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A new mathematical model for cellular manufacturing system with productivity consideration

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A B S T R A C T

In today's environment of escalating competition, companies are adapting their management and production strategies, and product diversity is rapidly increasing. Companies require cellular manufacturing systems to produce products with high diversity in a short amount of time, ensuring the desired quality and meeting customer expectations. Cellular manufacturing systems, which have a more flexible structure compared to traditional production systems, are a good and effective solution for managers. Cellular manufacturing is an approach that aims to produce products with varying diversity in the shortest possible time and at the lowest cost, targeting an increase in efficiency. In this study, a cell manufacturing system proposal is made and cell formation is carried out to increase efficiency and effectiveness in a company that manufactures industrial refrigeration cabinets. A productivity-based 0-1 integer mathematical programming model is prepared that facilitates the simultaneous grouping of part and machine families in cell formation. In addition to the intracellular and intercellular transportation costs found in productivity-based models in the literature, labor costs, maintenance costs, the depreciation costs of the machines used in the cells, and the waiting costs of the machines are also added to the prepared model. The model is solved with the help of the GAMS 23.5.1 software package, creating part families and machine groups. Group efficiency values are measured, and the current and proposed situations are compared.

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1. Introduction

Group Technology is a manufacturing and engineering management approach that helps manage diversity by leveraging fundamental similarities in products and activities (Selim et al., 1998). Group technology, in a general sense, is based on the principle that 'similar things should be done similarly'. The term 'things' can encompass all activities, including product design, process planning, manufacturing, assembly, and production control, as well as administrative tasks (Askin & Standridge, 1993). Cellular manufacturing is an application of the group technology philosophy used in production (Selim et al., 1998). In cellular manufacturing, machines with varying functions are organized into clusters known as cells. Each cell focuses on manufacturing a specific group of parts, termed a part family, which includes various parts sharing similar processing needs.

There are four processes involved in designing a cellular manufacturing system (CMS) (Vafaeinezhad et al., 2016; Mehdizadeh et al., 2020a).

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1. Cell Formation (CF): Assigning parts to families and machines to corresponding cells based on some characteristic, such as similar geometric design or machining requirements.
2. Group Layout (GL): Determining the location of machines and cells in the workshop by arranging the inter-cells and intra-cells.
3. Group Scheduling (GS): Scheduling parts within part families.
4. Resource Allocation (RA): Assigning required resources (such as labor and material handling devices) to manufacturing cells.

The first step in cellular manufacturing systems is cell formation. During the cell formation process, the processing requirements of part types, the demand for part types, and available resources (such as machines, equipment, etc.) are taken into consideration. Cell formation solution approaches are typically divided into three main categories:

- part families are created on a priority basis,
- machine groups are created according to part families and
- part families and machine groups are created simultaneously (Papaioannou & Wilson, 2010).

Different approaches have been applied and continue to be applied in the literature to solve the cell formation problem. In general, Papaioannou and Wilson (2010) presented the solution approaches to the cell formation problem as shown in Fig. 1.

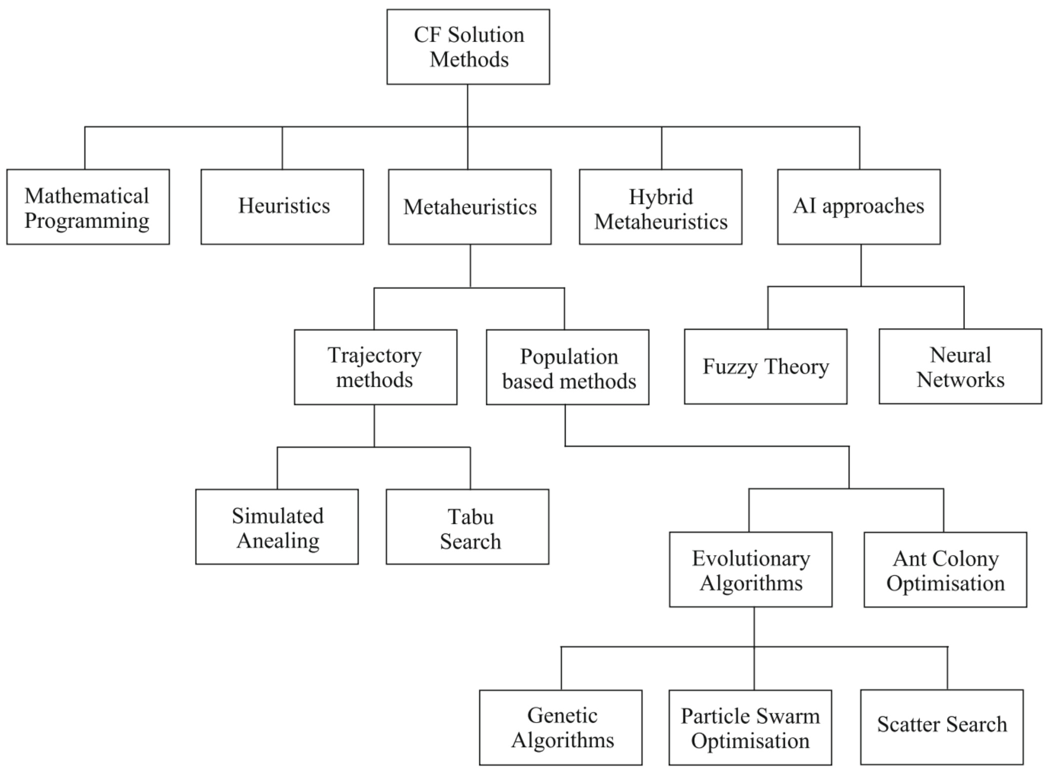

Fig. 1. Solution methods for the cell formation problem (Papaioannou & Wilson, 2010).

The study addressed the cell formation problem, which is the initial stage of the cellular manufacturing system. A new 0-1 integer mathematical model has been proposed for the cell formation problem, based on the efficiency-based mathematical model developed by Abduelmola and Toboun (2010). This proposed model is tested on a real set of problems from a firm that makes refrigeration cabinets.

The rest of the paper is structured as follows. Section 2 deals with studies in the literature that use mathematical programming to solve the cell formation problem. Section 3 introduces the addressed problem, and Section 4 proposes a mathematical model for cell formation. Section 5 introduces the real data, and then Section 6 tests the proposed model on the real data set and presents the results. Finally, Section 7 concludes and suggests future research.

2. Literature Review

Mathematical models have been extensively used for the solution of manufacturing problems. In this study, a 0-1 integer mathematical programming model is developed to solve the cell formation problem for a manufacturing system. Therefore, in this section, a literature review of mathematical programming is given.

In cell formation, the first usage of mathematical programming was made in 1974-1975 (Purcheck, 1975). In the beginning, it can solve only small size problems and group parts and machines one by one. Han and Ham (Han & Ham, 1986) developed an efficient computerized method for forming group technology part families using a goal programming based multi-objective clustering analysis with a group technology classification and coding system. It can be used only by families. Kusiak (1987) developed the P-median method with integer programming to make part families. It cannot be used for machine cells. Gunasingh and Lashkari (1989) introduced a sequential modeling approach to address the cell formation problem in cellular manufacturing systems. Initially, machines are grouped into cells according to their similarity in processing parts, followed by the allocation of parts to the appropriate machine groups based on their processing needs. The machine grouping and part allocation problems are formulated as a 0–1 integer programming model. However, the model is limited in its applicability to larger-scale problems. Askin and Chiu (1990) develop a mathematical model and solution procedure for grouping machines into cells and routing components to machines within those cells. The model incorporates costs such as inventory, machine depreciation, machine setup, and material handling into a mathematical programming formulation. To simplify the solution process, the formulation is divided into two subproblems. After that, mathematical programming models are improved to solve large-size problems and simultaneous grouping with lots of system constraints. Damodaran et al. (1992) develop a mixed integer linear model for the problem of assigning operations of part types to one or more machines in a cellular manufacturing system, considering the trade-off between refixturing and material handling movement. Examples are included to illustrate the applications of the models developed.

Dahel (1995) develops an optimal 0-1 integer programming model aimed at determining which machines and parts should be allocated to specific cells and defining the relative placement of these cells within the factory's material flow layout. The model organizes the manufacturing system into cells by minimizing intercell traffic, while also considering constraints such as machine capacity and operation sequences. Additionally, part setup and run times are taken into account to assess the capacity requirements. Abduelmola and Taboun (2000) present an efficiency-based mathematical model and then a simulated annealing algorithm. The efficiency measure is the ratio of sales to intra-cell and inter-cell transportation costs and is intended to be maximized. Important parameters, including production volume, sales price, and the maximum number of machines in each cell, are considered in the proposed algorithm when part families and machine cells are created simultaneously. Uddin and Shanker (2002) tackle a generalized grouping problem in which each part can follow multiple process routes. The challenge of assigning both machines and parts to cells is formulated as an integer programming problem. The goal of minimizing intercell movements is accomplished by reducing the number of visits a process route needs to make to different cells for processing the associated part. A genetic algorithm-based approach is proposed as the solution methodology. Defersha and Chen (2006) developed a comprehensive integer mathematical model for the design of cellular manufacturing systems (CMS). Workload balancing, machine adjacency requirements, lot splitting, sequence of operations, dynamic cell configuration, and alternative routings are all considered simultaneously during the model's development. Mahdavi et al. (2007) present a new mathematical model for cell formation in cellular manufacturing systems (CMS), focusing on the concept of cell utilization. The model aims to enhance cell performance by minimizing exceptional elements (EE) and reducing voids within cells. Several benchmark problems from the literature are used to demonstrate the effectiveness of the proposed model in forming part–machine groupings, showing improved results compared to previous models. Eğilmez et al. (2012) develop a non-linear mathematical model to address the design problem of a stochastic cellular manufacturing system (CMS). The model considers both machine and labor-intensive cells, with probabilistic operation times and uncertain customer demand. It is assumed that processing times and customer demand follow a normal distribution. The objective is to design a CMS with product families that consist of the most similar products, while minimizing the number of cells and machines for a given risk level. Various experiments are conducted to analyze the effect of risk level on CMS design. As the risk level increases, fewer cells and product families are formed, and average cell utilization improves. Forghani et al. (2013) presented a new robust approach to address demand uncertainty in cellular manufacturing systems. Instead of using predefined scenarios, they employed a more realistic and practical interval approach to handle the uncertainty in part demands, aiming to minimize the total material handling cost. The model identifies machine cells and optimizes the layout configurations to be controllable based on the level of robustness. Bagheri and Bashiri (2014) introduce a new mathematical model to simultaneously address cell formation, operator assignment, and inter-cell layout problems. The model aims to minimize inter- and intra-cell part transportation, machine relocation costs, and operator-related factors. To validate the model, several randomly generated numerical examples were solved using the branch and bound technique.

Mohammadi and Forghani (2014) proposed a new approach for the design of cellular manufacturing systems (CMS). In order to extend the applicability to real-world scenarios, various design factors such as alternative processing routes, aisle distances, and machine dimensions were considered. Material handling costs were evaluated more accurately by accounting for the positions of machines within cells. Additionally, a subcontracting approach was proposed, considering production volume, material handling, and outsourcing costs, along with demand and machine capacity constraints. A genetic algorithm with a special chromosome representation was developed to solve the problem efficiently. Erenay et al. (2015) propose a mathematical programming approach to design a layered cellular manufacturing system in a highly fluctuating demand environment. The model aims to minimize the number of cells by creating shared and remainder cells. Unlike traditional cellular manufacturing systems, in layered systems, certain cells can serve multiple part families. A five-step hierarchical methodology is employed, and the results indicate that designs with a greater number of part families tend to require fewer machines. Sakhaii et al. (2016) propose a novel integrated mathematical model to address the dynamic cellular manufacturing system (DCMS) for solving production planning problems. The model incorporates real-life manufacturing challenges such as alternative processing routes, inter-cell layouts, and system reconfiguration, while also considering uncertainty in part processing times. A robust optimization approach is employed to solve the problem efficiently. Aalaei and Davoudpour (2017) introduce a new mathematical model for a cellular manufacturing system within supply chain design that incorporates labor assignment. This model comprehensively addresses key manufacturing characteristics, including multiple plant locations, multi-market allocations with production planning, and diverse part mixes. Its objective is to minimize the total costs associated with inventory holding, inter-cell material handling, external transportation, fixed production costs for each part at each plant, as well as machine and labor wages. Aljuneidi and Buldak (2017) address a classical cell formation problem in Cellular Manufacturing Systems, bridged it with a production planning problem in Hybrid Manufacturing-Remanufacturing Systems, and touched on the reconfiguration of cellular manufacturing for different production periods. For this purpose, they developed a mixed-integer linear programming (MILP) model. Kheirkhah and Ghajari (2018) explore the development of a new mixed integer non-linear mathematical programming model. This model integrates various design characteristics and is intended for use in the dynamic design of cellular manufacturing systems (CMS). These characteristics encompass material transfer between machines, the presence of duplicate machines, lot splitting, alternative routing options, system reconfiguration, limitations on cell size, and multi-period production planning. The main objective of this model is to optimize the configuration of cellular systems, which may not maintain their optimality across different periods due to fluctuations in product mix and demand. Golmohammadi et al. (2019) propose nonlinear mixed integer programming considering machine unreliability as well as intra- and inter-cell movements of parts and machines. Genetic Algorithm (GA) and two more modern nature-inspired algorithms, Keshtel Algorithm (KA) and Red Deer Algorithm (RDA), are used to solve the given problem. Mehdizadeh et al. (2020b) explore an integrated integer nonlinear programming model that was developed in which cell formation and production planning problems are addressed simultaneously for a dynamic cellular manufacturing system (DCMS) with limited resources. This model aims to minimize the total costs of production planning, cell construction, and formation (including cell preparation and setup costs) under a dynamic system. Depreciation, purchase, rental, and installation costs are used as fixed expenses for the machine. Two meta-heuristic algorithms, the genetic algorithm (GA) and particle swarm optimization (PSO) algorithms, are used to solve large-scale problems. Ranjbar et al. (2022) present a mathematical formulation that captures the relationships between the different design elements. The main objective of the research is to provide a comprehensive mathematical framework that allows for the efficient design of cellular production systems, emphasizing the simultaneous consideration of cell formation, cellular layout, and material handling. To address this problem, a branch-and-bound algorithm is also given. Zhang et al. (2023) address the optimization of cellular manufacturing systems in dynamic market conditions characterized by multi-product and small-batch production. A two-phase dynamic virtual cell formation (DCF) model is developed to solve the problem. The first stage is distributing the workload evenly throughout the system and optimizing processing similarity between parts. Reconfiguration stability is the goal of the second phase, which minimizes the number of changes required to adjust to new demands. To solve the DVCF model, a hybrid metaheuristic algorithm combining Lévy-NSGA-II and a discrete Lévy flight search strategy is proposed.

In Table 1, studies about cellular manufacturing systems that use a mathematical programming approach are given. Some studies in the literature are summarized directly in Table 1 (Alimian et al., 2020; Amirahmadi & Choobineh, 1996; Boctor, 1991, 1996; Brown, 2015; Buruk Sahin & Alpay, 2019; Choobineh, 1988; Gupta & Seifoddini, 1990; H., 1989; Jain et al., 1990; Kılıç, 2008; Logendran, 1993; Rajamani et al., 1990; Shiyas & Pillai, 2014; Taboun et al., 1998; Tariq et al., 2009). They are categorized into some modeling features such as solution methodology, grouping, transportation cost, labor cost, maintenance cost, demand, etc.

Table 1
Classification of studies from literature in terms of model features and different factors

Reference / Year	Objective		Solution		Grouping			Transportation Cost		Maintenance Cost	Labor Cost	Waiting cost	Depreciation cost	Demand/ Production
	Min	Max	Exact	Heuristic	Parts	Machines	Simultaneous	Intracellular	Intercellular					
Purcheck, 1974-1975	✓		✓				✓							
Han and Ham, 1986	✓		✓		✓									
Kusiak, 1987	✓			✓	✓									
Choobineh,1988	✓		✓	✓						✓				✓
Seifoddini, 1989	✓		✓				✓		✓					✓
Gunasingh and Lashkari, 1989	✓		✓		✓	✓								
Gupta and Seifoddini, 1990	✓			✓	✓									✓
Rajamani et al.,1990	✓		✓				✓							✓
Askin and Chiu, 1990	✓		✓		✓	✓			✓				✓	
Jain et al., 1990	✓		✓				✓							✓
Boctor, 1991	✓		✓				✓		✓					✓
Damodaran et al., 1992	✓		✓				✓	✓	✓					
Logendran,1993		✓	✓				✓	✓						✓
Dahel, 1995	✓		✓				✓		✓					✓
Amirahmadi and Choobineh, 1996	✓		✓	✓			✓		✓					
Boctor, 1996	✓		✓				✓		✓				✓	
Heragu and Chen, 1997	✓		✓				✓							
Taboun et al., 1998	✓		✓				✓		✓					✓
Abduelmola and Taboun, 2000		✓	✓	✓			✓	✓	✓					✓
Uddin and Shanker, 2002	✓		✓	✓			✓		✓					
Defersha and Chen, 2006	✓		✓				✓		✓	✓				✓
Mahdavi et al., 2007	✓		✓				✓							
Kılıç, 2008	✓		✓				✓		✓	✓	✓			✓
Tariq et al., 2009	✓		✓				✓		✓					✓
Eğilmez et al., 2012	✓		✓				✓							✓
Forghani et.al., 2013	✓		✓				✓	✓	✓					✓
Shiyas and Pillai, 2014	✓		✓	✓			✓		✓			✓		
Bagheri and Bashiri, 2014	✓		✓			✓		✓	✓		✓			
Mohammadi and Forghari, 2014	✓		✓				✓							✓
Brown, 2015	✓		✓				✓		✓					✓
Erenay et al., 2015	✓		✓				✓	✓						✓
Sakhaii et al., 2016	✓		✓				✓	✓	✓					
Aalaei and Davoudpour, 2017	✓		✓				✓	✓	✓					✓
Aljuneidi and Buldak, 2017	✓		✓				✓			✓				✓
Kheirkhah and Ghajari, 2018	✓		✓	✓			✓	✓		✓			✓	✓
Golmohammadi et al.,2019	✓		✓	✓			✓	✓	✓					✓
Buruk Sahin and Alpay, 2019	✓		✓				✓							
Mehdizadeh et al., 2020	✓		✓	✓			✓		✓				✓	✓
Alimian et al., 2020	✓		✓				✓		✓	✓				✓
Ranjbar et al., 2022	✓		✓				✓	✓						
Zhang et al., 2023	✓		✓	✓			✓							✓
This study, 2024		✓	✓				✓	✓	✓	✓	✓	✓	✓	✓

When Table 1 is reviewed, it is observed that the objective function is minimized in most of the studies. As for the solution methodology, a heuristic method has been developed alongside the mathematical model in a few studies. Transportation costs and demand/production volumes have been considered in most studies. On the other hand, maintenance costs, labor costs, depreciation costs, and waiting costs have been considered in some studies. Lastly, in almost all of the studies, parts and machines are considered simultaneously while forming cell groups. Papaioannou and Wilson (Papaioannou & Wilson, 2010) highlight that for cell formation methodologies to be applicable in an industrial setting, production cells that consider workers and tools in addition to parts and machines must be created. Table 1 shows that, except for a few studies, the waiting costs and machine depreciation costs are not compensated. We were able to make our study more realistic by considering these costs. In our study, we have included the intra-cell and inter-cell transportation costs, as well as the depreciation cost of the machines, the waiting costs of the machines, labor costs, and maintenance costs of the machines in the model.

3. Problem Description

Cellular manufacturing systems are used for grouping parts and machine families by designs of parts and similarity of production characteristics. Thus, the increase of flexibility and efficiency is achieved while transportation and waiting between machines, labor force, inventory levels of work in process, and production costs are decreasing. In this study, a company is selected because it is affected by classical (traditional) production system characteristics (functional location layout, production of wide-ranging products, and multi-objective machine usage). This traditional production system causes an increase in standard production time, waiting time between machines, and transportation time for parts, decreasing customer satisfaction, efficiency, and the number of final products. To constitute a flexible, quick response (for changing conditions) and efficient system, a new system structure is needed. CMS is selected to be implemented as a new system structure to gain all the goals of system characteristics. In the company's existing system, there are more than one machines and each one of the parts has more than one operation (process). The machine-part incidence matrix indicates which part is processed on which machine. The operation times of parts are predetermined. The study aims to assign parts and machines simultaneously to cells based on machine and labor constraints and maximize the efficiency of the existing system. In this context, for establishing the cell groups with system constraints on the company's current system, a mathematical programming model that provides simultaneous grouping of parts and machines is proposed and then solved by the GAMS 23.5.1. software package, and the optimum assignments for cells are obtained.

4. Mathematical Model

In this study, the aim is to optimize the general system structure by raising the efficiency of cells. Accordingly, the mathematical model needs to have some properties about the system:

- Simultaneous grouping
- Guarantee of an optimum solution
- Routing
- Intra and inter cellular transportation costs
- Labor and maintenance costs
- Depreciation costs
- Waiting costs of machines
- Efficiency of the system
- Demand for products and sales

Considering these properties, the mathematical model can determine the efficiency of all systems. Therefore, a 0-1 integer mathematical model from the literature by Abduelmola and Taboun (2000) is selected to exemplify for development of a new model with system characteristics.

4.1. Abduelmola and Taboun's model for cell formation

In this 0-1 integer programming model, cells are established with parts and machines families. The objective function is based on efficiency and only intra and inter cellular transportations are taken as cost.

<u>*Index set*</u>

$i = 1, 2, \ldots, p$ part index

$j = 1, 2, \ldots, m$ machine index

$k = 1, 2, \ldots, k$ cell index

<u>*Coefficients*</u>

D_i: demand of part i

S_i: sale price of part i

NM_i: number of machines required by part type i

IMC: cost for intra cellular transportation

EMC: cost for inter cellular transportation

<u>*Decision variables*</u>

b_{ij}: 1; if part i needs machine j

0; otherwise

X_{jk}: 1; if machine type j is used in cell k

0; otherwise

Y_{ik}: 1; if part i belongs to cell k

0; otherwise

$$Z_{max} = \frac{\sum_i \sum_k D_i S_i Y_{ik}}{\sum_i \sum_k NM_i \, IMC \, D_i Y_{ik} + \sum_i \sum_j \sum_k (1 - X_{jk}) b_{ij} \, EMC \, D_i \, Y_{ik}} \quad (4.1)$$

subject to

$$\sum_j X_{jk} \leq M_{max} \qquad \forall k \quad (4.2)$$

$$\sum k \, X_{jk} = 1 \qquad \forall j \quad (4.3)$$

$$\sum k \, Y_{ik} = 1 \qquad \forall i \quad (4.4)$$

$$X_{jk} = 0, 1 \qquad \forall (j, k) \quad (4.5)$$

$$Y_{ik} = 0,1 \qquad \forall (i, k) \quad (4.6)$$

In Abduelmola and Taboun's model, because objective function takes only intra and inter cellular transportations as inputs, it is insufficient to maximize all system efficiency. Labor, maintenance, depreciation, and waiting costs of machines have a considerable effect on the production system and all costs of the production system must be integrated into the model to get more realistic results. According to these assumptions, in the objective function of a mathematical model, sales are accounted output, intra and inter cellular transportation costs, maintenance costs, labor costs, depreciation costs, and waiting costs of machines are accounted inputs. In this way, efficiency is determined by output divided by input and increased with the minimization of costs and maximization of outputs.

4.2. The proposed mathematical model for cell formation

Assumptions

- The number and type of machines are available for production.
- For one product, sale price, cost of labor, depreciation, waiting, intra and inter cellular transportation, maintenance, capacity of labor and machines, and demand of products are known at the beginning of the planning period.

Notation

Index set

i = 1, 2, …, p part index

j = 1, 2, …, m machine index

k = 1, 2, …, k cell index

Parameters

D_i: demand of part i

S_i: sale price of part i

NM_i: number of machines required by part type i

IMC: cost for intra cellular transportation

EMC: cost for inter cellular transportation

M_{max}: maximum number of machines in each cell

b_{ij}: 1; if part i needs machine j

0; otherwise

L_{ij}: processing time of part i on machine j

CO_{ij}: labor cost of part i on machine j

L_{max}: maximum labor time in each cell

N_j: the number of available types of machine j in cells

M_{min} : minimum number of machines in cells

M_j: maintenance cost of each machine in the cells

W_j: total waiting time during breaks of machine j

P_j: quantity of product produced per minute by machine j

C_j: acquisition cost of machine j

A_j: the lifetime of machine j

R_j: salvage sell price of machine j

U_{max}: maximum number of parts in cells

U_{min} : minimum number of parts in cells

Decision variables

X_{jk}: 1; if machine type j is used in cell k

0; otherwise

Y_{ik}: 1; if part i belongs to cell k

0; otherwise

V_{ijk}: 1; if machine j and part i are assigned to cell k

0; otherwise

The mathematical formulation of the cell formation problem is:

$$Z_{max} = \frac{\sum_i \sum_k D_i S_i Y_{ik}}{\sum_i \sum_k NM_i\, IMC\, Y_{ik} + \sum_i \sum_j \sum_k (1-X_{jk}) b_{ij} EMC\, D_i\, Y_{ik} + \sum_i \sum_j \sum_k D_i CO_{ij} b_{ij} X_{jk} + \sum_i \sum_k N_j M_j X_{jk} + \sum_i \sum_j W_j P_j S_i + \sum_j \frac{C_j - R_j}{A_j}} \quad (4.7)$$

subject to

$$\sum_j X_{jk} \leq M_{max} \qquad \forall\, k \qquad (4.8)$$

$$\sum k\, X_{jk} = 1 \qquad \forall\, j \qquad (4.9)$$

$$\sum k\, Y_{ik} = 1 \qquad \forall\, i \qquad (4.10)$$

$$\sum k\, D_i . L_{ij} . b_{ij} . X_{jk} \leq L_{max} \qquad \forall\, i, j \qquad (4.11)$$

$$\sum j\, X_{jk} \geq M_{min} \qquad \forall\, k \qquad (4.12)$$

$$\sum k\, X_{jk} \leq N_j \qquad \forall\, j \qquad (4.13)$$

$$V_{ijk} \geq X_{jk} + Y_{ik} - 1 \qquad \forall\, i, j, k \qquad (4.14)$$

$$\sum j\, W_j \leq 2500 \qquad (4.15)$$

$$\sum i\, Y_{ik} \leq U_{max} \qquad \forall\, k \qquad (4.16)$$

$$\sum i\, Y_{ik} \geq U_{min} \qquad \forall\, k \qquad (4.17)$$

$$X_{jk} \in [0,1] \qquad \forall\, j, k \qquad (4.18)$$

$$Y_{ik} \in [0,1] \qquad \forall\, i, k \qquad (4.19)$$

$$V_{ijk} \in [0,1] \qquad \forall\, i, j, k \qquad (4.20)$$

The objective function used in the new model is given in Eq. (4.7) and is calculated based on efficiency. Total sales are represented as the output in the objective function. The input includes not just the costs of material transportation but also a variety of other cost components. The first term represents total intra cellular material handling costs; the second term represents total inter cellular material handling costs; the third term represents total labor costs; the fourth term represents total maintenance costs; the fifth term represents total waiting costs; and the sixth term represents total depreciation costs. Eq. (4.8) shows that the number of machines in each cell will not exceed the maximum number of machines. Eq. (4.9) ensures each machine can be assigned to only one cell. Eq. (4.10) ensures each part is assigned to only one cell. Eq. (4.11) shows that the maximum labor time in the cells cannot be exceeded. Eq. (4.12) guarantees that the number of machines in the cells is equal to or greater than the minimum number of machines required to produce the parts. Eq. (4.13) guarantees that the number of machines in each cell must be less than or equal to the number of machines of type j available. Eq. (4.14) ensures that a specific part is assigned to a specific machine and that the machine is assigned to a specific cell. Eq. (4.15) guarantees that the total breaks of the machines are less than 2500. Eq. (4.16) ensures that the total number of parts in each cell is less than or equal to the maximum number of parts allowed in the cells. Eq. (4.17) ensures that the total number of parts in each cell is greater than or equal to the minimum number of parts allowed in the cells. The sign constraints of decision variables are demonstrated in equations (4.18), (4.19), and (4.20).

5. Application on Real Data

In the production system, for cell formation, 25 parts and 40 machines are selected to be grouped. At first, a machine-part incidence matrix is established to determine which part is processed on which machine. Then, the monthly demand for selected parts, sale prices, and number of machines are determined. Also, the processing time of parts on the machines, labor costs, and monthly maintenance costs of the machines are estimated. All required data are collected and transferred to the tables.

5.1. Input Parameters

The parameter values utilized in the model were obtained from the company, and the relevant details are provided in the appendices. Specifically, Appendix A contains the machine-part incidence matrix, while Appendix B presents the processing times of parts on machines. Labor costs are detailed in Appendix C, and Appendix D includes the demand for parts, sale prices, and the required number of machines. Additionally, Appendix E provides information on machine maintenance costs, acquisition costs, salvage sell prices, production quantities, and machine lifetimes. Other data and constants necessary for solving the mathematical model are summarized in Table 2

Table 2
Input parameters and constants for the solution of the mathematical model

Notation	Data	Notation	Data
i	U1, U2, U3, … ,U25	U_{max}	25 parts
j	M1, M2, M3, …. , M40	U_{min}	1 part
k	C1, C2, C3, C4	L_{max}	43000
IMC	82 tl	EMC	54 tl
N_j	M1: 1, M2: 1, …, M40: 1 (Every type of machine is ‘1’)	W_j	M1: 60, M2: 60, …, M40: 60 (Every type of machine is ’60’)
M_{min}	8 machines	M_{max}	12 machines

6. Solution of Mathematical Model with Real Data

To solve the 0-1 integer mathematical programming model, the GAMS 23.5.1 software package is used on a Dell INSPIRON N4050 Intel Core 2.1 GHz computer. In the model, a simultaneous grouping of parts and machines families is provided by minimizing the cost of intra and inter cellular transportation, labor, and maintenance. Thus, the efficiency of all systems can be considered, and machine and labor constraints are not exceeded.

The optimum solution of the model is obtained as Z_{max}= 13.6823 within 0.031 seconds.

The assignment of parts and machines is given in Table 3 (groups of cells are shown in red):

- In cell one (C1), parts: {U1, U6, U12, U19, U20, U21} and machines: {M6, M9, M10, M12, M17, M21, M34, M38} are assigned.
- In cell two (C2), parts: {U5, U9, U13, U14, U18, U22, U25} and machines: {M1, M4, M5, M7, M13, M16, M18, M22, M27, M29, M35, M39} are assigned.
- In cell three (C3), parts: {U3, U7, U8, U11, U15, U23, U2, U4, U10, U16, U17, U24} and machines: {M3, M8, M11, M14, M19, M20, M28, M33, M36, M40} are assigned.
- In cell four (C4), parts: {U2, U4, U10, U16, U17, U24} and machines: {M2, M15, M23, M24, M25, M26, M30, M31, M32, M37} are assigned.

6.1. Efficiency Measures Computation of the Proposed System

One of the most important stages in cell formation is evaluating the differences between the existing system and the proposed cell design. With a cellular manufacturing approach, a grouping of parts and machine integrity are provided. However, it is important to determine to what extent this grouping is effective in terms of production. Therefore, it is necessary to determine the effectiveness and efficiency of grouping and also what gains are rendered by the proposed cell design in terms of efficiency.

Table 3
Proposed cell formation with optimal solution assigned parts and machines

	U1	U6	U12	U19	U20	U21	U5	U9	U13	U14	U18	U22	U25	U3	U7	U8	U11	U15	U23	U2	U4	U10	U16	U17	U24
M6	1	1	1	1	1	1																			
M9	1	1	1	1	1	1																			
M10	1	1	1	1	1	1																			
M12	1	1	1	1	1	1										1									
M17	1	1	1	1	1	1																			
M21	1	1	0	1	1	1																			
M34	1	1	1	1	1	0																			
M38	1	1	1	1	1	1																			
M1							1	1	1	1	1	1	1												
M4							1	1	1	1	1	1	1					1							
M5							1	1	1	1	1	1	1												
M7							1	1	1	1	1	1	1												
M13			1				1	1	1	1	1	1	1												
M16							1	1	1	0	1	1	1									1			
M18							1	1	1	1	1	1	1						1						
M22							1	1	1	1	1	1	1												
M27							1	1	1	1	1	1	1												
M29							1	1	1	1	1	1	1												
M35							1	1	1	1	1	1	1												
M39							1	1	1	1	1	0	1												
M3														1	1	1	1	1	1						
M8				1										1	1	1	1	1	1			1			
M11														1	1	1	1	1	1						
M14														1	1	1	1	1	1						
M19														0	1	1	1	1	1						
M20														1	1	1	1	1	1						
M28			1											1	1	1	1	1	1						
M33														1	1	1	0	1	1						1
M36														1	1	1	1	1	1						
M40														1	1	1	1	1	0						
M2										1										1	1	1	1	1	1
M15																				1	1	1	1	1	1
M23																				1	1	1	1	1	1
M24																				1	1	1	1	0	1
M25																				1	1	1	1	1	1
M26																				1	1	1	1	1	1
M30																				1	1	1	1	1	1
M31		1														1				1	0	1	1	1	1
M32																				1	1	1	1	1	0
M37																				1	1	1	1	1	1

For evaluation of the effectiveness of cells, grouping measures from the literature (Murugan & Selladurai, 2011) are selected and applied to developed cells:

- **Percentage of exceptional components:**

  PE=(Number of exceptional components/number of total process)×100

- **Usage of machines:**

$$MK = 100 \times \frac{N}{\sum m_k p_k}$$

N: total number of 1's in parts-machine matrix
m_k: number of machines in cell k
p_k: number of parts in cell k

- **Effectiveness of grouping:**

GE= n =[qn_1+ (1-q)n_2]×100
n_1 number of 1's in diagonal blocks of parts-machine matrix: n_1= N/$\sum m_k p_k$.
n_2: number of 0's outside of diagonal blocks: n_2= 1- [NE/ (MN-$\sum m_k p_k$)].
NE: number of exceptional components, MN: the size of the parts-machine matrix
q: Generally it is taken 0,5.

- **Sufficiency of grouping:**
-

GY= $\phi = \frac{u-e}{u+v}$
$\varphi = \frac{e}{u}$: ratio of the number of exceptional components to the total number of process

$\Phi = \frac{v}{u}$: ratio of number of space components to total number of process
u: number of 1's in matrix
e: number of exceptional components
v: number of space components

Results from methods for effectiveness measurement are given in Table 4. In the existing system, only the usage of machines and the effectiveness of grouping can be measured, but with cell formation, different measures of effectiveness can be measured for being knowledgeable about the statements of all systems.

Table 4
Measures of effectiveness

Methods for effectiveness measurement	Existing system	With cell formation	Targeted values
Percentage of exceptional components	-	4,96%	0%
Usage of machines	82%	96.82%	100%
Effectiveness of grouping	-	98%	100%
Sufficiency of grouping	-	99.1%	100%
Total cost	5681401	4728525,585	-
Efficiency	11,38	13,68	>1

According to Table 4, with developed cells, usage of machines increased from 82% to 96.82%, and the effectiveness of grouping was measured at 98%. In addition, too approximate values against targeted values for other effectiveness measurements are obtained: sufficiency of grouping is 0,991, while the targeted value is 1. Considering all these results from the mathematical model and effectiveness measurements, the proposed cell formation is acceptable in terms of utility and applicability.

6.2. Gains

This study has made a lot of contributions to the literature in terms of mathematical models, optimum solutions, and efficiency measures. First of all, it is important to emphasize the efficiency of the system. Because instead of minimization, very few studies in the literature are interested in maximizing efficiency. In this study, trying to minimize cost, efficiency is increased. Then, the proposed mathematical model is strong because it guarantees the optimum solution. Also, it holds cost items, which can cause the most losses in manufacturing systems. Transportation because of unsuitable machine layout and the dimension of raw materials, labor costs, maintenance costs, depreciation, and waiting costs of machines are combined to represent the general system structure and evaluate the efficiency of the system. Finally, after obtaining the mathematical model optimum solution, it is analyzed how developed cells are effective with using efficiency measures from literature such as usage of machines, effectiveness of grouping, etc. Thus, before cell formation and with cell formation production systems can be compared. Also, it helps to evaluate new cell formation.
Analyzing all results from the model and efficiency measures, it is observed that the developed cell structure applies to the manufacturing system.

7. Conclusion

In the study conducted, it was decided to implement cell formation to address the deficiencies in the existing production system of a company manufacturing refrigeration cabinets, and the mathematical programming method was chosen as the cell formation method. A 0-1 integer mathematical model is developed based on the efficiency factor, in line with the company's requests and the constraints of the current system. In considering efficiency, the model is augmented with costs affecting the cost formation within the cell, including intra cellular and inter cellular material handling costs, labor costs, maintenance costs, depreciation costs of the machines, and the waiting costs of the machines. In this respect, the study has contributed to the literature and made the solution to the problem more realistic.

The model is solved with the help of a software package, resulting in the creation of part families and machine groups. The current system and the proposed system are compared in terms of group efficiency, cost, efficiency, and machine utilization. Compared to the current system, it has been observed that with the proposed system, costs decreased by 16.77%, efficiency increased by 20.21%, and the machine utilization rate increased from 82% to 96.82%. The existing structure of the manufacturing system will be changed because of the new location of the company. In this context, for future research; alternative operation routes, the uncertainty of demand, etc. will be considered for the new system and can be adapted to the mathematical model. Also, to cope with NP-hardness, heuristic and meta-heuristic algorithms will be used (like Tabu Search and Genetic) to solve the problems.

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Appendix

Appendix A: Machine-part incidence matrix

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20	U21	U22	U23	U24	U25
M1	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M2	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M3	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M4	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M5	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M6	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M7	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M8	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M9	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M10	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M11	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M12	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M13	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M14	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M15	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M16	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M17	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M18	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M19	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M20	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M21	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M22	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M23	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M24	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M25	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M26	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M27	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M28	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M29	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M30	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M31	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M32	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M33	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M34	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M35	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M36	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0
M37	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
M38	1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0
M39	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1
M40	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0

Appendix B

The processing time of parts on machines

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20	U21	U22	U23	U24	U25
M1	0	0	0	0	2,5	0	0	0	3,4	0	0	0	2,7	2,6	0	0	0	2,4	0	0	0	1,4	0	0	1,8
M2	0	3,6	0	2,8	0	0	0	0	0	4	0	0	0	0	0	3,1	3,5	0	0	0	0	0	0	1,9	0
M3	0	0	2,2	0	0	0	2	3	0	0	1,8	0	0	0	2,1	0	0	0	0	0	0	0	3,2	0	0
M4	0	0	0	0	3,7	0	0	0	3,5	0	0	0	3,2	2,2	0	0	0	2,4	0	0	0	1,9	0	0	2,3
M5	0	0	0	0	1,5	0	0	0	2,5	0	0	0	1,8	2,6	0	0	0	5,8	0	0	0	5,2	0	0	4,6
M6	3,2	0	0	0	0	4,4	0	0	0	0	0	1,9	0	0	0	0	0	0	2,3	3,1	1,6	0	0	0	0
M7	0	0	0	0	3	0	0	0	2,5	0	0	0	1,7	2,6	0	0	0	2	0	0	0	2,4	0	0	1,1
M8	0	0	1,6	0	0	0	2,4	3,1	0	0	2,7	0	0	0	1,5	0	0	0	0	0	0	0	2,1	0	0
M9	2,2	0	0	0	0	4	0	0	0	0	0	2	0	0	0	0	0	0	2	3,1	1,9	0	0	0	0
M10	3	0	0	0	0	3,1	0	0	0	0	0	2,8	0	0	0	0	0	0	1,6	3,5	3,2	0	0	0	0
M11	0	0	1,8	0	0	0	3	2	0	0	3,1	0	0	0	1,4	0	0	0	0	0	0	0	1,9	0	0
M12	2,4	0	0	0	0	1,3	0	0	0	0	0	2,2	0	0	0	0	0	0	2,4	2,1	3,7	0	0	0	0
M13	0	0	0	0	4,9	0	0	0	3,8	0	0	0	2,6	5,4	0	0	0	5,5	0	0	0	1,4	0	0	2
M14	0	0	4,5	0	0	0	5,3	1,9	0	0	3,3	0	0	0	4,1	0	0	0	0	0	0	0	2,4	0	0
M15	0	5,6	0	2,5	0	0	0	0	0	4,3	0	0	0	0	0	3,4	4,8	0	0	0	0	0	0	3,3	0
M16	0	0	0	0	14	0	0	0	14	0	0	0	16	23	0	0	0	21	0	0	0	22	0	0	15
M17	11,1	0	0	0	0	17	0	0	0	0	0	11,4	0	0	0	0	0	0	15	14	16,6	0	0	0	0
M18	0	0	0	0	24	0	0	0	23,8	0	0	0	25	27	0	0	0	30,4	0	0	0	23,8	0	0	31
M19	0	0	30,4	0	0	0	29,1	33	0	0	24	0	0	0	28,1	0	0	0	0	0	0	0	35	0	0
M20	0	0	19,5	0	0	0	21	20,5	0	0	23	0	0	0	26,7	0	0	0	0	0	0	0	26,4	0	0
M21	27	0	0	0	0	26,7	0	0	0	0	0	28,1	0	0	0	0	0	0	28	25	26,9	0	0	0	0
M22	0	0	0	0	11,1	0	0	0	12	0	0	0	13	12,8	0	0	0	14,6	0	0	0	17,7	0	0	19
M23	0	23,5	0	23	0	0	0	0	0	21	0	0	0	0	0	19,5	23,7	0	0	0	0	0	0	23,3	0
M24	0	26,3	0	25	0	0	0	0	0	28	0	0	0	0	0	28,1	26	0	0	0	0	0	0	21,2	0
M25	0	25	0	18	0	0	0	0	0	19	0	0	0	0	0	16,6	19	0	0	0	0	0	0	20	0
M26	0	5,6	0	21	0	0	0	0	0	11	0	0	0	0	0	19	21	0	0	0	0	0	0	27	0
M27	0	0	0	0	28,7	0	0	0	19,5	0	0	0	27,2	30,4	0	0	0	25	0	0	0	31	0	0	30,4
M28	0	0	14	0	0	0	16	20	0	0	23	0	0	0	11	0	0	0	0	0	0	0	14	0	0
M29	0	0	0	0	5,3	0	0	0	5,6	0	0	0	5,3	6,2	0	0	0	5,6	0	0	0	6	0	0	6,3
M30	0	5,4	0	5,5	0	0	0	0	0	6,1	0	0	0	0	0	5,3	5,9	0	0	0	0	0	0	5,5	0
M31	0	6	0	5,4	0	0	0	0	0	5,6	0	0	0	0	0	5,6	5,4	0	0	0	0	0	0	6	0
M32	0	5,6	0	6,4	0	0	0	0	0	5,9	0	0	0	0	0	6,4	6,3	0	0	0	0	0	0	5,3	0
M33	0	0	19	0	0	0	21	20	0	0	21,2	0	0	0	20,5	0	0	0	0	0	0	0	14,6	0	0
M34	26,7	0	0	0	0	28,1	0	0	0	0	0	19,5	0	0	0	0	0	0	23,8	25	33	0	0	0	0
M35	0	0	0	0	21	0	0	0	23,5	0	0	0	28	20,3	0	0	0	27	0	0	0	26,7	0	0	28,1
M36	0	0	5,7	0	0	0	6,4	5,3	0	0	5,4	0	0	0	6,1	0	0	0	0	0	0	0	6,2	0	0
M37	0	4,9	0	5,3	0	0	0	0	0	6,1	0	0	0	0	0	5,6	5,6	0	0	0	0	0	0	5,9	0
M38	30	0	0	0	0	30,4	0	0	0	0	0	28	0	0	0	0	0	0	19,5	17,7	23,8	0	0	0	0
M39	0	0	0	0	30,4	0	0	0	29,1	0	0	0	23,8	23,5	0	0	0	22,5	0	0	0	26,7	0	0	25
M40	0	0	29,1	0	0	0	28,7	30	0	0	22,5	0	0	0	21	0	0	0	0	0	0	0	24,4	0	0

Appendix C

Labor cost of parts on machines

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	U18	U19	U20	U21	U22	U23	U24	U25
M1	0	0	0	0	1,7	0	0	0	0,95	0	0	0	0,88	1,1	0	0	0	0,65	0	0	0	0,53	0	0	0,67
M2	0	1,3	0	0,86	0	0	0	0	0	1,43	0	0	0	0	0	0,78	0,57	0	0	0	0	0	0	0,83	0
M3	0	0	0,68	0	0	0	1,3	0,95	0	0	1,1	0	0	0	0,86	0	0	0	0	0	0	0	0,68	0	0
M4	0	0	0	0	1,68	0	0	0	0,85	0	0	0	0,76	0,85	0	0	0	0,63	0	0	0	1,78	0	0	1,57
M5	0	0	0	0	1,42	0	0	0	0,73	0	0	0	1,37	1,22	0	0	0	0,83	0	0	0	1,47	0	0	0,98
M6	1,1	0	0	0	0	0,88	0	0	0	0	0	0,9	0	0	0	0	0	0	1,03	1,6	1,48	0	0	0	0
M7	0	0	0	0	1,86	0	0	0	1,75	0	0	0	0,91	1,65	0	0	0	1,22	0	0	0	0	0	0	0,85
M8	0	0	1,38	0	0	0	0,72	0,88	0	0	1,3	0	0	0	0,42	0	0	0	0	0	0	0	1,86	0	0
M9	1,1	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0	0	0	1,13	0,74	0,85	0	0	0	0
M10	1,87	0	0	0	0	0,68	0	0	0	0	0	0,95	0	0	0	0	0	0	1,25	0,85	0,73	0	0	0	0
M11	0	0	0,88	0	0	0	1,68	0,96	0	0	1,3	0	0	0	0,63	0	0	0	0	0	0	0	0,74	0	0
M12	1,2	0	0	0	0	0,55	0	0	0	0	0	1,1	0	0	0	0	0	0	0,86	1,12	0,73	0	0	0	0
M13	0	0	0	0	0,77	0	0	0	1,3	0	0	0	0,59	1,15	0	0	0	0,97	0	0	0	1,58	0	0	1,43
M14	0	0	1,58	0	0	0	1,4	0,86	0	0	1,25	0	0	0	0,82	0	0	0	0	0	0	0	0,66	0	0
M15	0	1,18	0	1,2	0	0	0	0	0	1,48	0	0	0	0	0	0,62	1,63	0	0	0	0	0	0	0,48	0
M16	0	0	0	0	1,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M17	1,62	0	0	0	0	0,85	0	0	0	0	0	1,25	0	0	0	0	0	0	1,44	0,64	0,74	0	0	0	0
M18	0	0	0	0	1,45	0	0	0	1,9	0	0	0	0,72	0,69	0	0	0	1,96	0	0	0	1,43	0	0	0
M19	0	0	1,26	0	0	0	0,49	1,22	0	0	0,76	0	0	0	0,59	0	0	0	0	0	0	0	1,39	0	0
M20	0	0	1,59	0	0	0	0,86	0,68	0	0	0,86	0	0	0	1,1	0	0	0	0	0	0	0	1,28	0	0
M21	0,86	0	0	0	0	0,87	0	0	0	0	0	1,48	0	0	0	0	0	0	1,25	1,38	0,87	0	0	0	0
M22	0	0	0	0	1,23	0	0	0	0,56	0	0	0	1,6	0,48	0	0	0	1,48	0	0	0	1,45	0	0	0
M23	0	0,85	0	1,36	0	0	0	0	0	0,66	0	0	0	0	0	1,5	1,56	0	0	0	0	0	0	1,36	0
M24	0	0,72	0	0,88	0	0	0	0	0	0,98	0	0	0	0	0	1,74	1,5	0	0	0	0	0	0	0,86	0
M25	0	1,1	0	1,36	0	0	0	0	0	1,46	0	0	0	0	0	1,32	1,85	0	0	0	0	0	0	1,68	0
M26	0	1,3	0	0,73	0	0	0	0	0	0,85	0	0	0	0	0	1,9	1,8	0	0	0	0	0	0	1,66	0
M27	0	0	0	0	0,87	0	0	0	1,56	0	0	0	1,87	0,99	0	0	0	1,68	0	0	0	2,13	0	0	1,78
M28	0	0	1,26	0	0	0	1,48	1,2	0	0	0,96	0	0	0	1,68	0	0	0	0	0	0	0	1,89	0	0
M29	0	0	0	0	1,26	0	0	0	2,23	0	0	0	1,97	0,96	0	0	0	1,87	0	0	0	1,59	0	0	0
M30	0	1,26	0	0,86	0	0	0	0	0	1,5	0	0	0	0	0	2,8	1,85	0	0	0	0	0	0	1,24	0
M31	0	1,2	0	0,68	0	0	0	0	0	1,85	0	0	0	0	0	1,56	1,75	0	0	0	0	0	0	0,96	0
M32	0	1,26	0	0,75	0	0	0	0	0	2,06	0	0	0	0	0	1,65	1,74	0	0	0	0	0	0	1,26	0
M33	0	0	2,15	0	0	0	1,29	0,95	0	0	0,85	0	0	0	1,26	0	0	0	0	0	0	0	1,59	0	0
M34	1,58	0	0	0	0	1,69	0	0	0	0	0	1,9	0	0	0	0	0	0	1,56	0,94	1,28	0	0	0	0
M35	0	0	0	0	0,86	0	0	0	1,49	0	0	0	1,86	0,96	0	0	0	1,71	0	0	0	1,66	0	0	2,1
M36	0	0	1,25	0	0	0	0,76	1,29	0	0	0,86	0	0	0	1,56	0	0	0	0	0	0	0	1,69	0	0
M37	0	1,55	0	1,62	0	0	0	0	0	1,6	0	0	0	0	0	1,82	1,96	0	0	0	0	0	0	2,1	0
M38	1,45	0	0	0	0	0,76	0	0	0	0	0	1,75	0	0	0	0	0	0	1,69	1,3	1,69	0	0	0	0
M39	0	0	0	0	1,68	0	0	0	1,46	0	0	0	1,85	0,99	0	0	0	1,6	0	0	0	0,85	0	0	1,69
M40	0	0	1,83	0	0	0	1,86	0,86	0	0	1,28	0	0	0	1,45	0	0	0	0	0	0	0	1,67	0	0

Appendix D

Demand, sales price, and required number of machines for the part

Part	Demand of part	Sales Price	Required number of machines	Part	Demand of part	Sales Price	Required number of machines
U1	776	6972	21	U14	180	7636	17
U2	335	7470	20	U15	102	5976	15
U3	1032	7470	20	U16	344	5810	17
U4	166	8300	21	U17	250	8798	16
U5	320	8798	23	U18	118	5478	13
U6	138	8134	19	U19	110	5810	12
U7	742	8300	20	U20	200	5976	12
U8	721	8798	20	U21	166	6450	14
U9	287	7636	17	U22	220	7220	18
U10	473	7636	17	U23	150	5525	18
U11	600	7802	17	U24	310	6800	15
U12	135	6308	17	U25	620	5900	15
U13	308	6474	17				

Appendix E:

Maintenance cost, acquisition cost, salvage sell price, quantity of product produced, and lifetime for the machines

Machine	Maintenance cost	Acquisition cost	Salvage sell price	Quantity of product produced	Lifetime	Machine	Maintenance cost	Acquisition cost	Salvage sell price	Quantity of product produced	Lifetime
M1	2110	1670000	668000	0,15	12	M21	3100	1830000	549000	0,07	14
M2	1860	1840000	552000	0,2	18	M22	1620	1735000	520500	0,2	20
M3	1455	1500000	450000	0,09	13	M23	2110	1870000	374000	0,1	15
M4	1865	1590000	477000	0,1	15	M24	1600	1830000	366000	0,1	17
M5	2040	1770000	531000	0,22	13	M25	1980	1900000	380000	0,2	13
M6	2630	1910000	382000	0,06	20	M26	1940	1000000	300000	0,23	15
M7	1620	1490000	447000	0,065	21	M27	2005	1480000	444000	0,1	17
M8	1700	1880000	64000	0,11	19	M28	1870	1907000	381400	0,085	13
M9	2460	1850000	555000	0,1	21	M29	1920	1750000	525000	0,15	20
M10	2910	1590000	636000	0,24	15	M30	1650	1700000	510000	0,22	18
M11	2320	1860000	558000	0,05	14	M31	3040	1790000	537000	0,076	17
M12	1750	1940000	776000	0,2	16	M32	2500	1200000	240000	0,08	11
M13	1680	1750000	525000	0,81	12	M33	3100	1250000	375000	0,25	22
M14	2005	1690000	338000	0,09	11	M34	1850	1740000	522000	0,19	13
M15	1460	1580000	474000	0,07	12	M35	1460	1390000	417000	0,2	17
M16	1850	1870000	561000	0,15	14	M36	1200	1680000	504000	0,07	15
M17	2340	1880000	564000	0,2	17	M37	2600	1570000	628000	0,01	13
M18	3000	1995000	598500	0,16	18	M38	1800	1860000	558000	0,13	15
M19	2070	1690000	676000	0,1	17	M39	1750	1760000	528000	0,31	18
M20	2360	2100000	630000	0,04	12	M40	2300	1680000	672000	0,2	14

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