.00048	0.00022	0.00071	0.0096	0.2101
City 43	0.00100	0.00061	0.00051	0.00086	0.00015	0.00000	0.00018	0.00055	0.00114	0.00009	0.00091	0.00007	0.00124	0.0073	0.1591
City 44	0.00087	0.00041	0.00017	0.00045	0.00012	0.00000	0.00047	0.00092	0.00068	0.00009	0.00078	0.00008	0.00094	0.0060	0.1303
City 45	0.00088	0.00068	0.00153	0.00071	0.00011	0.00385	0.00034	0.00045	0.00149	0.00004	0.00099	0.00002	0.00072	0.0118	0.2575
City 46	0.00102	0.00061	0.00076	0.00066	0.00027	0.00000	0.00117	0.00043	0.00064	0.00020	0.00091	0.00009	0.00115	0.0079	0.1726
City 47	0.00106	0.00019	0.00007	0.00106	0.00140	0.00000	0.00015	0.00025	0.00013	0.00043	0.00072	0.00036	0.00090	0.0067	0.1461
City 48	0.00102	0.00036	0.00076	0.00086	0.00015	0.00000	0.00235	0.00109	0.00060	0.00007	0.00071	0.00009	0.00120	0.0093	0.2019
City 49	0.00076	0.00044	0.00010	0.00086	0.00021	0.00000	0.00039	0.00062	0.00071	0.00017	0.00102	0.00011	0.00135	0.0067	0.1471
City 50	0.00077	0.00038	0.00153	0.00078	0.00008	0.00385	0.00117	0.00265	0.00088	0.00010	0.00084	0.00007	0.00110	0.0142	0.3099
City 51	0.00052	0.00102	0.00153	0.00083	0.00006	0.00000	0.00078	0.00098	0.00123	0.00003	0.00060	0.00002	0.00066	0.0083	0.1801
City 52	0.00083	0.00010	0.00003	0.00108	0.00105	0.00385	0.00016	0.00015	0.00017	0.00043	0.00084	0.00054	0.00102	0.0102	0.2233
City 53	0.00111	0.00012	0.00012	0.00050	0.00142	0.00385	0.00008	0.00136	0.00008	0.00069	0.00059	0.00063	0.00090	0.0114	0.2496
City 54	0.00104	0.00024	0.00025	0.00064	0.00041	0.00000	0.00034	0.00047	0.00042	0.00023	0.00081	0.00015	0.00129	0.0063	0.1370
City 55	0.00105	0.00044	0.00010	0.00084	0.00051	0.00385	0.00020	0.00043	0.00038	0.00022	0.00077	0.00017	0.00100	0.0099	0.2168
City 56	0.00098	0.00015	0.00003	0.00055	0.00153	0.00000	0.00015	0.00069	0.00014	0.00059	0.00061	0.00044	0.00103	0.0069	0.1500
City 57	0.00043	0.00032	0.00051	0.00221	0.00013	0.00385	0.00021	0.00039	0.00048	0.00009	0.00032	0.00005	0.00060	0.0096	0.2091
City 58	0.00068	0.00013	0.00010	0.00133	0.00151	0.00385	0.00009	0.00043	0.00010	0.00060	0.00077	0.00076	0.00092	0.0113	0.2458
City 59	0.00077	0.00022	0.00015	0.00078	0.00084	0.00385	0.00018	0.00038	0.00014	0.00054	0.00095	0.00047	0.00115	0.0104	0.2272
City 60	0.00063	0.00123	0.00153	0.00111	0.00005	0.00000	0.00023	0.00105	0.00140	0.00002	0.00060	0.00002	0.00055	0.0084	0.1834
City 61	0.00070	0.00056	0.00076	0.00064	0.00031	0.00385	0.00078	0.00127	0.00058	0.00013	0.00089	0.00010	0.00101	0.0116	0.2526
City 62	0.00068	0.00088	0.00038	0.00065	0.00027	0.00000	0.00047	0.00064	0.00063	0.00011	0.00100	0.00009	0.00086	0.0067	0.1450
City 63	0.00089	0.00028	0.00019	0.00065	0.00057	0.00000	0.00026	0.00056	0.00040	0.00026	0.00056	0.00015	0.00107	0.0058	0.1275
City 64	0.00065	0.00306	0.00153	0.00150	0.00047	0.00000	0.00047	0.00057	0.00033	0.00014	0.00077	0.00011	0.00077	0.0104	0.2259
City 65	0.00107	0.00051	0.00051	0.00072	0.00017	0.00000	0.00235	0.00048	0.00072	0.00029	0.00092	0.00006	0.00109	0.0089	0.1937
City 66	0.00063	0.00028	0.00007	0.00101	0.00064	0.00385	0.00023	0.00028	0.00022	0.00036	0.00065	0.00027	0.00107	0.0096	0.2083
City 67	0.00111	0.00017	0.00007	0.00071	0.00092	0.00385	0.00029	0.00059	0.00019	0.00064	0.00072	0.00037	0.00107	0.0107	0.2330
City 68	0.00066	0.00613	0.00153	0.00220	0.00002	0.00385	0.00059	0.00081	0.00114	0.00001	0.00051	0.00001	0.00053	0.0180	0.3922
City 69	0.00086	0.00102	0.00153	0.00066	0.00023	0.00000	0.00078	0.00121	0.00117	0.00011	0.00102	0.00005	0.00111	0.0098	0.2127
City 70	0.00135	0.00026	0.00031	0.00107	0.00034	0.00385	0.00023	0.00071	0.00040	0.00022	0.00089	0.00012	0.00101	0.0108	0.2347
City 71	0.00052	0.00019	0.00022	0.00175	0.00037	0.00385	0.00007	0.00043	0.00024	0.00020	0.00027	0.00014	0.00045	0.0087	0.1895
City 72	0.00050	0.00047	0.00153	0.00215	0.00009	0.00385	0.00059	0.00034	0.00094	0.00002	0.00037	0.00002	0.00047	0.0113	0.2473
City 73	0.00074	0.00022	0.00002	0.00077	0.00046	0.00000	0.00021	0.00016	0.00031	0.00037	0.00076	0.00030	0.00107	0.0054	0.1176
City 74	0.00110	0.00044	0.00019	0.00071	0.00040	0.00000	0.00078	0.00066	0.00037	0.00016	0.00082	0.00012	0.00109	0.0068	0.1489
City 75	0.00151	0.00024	0.00015	0.00079	0.00026	0.00385	0.00017	0.00046	0.00037	0.00027	0.00079	0.00015	0.00110	0.0101	0.2203
City 76	0.00075	0.00102	0.00153	0.00087	0.00009	0.00385	0.00117	0.00163	0.00322	0.00002	0.00144	0.00002	0.00122	0.0168	0.3669
City 77	0.00097	0.00061	0.00008	0.00058	0.00032	0.00385	0.00078	0.00023	0.00050	0.00030	0.00088	0.00015	0.00112	0.0104	0.2258
City 78	0.00078	0.00028	0.00153	0.00110	0.00014	0.00385	0.00013	0.00017	0.00039	0.00004	0.00029	0.00004	0.00054	0.0093	0.2024
City 79	0.00060	0.00038	0.00019	0.00118	0.00020	0.00000	0.00117	0.00014	0.00089	0.00011	0.00095	0.00013	0.00116	0.0071	0.1550
City 80	0.00079	0.00068	0.00153	0.00092	0.00030	0.00000	0.00117	0.00046	0.00053	0.00017	0.00067	0.00010	0.00102	0.0083	0.1820
City 81	0.00118	0.00038	0.00051	0.00081	0.00064	0.00000	0.00034	0.00035	0.00056	0.00018	0.00098	0.00022	0.00123	0.0074	0.1609
σ	0.00176	0.00613	0.00153	0.00251	0.00543	0.00385	0.00235	0.00265	0.00353	0.00626	0.00144	0.00709	0.00135	0.0459	
P1	0.00163	0.00551	0.00138	0.00230	0.00489	0.00346	0.00212	0.00239	0.00318	0.00563	0.00132	0.00638	0.00126	0.0415	0.9038
P2	0.00136	0.00429	0.00107	0.00189	0.00381	0.00269	0.00165	0.00187	0.00248	0.00439	0.00109	0.00496	0.00108	0.0326	0.7115
P3	0.00109	0.00307	0.00077	0.00148	0.00272	0.00192	0.00119	0.00135	0.00178	0.00314	0.00085	0.00355	0.00090	0.0238	0.5192
P4	0.00083	0.00185	0.00046	0.00107	0.00164	0.00115	0.00072	0.00083	0.00109	0.00189	0.00062	0.00213	0.00072	0.0150	0.3269
P5	0.00056	0.00063	0.00016	0.00066	0.00056	0.00038	0.00026	0.00031	0.00039	0.00064	0.00039	0.00071	0.00054	0.0062	0.1345

Table 8
Assignment Results

City	Class	City	Class	City	Class	City	Class
City 1	P4	City 22	P4	City 43	P5	City 64	P5
City 2	P5	City 23	P4	City 44	P5	City 65	P5
City 3	P5	City 24	P5	City 45	P4	City 66	P5
City 4	P5	City 25	P4	City 46	P5	City 67	P4
City 5	P5	City 26	P5	City 47	P5	City 68	P4
City 6	P4	City 27	P5	City 48	P5	City 69	P5
City 7	P4	City 28	P4	City 49	P5	City 70	P4
City 8	P4	City 29	P5	City 50	P4	City 71	P5
City 9	P4	City 30	P4	City 51	P5	City 72	P4
City 10	P4	City 31	P5	City 52	P5	City 73	P5
City 11	P4	City 32	P4	City 53	P4	City 74	P5
City 12	P4	City 33	P5	City 54	P5	City 75	P5
City 13	P4	City 34	P5	City 55	P5	City 76	P4
City 14	P4	City 35	P5	City 56	P5	City 77	P5
City 15	P4	City 36	P4	City 57	P5	City 78	P5
City 16	P4	City 37	P5	City 58	P4	City 79	P5
City 17	P5	City 38	P5	City 59	P5	City 80	P5
City 18	P4	City 39	P4	City 60	P5	City 81	P5
City 19	P5	City 40	P3	City 61	P4		
City 20	P5	City 41	P4	City 62	P5		
City 21	P4	City 42	P5	City 63	P5		

In order to validate the results, we repeated the calculations by considering equal weighting. In this additional application, CRITIC objective weights are replaced with equal weights. Since there are 13 attributes considered, each is weighted by 1/13. Table 9 shows the assignment results. As seen, the equally weighted ARASsort application finds a worse assignment scheme: 1 city in the third class, 10 cities in the fourth class, and 70 cities in the fifth class. The results are discussed in the next chapter.

Table 9
Assignment Results of Equal Weighting based ARASsort

City	Class	City	Class	City	Class	City	Class
City 1	P5	City 22	P5	City 43	P5	City 64	P5
City 2	P5	City 23	P4	City 44	P5	City 65	P5
City 3	P5	City 24	P5	City 45	P5	City 66	P5
City 4	P5	City 25	P5	City 46	P5	City 67	P5
City 5	P5	City 26	P5	City 47	P5	City 68	P4
City 6	P5	City 27	P5	City 48	P5	City 69	P5
City 7	P4	City 28	P5	City 49	P5	City 70	P5
City 8	P4	City 29	P5	City 50	P4	City 71	P5
City 9	P4	City 30	P5	City 51	P5	City 72	P5
City 10	P4	City 31	P5	City 52	P5	City 73	P5
City 11	P5	City 32	P5	City 53	P5	City 74	P5
City 12	P5	City 33	P5	City 54	P5	City 75	P5
City 13	P5	City 34	P5	City 55	P5	City 76	P4
City 14	P5	City 35	P5	City 56	P5	City 77	P5
City 15	P4	City 36	P5	City 57	P5	City 78	P5
City 16	P5	City 37	P5	City 58	P5	City 79	P5
City 17	P5	City 38	P5	City 59	P5	City 80	P5
City 18	P5	City 39	P5	City 60	P5	City 81	P5
City 19	P5	City 40	P3	City 61	P5		
City 20	P5	City 41	P4	City 62	P5		
City 21	P5	City 42	P5	City 63	P5		

5. Discussion

This study aimed to analyze the happiness levels of women by applying a MADM-sorting approach, taking the case of Türkiye. The research instrumentalized a data set and examined the happiness levels of women living in 81 provinces of Türkiye under five basic categories based on life satisfaction components. Happiness categories are identified as follows: very happy, happy, average, unhappy, and very unhappy. A MADM approach named ARASsort examines various factors on which life satisfaction is based, such as health, working life, income, security and justice, municipality, hope, personal development, transportation, social security, education, and demographics.

The results of the study revealed that only one city (City 40) belonged to the third class, which represented a medium level of women’s happiness. The rest of the cities were either in the fourth class (33 cities) or the fifth class (47 cities), which represented the lowest levels of women’s happiness. The overall analysis shows that women's happiness levels are extremely low in Türkiye. This indicates that the majority of women living in Türkiye are unhappy or very unhappy with their lives, and City 40 is the best performing city in terms of women's well-being.

The research also provides an analysis that allows us to draw conclusions about the various factors affecting the happiness levels of women in different cities of the selected country. In doing so, this study also allows for a detailed discussion based on these factors, main indicators, and selected representative data based on TUIK's life satisfaction data set. The data set factor and indicator have been analyzed in accordance with the key findings of the Better Life Index of OECD (2020).

If we attempt to analyze the findings deeper, it would be meaningful to consider the health factor as a significant indicator that affects women's happiness levels, as it includes women's mental and physical well-being as well as their access to quality health services. The number of health institutions per 100,000 people was taken as the representative data for this indicator. Another data that can be given in addition to the number of health institutions is that Türkiye has a life expectancy that is 2 years lower than the OECD average and the country has a lower satisfaction rate with the quality of consumed water compared to other OECD countries (OECD, 2020).

Working life is also an important indicator that has a significant impact on women's happiness levels. To be more specific, women's employment status, working conditions, and income levels affect their happiness rates. In this research, the number of bullying or harassment complaints and the number of work accidents were considered as data representing this indicator. Again, looking at the OECD Better Life Index (2020) data, it is observed that Türkiye has a lower average household income, lower employment rate, and lower education level than the OECD average in business life indicators among OECD countries.

Income level is another indicator that has an important impact on individuals' happiness levels (Weech-Maldonado *et al.*, 2017), as it determines their financial situation, purchasing power, and living standards. In this research, women's labour force participation rate was taken as representative data for this indicator. According to OECD data, the average household net adjusted disposable income in Türkiye is below the OECD average of 30,490 USD per year. In terms of employment, approximately 48% of people in the 15-64 age group in Türkiye are employed as wage earners, and this rate remains below the OECD employment average of 66%. Approximately 65% of paid employees in the country are men and 35% are women (OECD, 2020). Such income-related factors are important because they can negatively impact women's well-being by limiting their ability to meet their basic needs, have a good quality of life, and achieve higher standards of living.

Another essential indicator that has been utilized to determine women's happiness levels is security and justice. In this study, the number of concerned women was used as proxy data for this indicator. Türkiye also has lower personal safety and lower perceived quality of public services compared to the average of OECD countries (OECD, 2020). Personal security is fundamental to the well-being of individuals. According to the indicator results discussed in OECD's Better Life Index (2020) data, the rate of individuals feeling safe in Türkiye is below the OECD average. Individuals in Türkiye have a lower perception of personal safety than the OECD average when it comes to feeling safe walking alone at night. To be more precise, in Türkiye, 59% of people say that they feel safe walking alone at night, less than the OECD average of 74% (OECD, 2020).

While investigating the relationship between municipal services and individuals' declared happiness levels, it was found that access to certain municipal services such as garbage/environmental waste collection, air pollution quality, public safety, amount of green space, and health/fitness center facilities was associated with higher satisfaction and happiness levels (Çıtak and Çakır, 2022). Therefore, access to municipal services is a key indicator that affects women's happiness levels as it reflects women's access to quality public services and facilities. The existence of women's institutions was used as the data representing this indicator in the research, but it is important to keep in mind the importance of general access to the municipal services listed above when interpreting this data. Türkiye has a perceived quality of public services below the OECD average (OECD, 2020). Based on all these, it can be inferred that there are regional differences in the provision and use of public services and facilities such as education, health, transportation, and social security; access to organizations such as women's counseling centers, associations, or shelters, and the quality and accessibility of public services and facilities may affect women's well-being massively.

The other vital indicator in the analysis of women's happiness levels is the hope factor, as it reflects optimism, resilience, and coping skills. The findings of some academic studies show that studies aimed at increasing hope, such as hope therapy, are an effective intervention in increasing happiness and quality of life (Arkadani, 2018). This article used the number of suicides in women as proxy data for the hope indicator. Compared to the OECD average, Türkiye has lower life satisfaction and a lower sense of community (OECD, 2020). In other words, regional differences in happiness levels, difficulties, problems, and risks faced by women such as poverty, unemployment, violence, or isolation may negatively affect their mental health and affect their well-being.

The personal development indicator has taken its place among the indicators that affect women's happiness levels, as an important indicator that reflects women's personal and professional development as well as quality education and cultural access opportunities. Library usage per 1000 people was taken as representative data for the personal development indicator. Again, if we go through OECD data (2020), Türkiye has lower education levels and lower civic participation compared to the OECD average. The presence of libraries, museums, and theatres in cities has the potential to impact women's well-being positively or negatively by providing opportunities for learning, reading, and recreation.

The transportation indicator was included in the study as an indicator that affects women's happiness levels as an element that reflects accessibility, mobility, and comfort. In the current study, the number of traffic accidents was used as representative data for this indicator. The findings of some academic studies conducted in recent years reveal that policies aimed at improving transportation services may have a positive effect on happiness (Gim, 2020). Based on this, we can deduce that there may be regional differences in transportation levels due to more developed road, vehicle, and public transportation facilities in some cities. Accordingly, factors such as traffic accidents, congestion, air pollution, transportation safety, efficiency, and comfort may affect women's welfare levels.

The social security indicator affects women's happiness levels because it reflects their insurance, social security, and welfare status. In the existing study, the number of insured people in the policy was used as representative data for this indicator. When looking at the OECD average in terms of social security, we observe that individuals are more vulnerable to unexpected circumstances, especially in countries where social safety nets are weaker. Looking at the example of Türkiye, employees in the country are expected to face a 13% loss of earnings if they become unemployed, which is well above the OECD average of 5.1% (OECD, 2020). In other words, it is one of the highest rates in the OECD and this shows that individuals in the country have low confidence in the social security system (OECD, 2020). This data can be interpreted as social problems such as poverty, unemployment, or inequality can negatively impact income, security, and well-being. The degree of protection, insurance, and welfare received from the social security system can positively or negatively affect women's well-being.

One of the important indicators affecting women's happiness levels is demographic factors. Demographic conditions affect individuals' family structures, relationships, and fertility levels. In this study, the divorce rate and women's fertility rate were used as representative data within the scope of this indicator. Türkiye has a lower sense of community and lower life satisfaction data compared to the average of OECD countries (OECD, 2020). Demographic factors, marital status issues such as domestic violence, forced marriage (Gordon, 2018), early marriage, and family tensions can positively or negatively affect women's well-being.

This study utilized ARASsort method for evaluating the women's happiness levels in the cities of Türkiye. All cities are taken into consideration by ARASsort and classified into five groups. The most powerful aspect of the method is its success in providing comprehensive and rational inferences despite the uncertainty and inconsistency of the data. This article contributes to the relevant social sciences literature by introducing a simpler but powerful MADM-sorting method, named ARASsort, as an approach that can be used in the appropriate classification problems. Additionally, via analyzing the factors affecting women's well-being in Türkiye with a comparative approach, it has the potential to help policymakers design and implement more effective interventions to improve the quality of life of Turkish women.

6. Conclusion

In recent years, women's rights movements have been one of the most important factors in Turkish political life. A substantial body of research was conducted, and many policies have been developed in order to increase social-gender equality and strengthen the contributions of women to socio-economic development. Therefore, many women's institutions have spent considerable effort tracking the status of women's ideas and emotions and attempting to understand what actions are required in order to prevent emotional and physical harm to women. As an important life-satisfaction component, the happiness level of women directly addresses the sense of joy and pleasure that women take in their daily lives. Therefore, in this study, a supportive MADM system, which can be utilized easily by women's institutions and governmental corporations is proposed to predict women's happiness levels regionally. Organizations can reliably use the provided method as a measurement system that enables relevant foundations to determine any trends in the happiness levels of women.

It should be stated that the study has some limitations. First, as the study used a data set collected in 2013, it has limitations in reflecting the latest changes and developments in women's happiness levels in Türkiye. Additionally, the number of criteria utilized to evaluate cities is limited. Some other important data such as environmental, cultural, and political components that may directly affect women's happiness levels were not examined within the scope of the study. There is room for the forthcoming studies to address these limitations by assessing more recent and extensive data, integrating more pertinent and unbiased criteria, and applying more thorough and lucid methods to establish ratings and weights of criteria.

The directions for further research can include: (1) new life-satisfaction indicators, which can be added to the methodology, (2) a rerunning of the model by considering a new tangible happiness-level indicator, (3) the development of new sorting methods to measure and compare the performance of ARASsort relatively, and (4) conduct a new application process if the new studies are done for the following years.

References

Ardakani, Z. A. (2018). The Effectiveness of Hope Therapy on Happiness and Quality of Life of Older Women Residing in the Nursing Home. *Journal of Gerontology*, *3*(1), 36-44.

Alquwez, N., Cruz, J. P., Alotaibi, N. S., & Alshammari, F. (2021). Validity and reliability of the subjective happiness scale Arabic version among Saudi working women. *Journal of Taibah University Medical Sciences, 16*(6), 835-842.

Araz, C. & Ozkarahan, I. (2005). A multicriteria sorting procedure for financial classification problems: The case of business failure risk assessment. M. Gallagher, J.P. Hogan, F. Maire (Ed.), Intelligent Data Engineering and Automated Learning - IDEAL 2005, pp. 563-570. Berlin: Springer-Verlag.

Assumma, V., Bottero, M., Ishizaka, A., & Tasiou, M. (2021). Group Analytic Hierarchy Process Sorting II Method: An Application to Evaluate the Economic Value of a Wine Region Landscape. *Environmental Modeling & Assessment*, *26*, 355-369.

Bolonkin, A. (2012). Universe, human immortality and future human evaluation. Elsevier, London.

Brickman, P. & Campbell, D. T. (1971). Hedonic relativism and planning the good society. In M. H. Appley (Ed.), Adaptation level theory, Academic Press, New York.

Charles, S. T., Reynolds, C. A., & Gatz, M. (2001). Age-related differences and change in positive and negative affect over 23 years. *Journal of Personality and Social Psychology, 80*, 136–151.

Çıtak, F. & Çakır, C. (2022). Relationship Between Municipal Services and Happiness Level: An Empirical Study from TR90 Region in Turkey. *Dumlupınar Üniversitesi Sosyal Bilimler Dergisi, 73*, 1-14.

De Lima Silva, D.F., & Almeida Filho, A.T. (2020). Sorting with TOPSIS through boundary and characteristic profiles. *Computers & Industrial Engineering*, *141*, 106328.

De Lima Silva, D.F., Ferreira, L., & Almeida Filho, A.T. (2020). A new preference disaggregation TOPSIS approach applied to sort corporate bonds based on financial statements and expert's assessment. *Expert Systems with Applications*, *152*, 113369.

Demir, L., Akpınar, M.E., Araz, C., & Ilgın, M.A. (2018). A green supplier evaluation system based on a new multi-criteria sorting method: VIKORSORT. *Expert Systems with Applications*, *114*, 479-487.

Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The critic method. *Computers & Operations Research*, *22*(7), 763-770.

Diener, E. (1984). Subjective well-being. *Psychological Bulletin, 95*, 542–575.

Diener, E. & Suh, M. E. (1997). Subjective wellbeing and age: an international analysis. *Annual Review of Gerontology and Geriatrics, 17*, 304–324.

Diener, E., Suh, E. M., Lucas, R. E., & Smith H. L. (1999). Subjective well being: three decades of progress. *Psychological Bulletin, 125*(2), 276-302.

Diener, E. & Diener, R. B. (2002). Will money increase subjective well-being? *Social Indicators Research, 57,* 119–169.

Du, Y., Tu, Y., Li, Z., & Nie, L. (2022). Regional water resources security grading evaluation considering both visible and virtual water: a case study on Hubei province, China. *Environmental Science and Pollution Research*, *29*(17), 25824-25847.

Easterlin, R. A. (2006). Life cycle happiness and its sources intersections of psychology, economics and demography. *Journal of Economic Psychology, 27*, 463-482.

Ehrlich, C. (2022). Evaluation of the happiness through goal-setting training. *Psychological Reports,* 0(0). https://doi.org/10.1177/00332941211071007.

Extremera, N. & Fernández-Berrocal, P. (2014). The subjective happiness scale: translation and preliminary psychometric evaluation of a Spanish version. *Social Indicator Research, 119*, 473–481.

Gim, T. H. T. (2020). The relationship between overall happiness and perceived transportation services relative to other individual and environmental variables. *Growth and Change*, *51*(2), 712-733.

Gordon, A. (2018). Marriage, religion, and women's happiness. *Honors Theses*, 1701. 1

Gül, S. (2021a). Fermatean fuzzy set extensions of SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem. *Expert Systems*, *38*(8), e12769.

Gül, S. (2021b). Extending ARAS with Integration of Objective Attribute Weighting under Spherical Fuzzy Environment. *International Journal of Information Technology & Decision Making*, *20*(03), 1011-1036.

Gül, S. (2023). ARASsort: A new sorting based multiple attribute decision-making algorithm. *Journal of Multi-Criteria Decision Analysis*, *30*(3-4), 93-108.

Headey, B. & Wearing, A. (1989). Personality, life events, and subjective well-being: toward a dynamic equilibrium model. *Journal of Personality and Social Psychology, 57*, 731–739.

Ishizaka, A., Pearman, C., & Nemery, P. (2012). AHPSort: an AHP-based method for sorting problems. *International Journal of Production Research*, *50*(17), 4767-4784.

Ishizaka, A., Tasiou, M., & Martinez, L. (2020). Analytic hierarchy process-fuzzy sorting: An analytic hierarchy process–based method for fuzzy classification in sorting problems. *Journal of the Operational Research Society*, *71*(6), 928-947.

Kahneman, D., Schkade, D. A., Fischler, C., Krueger, A. B., & Krill, A. (2009). The structure of well-being in two cities: life satisfaction and experienced happiness in Columbus, Ohio; and Rennes, France. In Diener, E. Kahneman, D., & Helliwell (Ed), J. International differences in well-being.

Krejci, J. & Ishizaka, A. (2018). FAHPSort: A Fuzzy Extension of the AHPSort Method. *International Journal of Information Technology & Decision Making*, *17*(04), 1119-1145.

Labella, A., Ishizaka, A., & Martinez, L. (2021). Consensual Group-AHPSort: Applying consensus to GAHPSort in sustainable development and industrial engineering. *Computers & Industrial Engineering*, *152*, 107013.

Labella, A., Rodriguez-Cohard, J.C., Sanchez-Martines, J.D., & Martinez, L. (2020). An AHPSort II Based Analysis of the Inequality Reduction within European Union. *Mathematics*, *8*, 646.

Lane, R. E. (2000). The loss of happiness in market democracies. New Haven, CT, Yale University Press.

Liu, N. & Xu, Z. (2021). An overview of ARAS method: Theory development, application extension, and future challenge. *International Journal of Intelligent Systems*, *36*(7), 3524-3565.

Lopez, C. & Ishizaka, A. (2017). GAHPSort: A new group multi-criteria decision method for sorting a large number of the cloud-based ERP solutions. *Computers in Industry*, *92-93*, 12-24.

Lykken, D. & Tellegen, A. (1996). Happiness is a stochastic phenomenon. *Psychological Science, 7,* 186–189.

Lyubomirsky, S., Sheldon, K. M., & Schkade, D. (2005). Pursuing happiness: the architecture of sustainable change. *Review of General Psychology, 9,* 111–131.

Lucas, R. E. (2007). Long-term disability has lasting effects on subjective well-being: evidence from two nationally representative longitudinal studies*. Journal of Personality and Social Psychology, 92*, 717–730.

Miccoli, F. & Ishizaka, A. (2017). Sorting municipalities in Umbria according to the risk of wolf attacks with AHPSort II. *Ecological Indicators*, *73*, 741-755.

Mroczek, D. K. (2001). Age and emotion in adulthood. *Current Directions in Psychological Science, 10*, 87–90.

Myers, D. G. (2000). The funds, friends, and faith of happy people. *American Psychologist, 55,* 56–67.

Nemery, P. & Lamboray, C. (2008). Flowsort: a flow-based sorting method with limiting or central profiles. *TOP*, *16*(1), 90-113.

Ngamaba, K. H., Kandala, N. B., Ilenda, F. B., & Mupolo, P. K. (2022). Are men's happiness and life satisfaction linked to why men die earlier than women? A panel study from 1981 to 2020 in 102 countries. *Journal of Happiness and Health, 3*(1), 14-33.

Nunes, P. M., Proença, T., & Carozzo-Todaro, M. E. (2023). A systematic review on well-being and ill-being in working contexts: contributions of self-determination theory. *Personnel Review*. https://doi.org/10.1108/PR-11-2021-0812.

Ocampo, L., Tanaid, R.A., Tiu, A.M., Selerio Jr, E., & Yamagishi, K. (2021). Classifying the degree of exposure of customers to COVID-19 in the restaurant industry: A novel intuitionistic fuzzy set extension of the TOPSIS-Sort. *Applied Soft Computing*, *113*, 107906.

Ocampo, L. & Yamagishi, K. (2021). Multiple criteria sorting of tourist sites for perceived COVID-19 exposure: the use of VI-KORSORT. *Kybernetes*, *51*(11), 3121-3152.

OECD (2020). OECD Better Life Index. Oecdbetterlifeindex.org. https://www.oecdbetterlifeindex.org/#/11111111111

Oishi, S., Schimmack, U., & Colcomb, S. J. (2003). The contextual and systematic nature of life satisfaction judgments. *Journal of Experimental Social Psychology, 39*, 232–247.

Polat, O., Akçok, S.G., Akbay, M.A., Toplaoğlu, D., Arslan, S., & Kalayci, C.B. (2021). Classification of raw cow milk using information fusion framework. *Journal of Food Measurement and Characterization*, *15*, 5113–5130.

Sabbagh, P., Pourmohamad, R., Elveny, M., Beheshti, M., Davarpanah, A., Metwally, A.S.M., Ali, S., & Mohammed, A.S. (2021). Evaluation and Classification Risks of Implementing Blockchain in the Drug Supply Chain with a New Hybrid Sorting Method. *Sustainability*, *13*, 11466.

Sabokbar, H.F., Hosseini, A., Banaitis, A., & Banaitiene, N. (2016). A novel sorting method TOPSIS-sort: An application for Tehran environmental quality evaluation. *Ekonomika a Management*, *XIX*(2), 87–104.

Sheldon, K. M., Abad, N., Ferguson, Y., Gunz, A., Houser-Marko, L., Nichols, C. P., & Lyubomirsky, S. (2010). Persistent pursuit of need-satisfying goals leads to increased happiness: a 6-month experimental longitudinal study. *Motivation and Emotion, 34*, 39–48.

Shin, H. J. & Kim, J. (2021). Development and validation of the happiness scale for middle-aged women based on existence, relation, and growth theory. *Asian Nursing Research, 15*(2), 96-104.

Shmotkin, D. (2005). Happiness in the face of adversity: reformulating the dynamic and modular bases of subjective well-being. *Review of General Psychology, 9*(4), 291–325.

Suh, E., Diener, E., & Fujita, F. (1996). Events and subjective well-being: only recent events matter. *Journal of Personality and Social Psychology, 70*, 1091–1102.

The Ministry of Family and Social Services (2022). Aile ve Sosyal Hizmetler Bakanlığı Kadın Konukevlerini Yaygınlaştırıyor, https://www.aile.gov.tr/ksgm/haberler/aile-ve-sosyal-hizmetler-bakanligi-kadin-konukevlerini-yayginlastiri-yor/#:~:text=Aile%20ve%20Sosyal%20Hizmetler%20Ba-kanl%C4%B1%C4%9F%C4%B1%202022%2D2026%20Stratejik%20Plan%C4%B1'na,bu%20y%C4%B1l%20154'e%20y%C3%BCkseltilecek.

Toledo, R.Y., Alzahrani, A.A., & Martinez, L. (2019). A Food Recommender System Considering Nutritional Information and User Preferences. *IEEE Access*, *7*, 96695-96711.

TÜİK, Turkish Statistics Institute Gender Statistics (2022), Ankara.

TÜİK, Turkish Statistics Institute Life Satisfaction Index (2013), Ankara.

Voukelatou, V., Gabrielli, L., Miliou, I., Cresci, S., Sharma, R., Tesconi, M., & Pappalardo, L. (2021). Measuring objective and subjective well-being: dimensions and data sources. *International Journal of Data Science and Analytics, 11*, 279–309.

Weech-Maldonado, R., Miller, M. J., & Lord, J.C. (2017). The Relationships among Socio-Demographics, Perceived Health, and Happiness. *Applied Research Quality Life, 12*, 289–302.

Wilson, W. (1967). Correlates of avowed happiness. *Psychological Bulletin, 67,* 294-306.

Xie, K., Mei, Y., Gui, P., & Liu, Y. (2019). Early-warning analysis of crowd stampede in metro station commercial area based on internet of things. *Multimedia Tools and Applications*, *78*, 30141-30157.

Xu, Z., Qin, J., Liu, J., & Martinez, L. (2019). Sustainable supplier selection based on AHPSort II in interval type-2 fuzzy environment. *Information Sciences*, *483*, 273-293.

Yamagishi, K. & Ocampo, L. (2022). Utilizing TOPSIS-Sort for sorting tourist sites for perceived COVID-19 exposure. *Current Issues in Tourism*, *25*(2), 168-178.

Zavadskas, E.K. & Turskis, Z. (2010). A new additive ratio assessment (ARAS) method in multicriteria decision-making. *Technological and Economic Development of Economy*, *16*(2), 159-172.