Stochastic behavior of a cold standby system with maximum repair time

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C H R O N I C L E	A B S T R A C T
	The main aim of the present paper is to analyze the stochastic behavior of a cold standby system with concept of preventive maintenance, priority and maximum repair time. For this purpose, a stochastic model is developed in which initially one unit is operative and other is kept as cold standby. There is a single server who visits the system immediately as and when required. The server takes the unit under preventive maintenance after a maximum operation time at normal mode if one standby unit is available for operation. If the repair of the failed unit is not possible up to a maximum repair time, failed unit is replaced by new one. The failure time, maximum operation time and maximum repair time distributions of the unit are considered as exponentially distributed while repair and maintenance time distributions are considered as arbitrary. All random variables are statistically independent and repairs are perfect. Various measures of system effectiveness are obtained by using the technique of semi-Markov process and RPT. To highlight the importance of the study numerical results are also obtained for MTSF, availability and profit function.

1. Introduction

Economic growth of a country totally depends on its industrial and mechanical development and this growth gives emergence to new technologies. These advance technologies developed many complex industrial systems with the composition of many simultaneous components. Complexity of the system also affects its performance, cost and reliability. But, consumers expect highly reliable systems at minimum cost. Now, system designers and researchers face a great challenge to develop such systems which operate continually and increase their performance, reliability and safety. Many researchers such as Moghaddass et al. (2011), Zhang and Wang (2009) and Wu and Wu (2011) analysed reliability of repairable systems under different assumptions. Goel and Sharma (1989), Cao and Wu (1989), Chandrasekhar et al. (2004), Gopalan and Nagarwall (1985), Gopalan and Bhanu (1995), Chander (2005), Mahmoud and Moshref (2010), Mokaddis et al. (2008) and Kumar and Malik (2012) discussed several stochastic model under common assumptions that after a maximum operation time unit

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undergoes for preventive maintenance either standby unit available or not. Mokaddis et al. (2008) studied the effect of preventive maintenance on the systems of non-identical units. Jin et al. (2009) discussed option model for joint production and preventive maintenance. But, keeping in mind the economic aspects of the system sometimes this assumption is not practically feasible. It is also proved that preventive maintenance can slow the deterioration process of a repairable system and restore the system in a younger age or state. Malik and Nandal (2010) carried out the cost-benefit analysis of a stochastic model using the concept of preventive maintenance. Osaki and Asakura (1970) and Kapur and Kapoor (1974) developed reliability model for two-unit standby redundant system with repair and preventive maintenance. Thus, the method of preventive maintenance can be used to improve the reliability and profit of system. Nakagawa and Osaki (1975) analysed stochastic behaviour of a two-unit parallel redundant system with preventive maintenance.

Further, it is a known fact that frequency of failure of standby systems can be reduced up to a desired extent by the method of redundancy. Therefore, keeping a system in cold standby has attracted the attention of many researchers. Barak and Malik (2014), Kumar and Malik (2015), Kumar et al. (2012) and Barak and Barak (2013) suggested some reliability models using the concept of redundancy. Recently, Malik and Munday (2014) developed stochastic model for a computer system by using the concept of hardware redundancy.

Further, the reliability of a system can be increased by making replacement of the components by new one in case repair time is too long i.e., if it extends to a pre-specific time. Singh and Agrafiotis (1995), Malik and Gitanjali (2012) Malik and Kumar (2011) and Kumar and Malik (2014) analyzed stochastically two-unit cold standby systems subject to maximum operation and repair time.

In view of the above importance and practical application of cold standby systems, a stochastic model is developed here by using the concept of preventive maintenance, priority and maximum repair time. For this purpose, a stochastic model is developed in which initially one unit is operative and other is kept as cold standby. There is a single server who visits the system immediately as and when required. The server takes the unit under preventive maintenance after a maximum operation time at normal mode if one standby unit is available for operation. If the repair of the failed unit is not possible up to a maximum repair time failed unit is replaced by new one. The failure time, maximum operation time and maximum repair time distributions of the unit are considered as exponentially distributed while repair and maintenance time distributions are considered as arbitrary. All random variables are statistically independent. Repairs are perfect. Various measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure, steady state availability, busy period of the server due to replacements, repair and preventive maintenance, expected number of repairs, replacements and preventive maintenance, expected number of visits by the server and expected profit earned by the system in (0, t) are obtained by using the technique of semi-Markov process and RPT. To highlight the importance of the study numerical results are also obtained for MTSF, availability and profit function.

2. Model Description

(i) Initially system consists of two identical units- one operative and other is kept as cold standby.
(ii) Both units have three modes- normal, under repair due to failure and under preventive maintenance.
(iii) The failed unit is replaced by new one if the repair of the unit is not possible up to a maximum repair time.
(iv) The failure and maximum operation time distribution are exponentially distributed while repair and preventive maintenance times are distributed arbitrarily.
(v) There is a single server who visits the system immediately as and when required.
(vi) The switch devices, repairs and preventive maintenance are perfect.
(vii) All random variables are statistically independent.

3. Notations

O	The unit is operative and in normal mode,
Cs	The unit is in cold standby,
Λ	Constant failure rate of the unit,
α_0	Constant rate of Maximum Operation Time,
β_0	Constant rate of Maximum Repair Time,
f(t)/F(t)	pdf / cdf of preventive maintenance of the unit after maximum operation time,
h(t)/H(t)	pdf / cdf of replacement time of the unit,
Pm/PM	The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state,
FUr/FUR	The unit is failed and is under repair / under repair continuously from previous state,
FWr / FWR	The unit is failed and is waiting for repair/ waiting for repair from previous state,
Furp/FURP	The unit is failed and is under replacement / under replacement continuously from previous state,
g(t) / G(t)	pdf / cdf of repair time of the failed unit,
$q_{ij}(t)$/ $Q_{ij}(t)$	pdf/cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state,
$q_{ij.k}(t)$ / $Q_{ij.k}(t)$	pdf/cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t],
$M_i(t)$	Probability that the system is up initially in state $S_i \in E$ is up at time 't' without visiting to any other regenerative state,
pdf / cdf	Probability density function/ Cumulative density function,
$W_i(t)$	Probability that the server is busy in state S_i up to time t without making transition to any other regenerative state or returning to the same via one or more non regenerative states,
m_{ij}	Contribution to mean sojourn time in state S_i when system transits directly to state S_j (S_i, $S_j \in$ E) so that $\mu_i = \sum_j m_{ij}$ where $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^*$ and μ_i is the mean sojourn time in state $S_i \in$ E,
μ_i	The mean Sojourn time in state S_i this is given by $$\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$$, where T_i is the sojourn time in state S_i,
Ⓢ / ©	Symbol for Stieltjes convolution / Laplace convolution,
~ / *	Symbol for Laplace Stieltjes Transform (LST) / Laplace Transform (LT).

In view of the above notations and assumptions the system may be in one of the following states:
$S_0 = (O, Cs)$, $S_1 = (O, Pm)$, $S_2 = (O, Fur)$, $S_3 = (FUR, Fwr)$, $S_4 = (Furp, FWR)$
$S_5 = (O, Furp)$, $S_6 = (FURP, Fwr)$ and $S_7 = (PM, Fwr)$

Up state are: S_0, S_1, S_2, S_5 Down states are: S_3, S_4, S_6, S_7

Regenerative states: S_0, S_1, S_2, S_5 Non-regenerative states: S_3, S_4, S_6, S_7

4. Transition Probabilities and Mean Sojourn Times:

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{i\,j} = Q_i(\infty) = \int_0^\infty q_i(t)d \tag{1}$$

as

$$P_{01} = \frac{\alpha_0}{\alpha_0 + \lambda} , P_{02} = \frac{\lambda}{\alpha_0 + \lambda} , P_{10} = \frac{\alpha}{\alpha + \lambda} , P_{17} = \frac{\lambda}{\alpha + \lambda} , P_{25} = \frac{\beta_0}{\beta_0 + \theta + \lambda} , P_{23} = \frac{\lambda}{\beta_0 + \lambda + \theta} ,$$

$$P_{20} = g^*(\lambda + \beta_0) , P_{34} = 1 , P_{42} = 1 , P_{62} = 1 , P_{72} = 1 , P_{50} = h^*(\lambda_1) , P_{56} = [1 - h^*(\lambda)] , \tag{2}$$

$$P_{22.3.4} = [1 - g^*(\beta_0)] , P_{12.7} = [1 - f^*(\lambda)] , P_{52.6} = [1 - h^*(\lambda)] .$$

It can be easily verified that

$$P_{01} + P_{02} = P_{10} + P_{17} = P_{20} + P_{25} + P_{23} = P_{50} + P_{56} = P_{34} = P_{42} = P_{62} = P_{72} = P_{22.3.4} + P_{25} + P_{20} = \tag{3}$$

$$P_{12.7} + P_{10} = P_{52.6} + P_{50} = 1$$

The mean sojourn times (μ_i) of the state S_i are:

$$\mu_0 = \frac{1}{\lambda + \alpha_0} , \mu_1 = \frac{1}{\alpha + \lambda} , \mu_2 = \frac{1}{\beta + \theta + \lambda} , \mu_5 = \frac{1}{\lambda + \beta} , \mu_1^{'} = \frac{1}{\alpha} , \mu_5^{'} = \frac{1}{\beta} , \tag{4}$$

$$\mu_2^{'} = \frac{\lambda + \beta - (\lambda\beta)}{\beta(\beta + \lambda + \theta)}$$

5. Reliability and Mean Time to System Failure (MTSF)

Let $\varphi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\varphi_i(t)$:

$$\varphi_i(t) = \sum_j Q_{i,j}(t) \circledR \varphi_j(t) + \sum_k Q_{i,k}(t), \tag{5}$$

where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly. Taking LST of Eq. (5) and solving for $\tilde{\phi}_0(s)$, we have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} . \tag{6}$$

The reliability of the system model can be obtained by taking inverse Laplace transform of Eq. (6). The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \to o} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad \text{where} \tag{7}$$

$$N_1 = \mu_0 + \mu_2 P_{02} + \mu_1 P_{01} + P_{02} P_{25} \mu_5 \quad \text{and} \quad D_1 = 1 - P_{02} P_{02} - P_{50} P_{25} P_{02} - P_{01} P_{10}$$

6. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright A_j(t), \tag{8}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\lambda+\alpha_0)t}, M_1(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)}, M_2(t) = e^{-(\lambda+\alpha_0)t}\overline{H(t)}, M_4(t) = e^{-(\lambda+\alpha_0)t}\overline{G(t)}$$

$$M_3(t) = e^{-(\lambda+\alpha_0)t}\overline{M(t)}$$

(9)

Taking LT from Eqs. (8) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \text{ , where}$$

(10)

$$N_2 = [(\mu_0 + \mu_1 p_{01})(1 - p_{22.3.4} - p_{52.6}p_{25})] + [(\mu_2 + \mu_5 p_{25})(p_{02} + p_{12.7}p_{01})] \text{ and}$$

$$D_2 = [(\mu_0 + \mu_1' p_{01})(1 - p_{22.3.4} - p_{52.6}p_{25})] + [(\mu_2' + \mu_5' p_{25})(p_{02} + p_{12.1}p_{01})]$$

7. Busy Period Analysis of the Server

(a) Due to Preventive Maintenance (PM)

Let $B_i^P(t)$ be the probability that the server is busy in preventive maintenance of the system (unit) at an instant 't' given that the system entered state S_i at t = 0. The recursive relations for $B_i^P(t)$ are as follows:

$$B_i^P(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^P(t),$$

(11)

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t} \copyright 1)\overline{F}(t) + (\lambda e^{-(\lambda+\alpha_0)t} \copyright 1)\overline{F}(t)$$

(b) Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to failure at an instant 't' given that the system entered state S_i at t = 0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \copyright B_j^R(t),$$

(12)

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to repair of the unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2 = e^{-(\lambda+\beta_0)t}\overline{G(t)} + +(\beta_0 e^{-(\lambda+\beta_0)t} \copyright 1)\overline{G}(t) + (\lambda e^{-(\lambda+\beta_0)t} \copyright 1)\overline{G}(t)$$

(c) Due to Replacement

Let B_i^{Rp}(t) be the probability that the server is busy in the replacement of the unit due to failure at an instant 't' given that the system entered state S_i at t = 0. The recursive relations for B_i^{Rp}(t) are as follows:

$$B_i^{Rp}\left(t\right) = W_i\left(t\right) + \sum_j q_{i,j}^{(n)}\left(t\right) © B_j^{Rp}\left(t\right), \tag{13}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to repair of the unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_5 = e^{-\lambda t}\overline{H}(t)$$

Taking LT of above relations and solving for B_0^{*P} (s), B_0^{*R} (s) and B_0^{*Rp} (s). The time for which server is busy due to preventive maintenance, repair and replacement is

$$B_0^P = \lim_{s \to 0} sB_0^{*P}(s) = \frac{N_3^P}{D_2}, \ B_0^R = \lim_{s \to 0} sB_0^{*R}(s) = \frac{N_3^R}{D_2} \text{ and } B_0^{Rp} = \lim_{s \to 0} sB_0^{*Rp}(s) = \frac{N_3^{Rp}}{D_2} \tag{14}$$

where

$$N_3{}^P = W_1^*(0)(-p_{01})(1 - p_{22.3.4} - p_{25}p_{52.6}), N_3{}^R = W^*(0)[p_{01}p_{12.7} + p_{02}],$$
$$N_3{}^{RP} = (W_2{}^{rp}(0) + W_5^*(0)p_{25})(p_{02} + p_{01}p_{127}) \text{ and } D_2 \text{ is already defined.}$$

8. Expected Number of Preventive Maintenances

Let $R_i^p(t)$ be the expected number of preventive maintenances conducted by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $R_i^p(t)$ are given as

$$R_i^p\left(t\right) = \sum_j Q_{i,j}^{(n)}\left(t\right) ® \left[\delta_j + R_j^p\left(t\right)\right], \tag{15}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and $\delta_j = 1$, if S_j is the regenerative state where the server does job afresh, otherwise $\delta_j = 0$. Taking LST of Eqs. (15) and solving for $\tilde{R}_0^p(s)$. The expected numbers of preventive maintenances per unit time are given by

$$R_0^p(\infty) = \lim_{s \to 0} s\tilde{R}_0^p(s) = \frac{N_4^p}{D_2} \tag{16}$$

where $N_4{}^P = (p_{01} + p_{12.7})(-p_{01})(1 - p_{22.3.4} - p_{25}p_{52.6})$ and D_2 is already mentioned.

9. Expected Number of Repairs

Let $R_i^r(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relationships for $R_i^r(t)$ are given as

$$R_i^r\left(t\right) = \sum_j Q_{i,j}^{(n)}\left(t\right) ® \left[\delta_j + R_j^r\left(t\right)\right], \tag{17}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and $\delta j = 1$, if S_j is the regenerative state where the server does job afresh, otherwise $\delta j = 0$. Taking LST of Eqs. (17) and solving for $\tilde{R}_0^r(s)$. The expected numbers of repairs per unit time are given by

$$R_0^r(\infty) = \lim_{s \to 0} s\tilde{R}_0^r(s) = \frac{N_4^r}{D_2}, \tag{18}$$

where $N_4^R = (p_{20} + p_{22.3.4})(p_{01}p_{12.7} + p_{02})$ and D2 is already mentioned.

10. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_i(t) \right], \tag{19}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and $\delta j = 1$, if j is the regenerative state where the server does job afresh, otherwise $\delta j = 0$. Taking LST of Eqs. (19) and solving for $\tilde{R}_0^I(s)$, the expected numbers of replacements per unit time are given by

$$N_0(\infty) = \lim_{s \to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2}, \tag{20}$$

where $N_0 = (p_{01} + p_{02})(1 - p_{22.3.4} - p_{25}p_{52.6})$ and D2 is already mentioned.

11. Expected number of replacements

Let $R_i^{rp}(t)$ be the expected number of replacements by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $R_i^{rp}(t)$ are given as

$$R_i^{rp}(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j^{rp}(t) \right], \tag{21}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and $\delta j = 1$, if S_j is the regenerative state where the server does job afresh, otherwise $\delta j = 0$. Taking LST of Eqs. (21) and solving for $\tilde{R}_0^r(s)$, The expected numbers of repairs per unit time are given by

$$R_0^r(\infty) = \lim_{s \to 0} s\tilde{R}_0^r(s) = \frac{N_4^r}{D_2} \tag{22}$$

$N_4^{RP} = (p_{22.3.4} + p_{25}p_{52.6} + p_{25}p_{50})(p_{02} + p_{12.7}p_{01})$ and D2 is already mentioned.

12. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0 A_0 - K_1 B_0^R - K_2 B_0^P - K_3 R_0^R - K_4 R_0^P - K_5 N_0 - K_6 R_0^{RP} \tag{23}$$

where
K_0 = Revenue per unit up-time of the system K_3 = Cost per unit time repair of the unit
K_1 = Cost per unit time for which server is busy K_4 = Cost per unit time preventive maintenance
due to repair   of the unit

K_2 = Cost per unit time for which server is busy K_5 = Cost per unit time visit by the server due to preventive maintenance

K_6 = cost per unit time replacement of unit

13. Conclusion

The graphical results for various reliability measures are obtained in Figs. 1-3 by considering the particular case $g(t) = \theta e^{-\theta t}$, $h(t) = \beta e^{-\beta t}$ and $f(t) = \alpha e^{-\alpha t}$ with cost $K_0 = 5000$, $K_1 = 250$, $K_2 = 300$, $K_3 = 200$, $K_4 = 190$, $K_5 = 210$, $K_6 = 160$ and $K_0 = 150$. From Figs. 1-3, it is revealed that MTSF, profit and availability decrease with the increase of failure rate (λ) and maximum operation time (α_0), but the value of steady state availability and MTSF increase with the increase of repair rate (θ) and preventive maintenance rate (α). The profit of the system declines with the increases of preventive maintenance rate (α). Thus finally it is concluded that a cod standby system in which a maximum repair time is given to server for repair can be made more reliable and profitable to use

(i)   By taking one more unit in cold standby,

(ii)   By ignoring the concept of priority to operation over preventive maintenance,

(iii)   By increasing the repair rate,
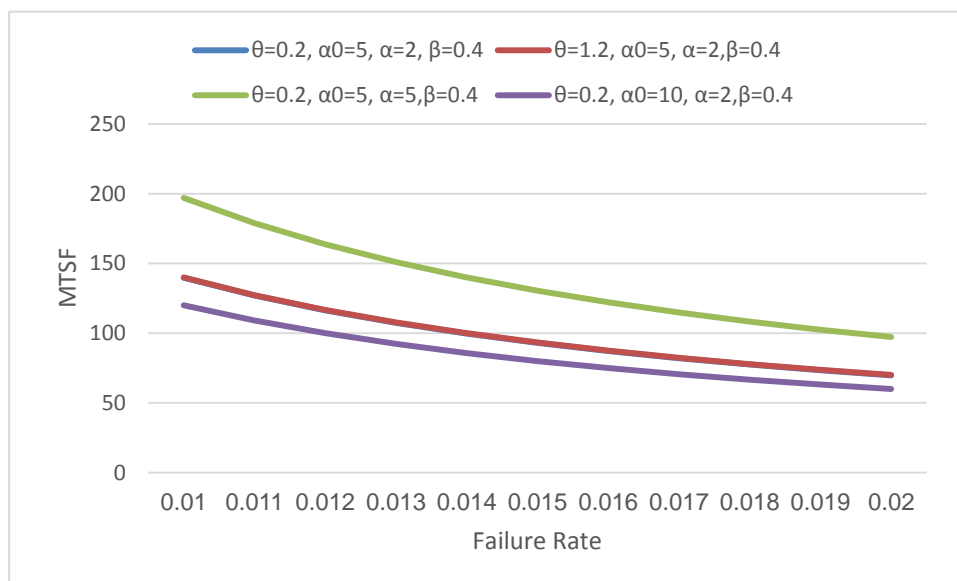
(iv)   By increasing the maximum repair time.
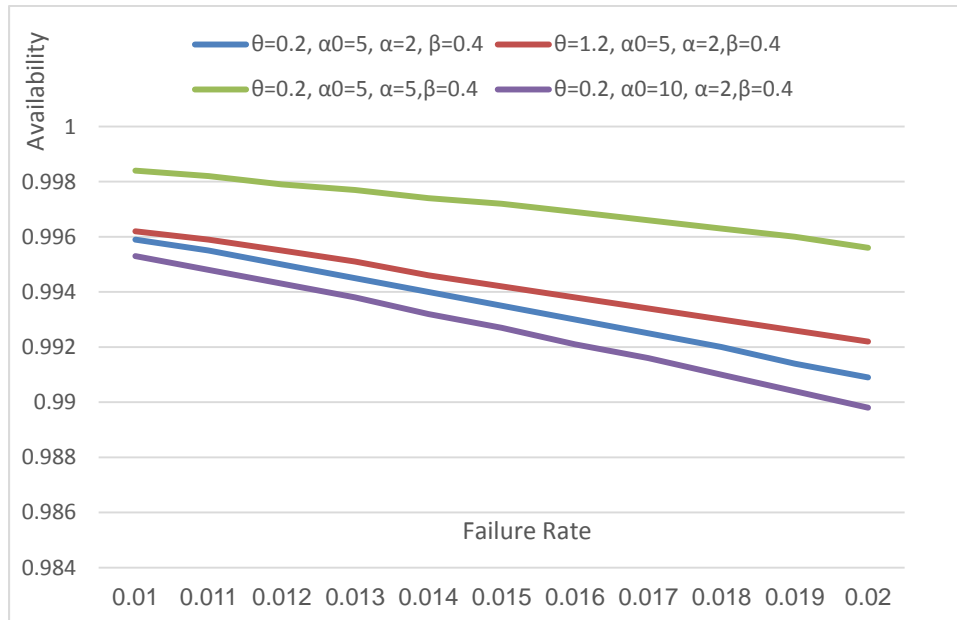

Fig. 1. MTSF vs. Failure Rate

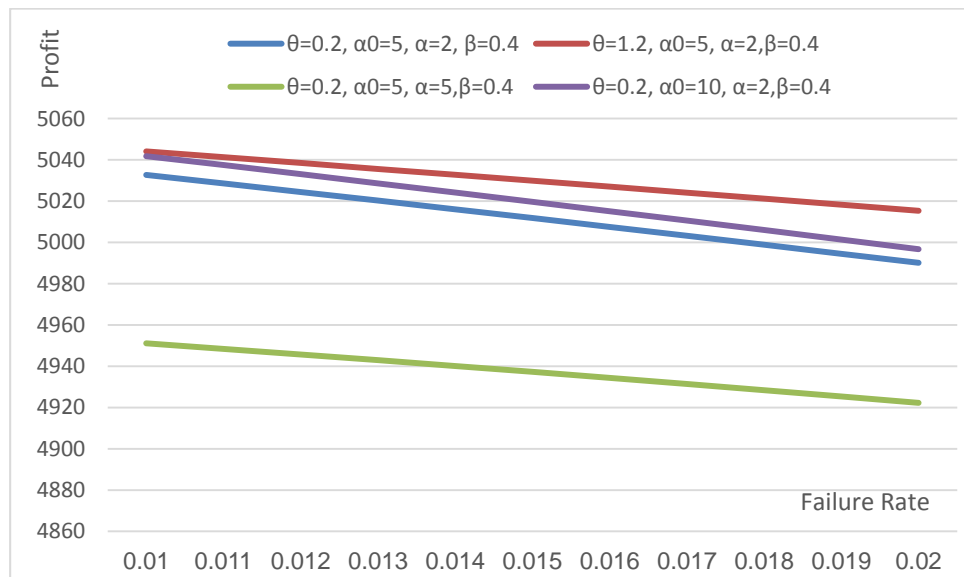**Fig. 2.** Availability vs. Failure Rate

Fig. 3. Profit vs. Failure Rate

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