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Dynamic multicriteria optimization for the nurse scheduling problem

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A B S T R A C T

This document addresses the Nurse Scheduling Problem (NSP) and presents a dynamic multi-criteria optimization model for its solution considering a predefined time horizon. The purpose is to maximize the level of "work well-being" of nurses formulated as the minimization of "aversion" which translates into costs or penalties for certain undesirable work shifts. For this, a series of criteria are defined to estimate the preference structure of nurses according to the hospital center specifications by assigning costs for undesirable shift assignments. The proposed methodology involves developing a heuristic to decompose the global problem into daily subproblems for which a dynamic algorithm is implemented that considers a cost accumulation process for all criteria and all nurses. Daily models are dynamically solved by modifying the coefficients of the well-being function to achieve equity throughout the planning period by updating and accumulating different averages. This methodology has shown satisfactory results for scheduling work shifts for doctors, paramedics, security guards, and drivers in numerous hospital centers in Colombia.

1. Introduction

The assignment problem is one of the most significant in the optimization field, where, for example, different people are assigned to various jobs to minimize or maximize a pre-established objective. Generally, each assignment implies a different performance and, therefore, a different contribution to the objective function. These assignments can be of people to machines, men to offices, or drivers to buses, among others. Multiple approaches have been developed to find optimal solutions.

In this work, a dynamic multi-criteria optimization model is developed for formulating and solving a classic operations research problem for assigning work shifts to different types of personnel over a specific planning period. The developed model is implemented for the case of nurses in several hospitals in Colombia, where time is freed up for the staff in charge of planning and scheduling nurses' work shifts, allowing them to dedicate themselves to patient care tasks as the planning can be done more quickly and better through an operations research model. For the case presented in this paper a very large number of criteria are used to balance the "work well-being" of nurses; it is equivalent to minimize the total aversion (the cost coefficients).

The problem is solved for a planning period as long as it is required (it could be several weeks) but in order to avoid the complexity of the problem for the whole period, the proposed algorithm solves the problem for each day consecutively, but for each new day an update of parameters and variables is made, as it is described later, in order to rebalance the accumulated welfare (maximization problem) or aversion (minimization problem, as is the case for this paper). The problem is solved through a hybrid methodology consisting of two stages for each day:

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i. Developing a dynamic multi-criteria model for maximizing nurses' "work well-being" based on a heuristic methodology comprising the following steps: forming the preference structure integrated by a set of "work well-being" criteria and estimating and calibrating the satisfaction coefficient matrix for each of these criteria.
ii. Solving the assignment problem by applying analytical techniques. The assignment of work shifts to nurses can be solved using a traditional procedure such as the hungarian algorithm for a square matrix, which is quite efficient for this problem, or using one of the specialized software packages. To facilitate and simplify the solution of a large problem, decomposition techniques such as Benders, Dantzig-Wolfe, Lagrangian Relaxation, or Cross Decomposition could be used.

Implementing the algorithm without dividing it into the two mentioned steps would increase the difficulty to formulate the constraints and, therefore, the execution time, as the model would no longer represent the classic assignment problem. Moreover, if it were forced to maintain the classic model, the number of variables would grow factorially since each combination of shifts for each day of all the planning horizon would be considered a variable. For these reasons, the proposed algorithm was developed in two steps.

This work focuses on the first stage, which is central and presents the main contribution: the development of a dynamic multi-criteria model. A detailed description is provided for each of the proposed criteria to form the nurses' preference structure so that shift assignments are equitable and fair, meaning that their shifts are balanced in terms of their "work well-being" throughout the planning period. A file, where the criteria values for each institution, is defined, recorded, and adjusted through trials with the hospital staff performing this task. The selected criteria are considered the most representative for the case study. However, other criteria could be defined as these concepts have a subjective and particular component for the staff of each institution.

Stages i) and ii) are repeated iteratively for the entire planning period once for each day. Therefore, the "work well-being" matrix of step 1 is different for each day because when a shift is assigned for a day, the personnel's situation changes, as the assigned shift may be desired or not, causing the "well-being" values to differ for each person the following day. If a nurse is assigned to an undesirable shift, the next day there should be a propensity to assign her to a more desired shift, and conversely, other nurses who were assigned to a more desired shift may have a less desired one the next day to seek equity for all personnel throughout the planning period. Additionally, not only the current shift but the accumulation of all assigned shifts up to the current day should be considered, updating a set of averages and accumulated values day by day as specified later.

The rest of this document is organized as follows. Section 2 presents a literature review of the most relevant approaches and methodologies for the nurse shift assignment problem. Section 3 outlines the proposed methodology, validated in Section 4 with computational results. Finally, Section 5 provides conclusions.

2. Literature Review

The Resource Allocation Problem is of great importance in organizations of all types, industrial and service, among others, as it is based on the concept of small-action big-effect, maximizing productivity and profitability. In hospital centers for medical care, administrators face significant challenges as efficient resource allocation means their programmers are under increasing pressure to control costs while ensuring high-quality service delivery. One way to alleviate this pressure is to develop better decision-support systems for the scheduling staff. This work focuses on the nurse shift assignment aiming to maximize their "work well-being" considering that their satisfaction level results in better service quality and greater patient well-being (Stimpfel et al., 2012). Simultaneously, the goal is to reduce costs for delivering high-quality hospital services.

The Nurse Scheduling Problem (NSP), also known as the Nurse Rostering Problem (NRP), is an NP-hard combinatorial optimization problem (Zolfagharinia et al., 2024) related to scheduling nurses' work shifts over a planning horizon to meet the hospital center's requirements or demands and workday-related constraints. Government regulations and labor laws, hospital policies, and nurses' status must be considered, as well as their "aversion" to certain work shifts. Although there are many possibilities for defining the objective function, this work aims to minimize the nurses' total "aversion" to certain shifts and the penalty costs for violating soft constraints, intending to maximize their "work well-being".

The optimal solution to the NSP can improve the hospital resource allocation efficiency, nurse and patient safety and satisfaction, and administrative workload. Due to variations among different hospital centers depending on their conditions, numerous hard and soft constraints, and various objective function possibilities, the NSP has a multitude of representations and a wide variety of solution procedures. Due to its complexity and practical relevance, many different procedures have been proposed, including various methods and models, as shown below.

Multi-Commodity Network Flow (MCF): In (El Adoly et al., 2018), a minimum cost MCF model is proposed, where source nodes are available nurses, sink nodes are shifts on different days of the planning horizon, and the demand is the number of nurses required per shift. The model was applied in an Egyptian hospital, improving the nurse satisfaction and reducing overtime costs by 36%. MCF has also been implemented for energy management (Adhikari et al., 2012), airline scheduling (Maharjan & Matis, 2012), material routing (Jiang et al., 2014), maintenance planning (Mesquita et al., 2015), the traveling salesman problem (Boland et al., 2016; Letchford & Salazar-González, 2016), and inventory management (Rudi et al., 2016).

Mixed Integer Programming (MIP): In (EL-Rifai et al., 2015), a stochastic MIP model for NSP in the emergency department of the Lille University Hospital, France, combined with discrete event simulation, is proposed. In (M'Hallah & Alkhabbaz, 2013), a MIP model is applied in health units in Kuwait to minimize the number of subcontracted nurses, considering their preferences, contributing to their satisfaction and performance, and providing a safer environment for patients. In (Lin et al., 2014), shift preference constraints and historical data are considered to maximize the nurse satisfaction. In (Yilmaz, 2012), the total nurse idle time is minimized over a week planning horizon. In (Yahia et al., 2016), a stochastic optimization model is proposed under three types of constraints: nurses, beds, and operating room time. In (Svirsko et al., 2019), service in the emergency department of the Children's Hospital of Pittsburgh is improved through a mathematical model minimizing the number of shifts needed to achieve the service level objective, considering nurses' shift durations and meal coverage. Additionally, fuzzy mathematical models have been implemented, such as in (Jafari et al., 2016), where the number of subcontracted nurses to cover daily demands is minimized, considering preferences under uncertainty modeled with fuzzy variables. In (Burke & Curtois, 2014), the NSP is solved using a Branch and Price algorithm and an ejection chain method.

Approximation Techniques: In (Ohki et al., 2010), a Cooperative Genetic Algorithm (CGA) is proposed, including a mutation operator and using a penalty function. In (Jafari & Salmasi, 2015), a mathematical model maximizes the nurses' preferences, and its optimal solution is compared with that generated through Simulated Annealing (SA). In (Legrain et al., 2015), supernumeraries are considered to cover regular nurse shortages; two easy-to-implement models in spreadsheets are proposed to minimize the total cost, using local search algorithms. In (Bilgin et al., 2012), a general high-level hyper-heuristic approach is proposed to solve the Patient Admission Scheduling Problem (PASP) and the NSP. In (Constantino et al., 2013), a two-phase heuristic is proposed to maximize nurse preference satisfaction and minimize soft constraint violations. In (Akbari et al., 2013), an SA and Variable Neighborhood Search (VNS) approach is developed, considering worker fatigue during their shift. In (Tassopoulos et al., 2015), a two-phase VNS heuristic is proposed. In (Rahimian et al., 2017), a hybrid IP and a VNS four-stage algorithm is presented to generate and improve solutions.

Hybrid Algorithms: In (X. Zhang et al., 2024), a multi-agent deep Q-network-based algorithm (MDQN-MA) for solving the problem with different strategies using neural networks to learn from their experiences is proposed. In (Ceschia et al., 2023), a MIP model is combined with a Simulated Annealing metaheuristic based on two neighborhood structures. The model was tested in 34 hospitals in Italy. In (Chen et al., 2023), a learning mechanism algorithm with a deep neural network is proposed to reconstruct the solution obtained from local optima. In (Amindoust et al., 2021), a hybrid genetic algorithm considering the fatigue factor is proposed. In (Zhang et al., 2011), a hybrid swarm-based optimization algorithm combining genetic algorithms and variable neighborhood search in three steps, dividing the problem into several subproblems, is proposed.

Multi-objective Optimization (MOO): In (Liang & Turkcan, 2016), two MOO models for appointment assignment are developed, aiming to minimize both total patients waiting time and total nurse overtime. In (Burke et al., 2010), MOO is proposed in two steps; first, integer programming generates a feasible solution, and then variable neighborhood search improves it. In (Hamid et al., 2020), MOO with three objectives is proposed: minimizing the total staffing cost, minimizing the sum of assignment incompatibilities, and maximizing the nurses' overall satisfaction with the assigned shifts. In (Di Martinelly & Meskens, 2017), a multi-objective model considering different staff skills aims to maximize group affinity and minimize idle time.

Previous works have proposed up to four criteria, generally using an MOO approach for their solution. These methods have several notable disadvantages: first, the computational complexity and resource intensity required to evaluate and balance multiple objectives; second, it can be challenging to identify a clear optimal solution as the process often results in a set of non-dominated solutions (Pareto front) rather than a single answer, making decision-making difficult; third, formulating appropriate objective functions and weighting their relative importance is highly subjective and context-dependent, potentially leading to biases; finally, interpreting and analyzing the results can be challenging due to the multidimensional nature of the outcomes. For these reasons, this work develops a Multi-Criteria Decision Making (MCDM) approach that includes 20 criteria (in other cases, up to 40 criteria) applied in several hospital centers in Colombia. This proposal decomposes the NSP into multiple easily solvable subproblems in each period, integrating solutions sequentially and dynamically into the complete problem's solution. The proposed methodology integrates two main components: formulating a 20-criteria preference matrix and its estimation based on learning and exact daily subproblem solution, allowing a high level of detail in the criteria. To the authors' knowledge, this approach has not been considered in other papers generally or for NSP specifically.

3. Proposed Dynamic Multi-Criteria Optimization Model

The assignment problem is a classic operations research problem that can be defined using a table or matrix where, in this case of assigning work shifts to nurses in a hospital center, the rows represent nurses, and the columns represent shifts. A cost or "aversion" coefficient is assigned to each matrix element since the criteria used may correspond to undesirable situations. Therefore, to maximize the nurses' "work well-being" a problem of minimizing costs associated with assigning certain shifts to each nurse is proposed.

In the assignment problem, for example, a worker can perform several tasks, or a task can be shared by several workers. This work solves a general version of the assignment problem through a balanced model where each nurse is assigned to a single shift, and each shift is assigned to a single nurse. If there are more nurses than shifts, excess rest shifts are created to balance the problem; similarly, if there are more shifts than nurses, fictitious nurses are created. This last situation implies insufficient nurses to cover the shifts. This way of balancing the problem ensures that the number of nurses always equals the number of shifts, which is the simplest way to solve the assignment problem.

To solve the resulting mathematical model of this assignment problem, one of three alternative approaches could be used: i) As a general Linear Programming problem, ii) As a Transportation Problem considered a special case of Linear Programming, iii) As an Assignment Problem, a special case of the Transportation Problem solvable by numerous methods such as the highly efficient "Hungarian Algorithm" for this problem.

In this work, the following procedure is developed to solve the problem of assigning shifts to nurses in a hospital center:

i) Selection of cost or "aversion" criteria for nurses' shifts.
ii) Estimation of cost coefficients. Heuristic rules are used for their estimation. For each combination of nurses and shifts, the cost or "aversion" level of the nurse for performing that shift is estimated by aggregating several criteria (multi-criteria problem) and dynamically considering accumulated values up to the programming day. Thus, in this phase, criteria and coefficients are recalculated considering past assignments up to the current period.
iii) Parameter estimation and calculation of relative weights of the criteria through an iterative procedure based on the experience and preferences of the health entity.
iv) Minimization of the global nurse cost or "aversion" during the current day based on the cost or "aversion" matrix using specialized software for the assignment problem.

This process is repeated iteratively from step ii) to iv), day by day, recalculating all averages and variables until assigning shifts for all days of the planning horizon. Step 1 is not repeated, because it is supposed that the aversion to a shift or welfare during the whole planning period remains constant. In this way, for different days the aversion (cost) coefficients change as long are shifts are assigned for each day, i.e. every time the model is solved.

3.1. Definition of the Preference Structure and Cost Coefficients

The proposed dynamic multi-criteria optimization model consists of three subsections. The first defines the sets and indices, parameters and variables, and the procedure for estimating the coefficient matrix used in the assignment problem. The second shows the integer linear programming model. The third describes the model parameter calibration.

Definition of Sets and Indices

$\{1, \dots, i, \dots, I\}$	Set of nurses. I is the total number of nurses.
$\{1, \dots, j, \dots, J\}$	Set of shifts. J is the maximum number of shifts. In this case, $J = 5$ (Table 1).
$\{1, \dots, s, \dots, S\}$	Set of weeks in the planning horizon. S is the total number of considered weeks.
$\{1, \dots, k, \dots, 7\}$	Set of days of the week. $k = 1, 2, 3, 4, 5, 6, 7$ corresponds to Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, and Sunday, respectively.
$\{1, \dots, h, \dots, H\}$	Set of criteria for the multi-criteria model. H is the total number of considered criteria.

Definition of Shifts (Case Study)

Table 1 shows the most commonly used shifts in hospital centers according to Colombian regulations. To ensure compliance with legal regulations regarding maximum daily working hours, overtime, or incompatible shifts according to law, the model uses high-value penalty coefficients to avoid them. However, when attempting to balance so many criteria simultaneously, some assignments may exceed the established limits. In such cases, the coefficients are adjusted, and the shifts are reassigned (a new model is solved).

Table 1
Work shifts considered

Symbol	Shift	Description	Shift Duration (hours/day)
1	Day	7:00 AM to 7:00 PM	12
2	Night	7:00 PM to 7:00 AM	12
3	Morning	7:00 AM to 1:00 PM	6
4	Afternoon	1:00 AM to 7:00 PM	6
5	Rest	No work	0

Parameters and Variables Definition

Abbreviated words, mostly three letters long, are used for each variable name, as described below. Number (Num), Hour (Hou), Shift (Shi), Day (Day), Dedication (Ded), Nurse (Nur), Average (Ave), Accumulated (Acu), Week (Wee), Period (Per), Night (Nig), Rest (Res), Saturday (Sat), Sunday (Sun), Weekend (Wen), Non-Weekend (Nwe), All (All), Current (Cur). Using this nomenclature, Table 2 presents the list of parameters, and Table 3 shows the list of variables for week s, day k for each nurse i (left side) and for all nurses (right side).

These parameters and variables are numbered consecutively for later referencing. They are defined in detail below. Two additional sub-index values are used: $CurWee$, which refers to the current week of programming during the planning horizon, and $ActDay$, referring to the current programming day of the current week. Additionally, to refer to a specific day k of a specific week s, the pair (s, k) will be used.

Table 2
List of Parameters

Symbol	Parameter
$V1_j$	$NumHouShi_j$
$V2_{sk}$	$AveHouDay_{sk}$
$V31_i$	$DedNur_i$
$V32_{sk}$	$NumNurDay_{sk}$
$V36$	$NumMaxNur$
$V39_{sk}$	$AveNumNurPer_{sk}$

Table 3
List of Variables

Symbol	For each nurse	Symbol	For all nurses
$V3_{isk}$	$AveHouNurWee_{isk}$	$V4_{sk}$	$AveHouAllSem_{sk}$
$V5_{isk}$	$AveHouEnfPer_{isk}$	$V6_{sk}$	$AveHouAllPer_{sk}$
$V7_{is}$	$AveNurResWenPer_{is}$	$V8_s$	$AveAllResWenPer_s$
$V9_{is}$	$AveNurResWenPer_{is}$	$V10_s$	$AveResWenPer_s$
$V11_{is}$	$AveNurResSunPer_{is}$	$V12_s$	$AveAllResSunPer_s$
$V13_{isk}$	$AveNurResNwePer_{isk}$	$V14_{sk}$	$AveAllResNwePer_{sk}$
$V15_{is}$	$AveNurResWenPer_{is}$	$V16_s$	$AveAllResWenPer_s$
$V17_{is}$	$AveNurNigWenPer_{is}$	$V18_s$	$AveAllNigWenPer_s$
$V19_{isk}$	$AveNurNigPer_{isk}$	$V20_{sk}$	$AveAllNigPer_{sk}$
$V21_{is}$	$AveNurNigSatPer_{is}$	$V22_s$	$AveAllNigSatPer_s$
$V23_{is}$	$AveNurNigSunPer_{is}$	$V24_s$	$AveAllNigSunPer_s$
$V25_{isk}$	$AveNurResNwePer_{isk}$	$V26_{sk}$	$AveAllResNwePer_{sk}$
$V27_{is}$	$AveNurResSatPer_{is}$	$V28_s$	$AveAllResSatPer_s$
$V29_{is}$	$AveNurResSunPer_{is}$	$V30_s$	$AveAllResSunPer_s$
$V37_{isk}$	$HouNurDay_{isk}$	$V35_{sk}$	$AveNumNurDay_{sk}$
$V38_{ijsk}$	$ShiNurDay_{ijsk}$		

- $V1_j$ ($NumHouShi_j$): numbers of hours of shift j, (Table 1).

- $V2_{sk}$ ($AveHouDay_{sk}$): average number of hours of all shifts for the day (s, k). For example, if a day requires 9 shifts: 3 days, 2 nights, 1 morning, 1 afternoon, and 2 rests, this average would be $(3 * 12 + 2 * 12 + 1 * 6 + 1 * 6 + 2 * 0)/9 = 8$ hours.

- $V38_{ijsk}$ ($ShiNurDay_{isk}$): shift j assigned to nurse i on day (s, k).

- $V37_{isk}\left(HouNurDay_{ijsk}\right)$: hours worked by nurse i on day (s, k) depending on the assigned shift $V38_{ijsk}$, according to last column of Table 1.

- $V3_{isk}$ ($AveHouNurWee_{isk}$): average daily hours of nurse i from the start of week s until day k.

$$V3_{i,s,CurDay} = \frac{\sum_{k=1}^{CurDay} V37_{i,s,k}}{CurDay}, \quad \forall i, \forall s, \forall CurDay \tag{1}$$

- $V4_{sk}$ $(AveHouAllWee_{sk})$: average daily hours worked by all nurses in week k until the last assigned day s.

$$V4_{s,CurDay} = \frac{\sum_{i=1}^{I} \sum_{k=1}^{CurDay} V37_{i,s,k}}{CurDay}, \quad \forall\, s, \forall\, CurDay \tag{2}$$

- $V5_{isk}$ $(HouAveNurPer_{isk})$: average daily hours of nurse i from the start of the planning period until day (s,k).

$$V5_{i,CurWee,CurDay} = \frac{\sum_{s=1}^{CurWee} \sum_{k=1}^{CurDay} V37_{isk}}{7 * (CurWee - 1) + CurDay}, \quad \forall i, \forall CurWee, \forall CurDay \tag{3}$$

- $V32_{sk}$ $(NumNurDay_{sk})$: nurse demand on day (s,k).

- $V39_{sk}$ $(AveNumNurPer_{sk})$: average daily number of nurses working from the start of the planning period until day (s,k).

$$V39_{CurWee,CurDay} = \frac{\sum_{s=1}^{CurWee} \sum_{k=1}^{CurDay} V32_{sk}}{(7 * (CurWee - 1) + CurDay)}, \qquad \forall CurWee, \forall CurDay \tag{4}$$

- $V35_{sk}(AveNumNurDay_{sk})$: average daily hours for all nurses working from the start of the planning period until day (s,k).

$$V35_{CurWee,CurDay} = \frac{\sum_{i=1}^{I} V5_{i,CurWee,CurDay}}{V39_{CurWee,CurDay}}, \qquad \forall CurWee, \forall CurDay \tag{5}$$

- $V36$ $(NumMaxNur)$: maximum number of nurses to schedule during any day, equal to the number of shifts for that day. The number of shifts may vary from day to day, implying a change in the assignment matrix size. This size is the maximum number of nurses determined by the day with the most shifts, meaning the day with the most nurses working.

- $V6_{sk}$ $(AveHouAllPer_{sk})$: average daily hours worked by all nurses throughout the planning period until day(s,k).

$$V6_{CurWee,CurDay} = \frac{\sum_{i=1}^{V36} V5_{i,CurWee,CurDay}}{V39_{CurWee,CurDay}}, \qquad \forall CurWee, \forall CurDay \tag{6}$$

- $V7_{is}$ $(AveNurResWenPer_{is})$: average weekly rests on saturdays or sundays for nurse i throughout the planning period until week s. $SatSunRes_{is}$ is a variable that takes the value 1 if the assigned shift to nurse i is rest and it is saturday or sunday; it takes the value of 0 otherwise.

$$V7_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} SatSun\mathrm{Res}_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{7}$$

- $V8_s(AveAll\mathrm{Res}WenPer_s)$: average weekly rests on saturdays or sundays for all nurses throughout the planning period until week s.

$$V8_{CurWee} = \frac{\sum_{i=1}^{I} V7_{i,CurWee}}{V39_{CurWee,7}}, \qquad \forall CurWee \tag{8}$$

- $V9_{is}$ $(AveNur\mathrm{Res}WenPer_{is})$: average weekly weekend rests for nurse i throughout the planning period. $WenResNur_{is}$ is a variable that takes the value 1 if the assigned shift to nurse i is rest on both saturday and sunday of week s; 0 otherwise.

$$V9_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} Wen\mathrm{Res}Nur_{i,CurWee}}{CurWee}, \ \forall i, \forall CurWee \tag{9}$$

- $V10_s$ $(AveResWenPer_s)$: average weekly weekend rests for all nurses throughout the planning period.

$$V10_{CurWee} = \frac{\sum_{i=1}^{I} V9_{i,CurWee}}{V39_{CurWee,7}}, \qquad \forall CurWee \tag{10}$$

- $V11_{is}$ ($AveNurResSunPer_{is}$): average weekly rests on sundays for nurse i throughout the planning period. $ResSunNur_{is}$ is a variable that takes the value 1 if the assigned shift to nurse i is rest on sunday of week s; 0 otherwise.

$$V11_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} \text{Re}sSunNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{11}$$

- $V12_s$ ($AveAllResSunPer_s$): average weekly rests on Sundays for all nurses throughout the planning period.

$$V12_{CurWee} = \frac{\sum_{i=1}^{I} V11_{i,CurWee}}{V39_{CurWee,7}}, \qquad \forall CurWee \tag{12}$$

- $V13_{isk}$ ($AveNurResNwePer_{isk}$): average daily rests for nurse i on non-weekend days during the planning period. $ResNweNur_{isk}$ is a variable that takes the value 1 if the day (s,k) is a rest day; 0 otherwise. This calculation is only done for days other than Saturday and Sunday.

$$V13_{i,CurWee,CurDay} = \frac{\sum_{s=1}^{CurWee} \sum_{k=1}^{CurDay} ResNweNur_{isk}}{(7 * (CurWee - 1) + CurDay)}, \qquad \forall i, \forall CurWee, \forall CurDay \tag{13}$$

- $V14_{sk}$ ($AveAllResNwePer_{sk}$): average daily rests for all nurses on non-weekend days during the planning period. This calculation is only done for days other than Saturday and Sunday.

$$V14_{CurWee,CurDay} = \frac{\sum_{i=1}^{I} V13_{i,CurWee,CurDay}}{V39_{CurWeeCurDay}}, \qquad \forall CurWee, \forall CurDay \tag{14}$$

- $V15_{is}$ ($Ave\text{NurRe}sWenPer_{is}$): average weekly rests for nurse i on Saturday or Sunday during the planning period. $ResWenNur_{is}$ is a variable that takes the value 1 if in week s, nurse i rests on saturday or sunday; 0 otherwise.

$$V15_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} ResWenNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{15}$$

- $V16_s$ ($AveAll\text{Re}sWenPer_s$): average weekly rests for all nurses on saturday or sunday during the planning period.

$$V16_s = \frac{\sum_{i=1}^{I} V15_{i,s}}{V39_{s,7}}, \qquad \forall s \tag{16}$$

- $V17_{is}$ ($AveNurNigWenPer_{is}$): average weekly nights worked on weekend days (Friday, Saturday, or Sunday) for nurse i during the planning period. Here, Friday is included as a weekend day because Friday night extends to Saturday in the morning. $NigWenNur_{is}$ is a variable that takes the value 1 if in week s, nurse i rests on Friday, Saturday, or Sunday; 0 otherwise.

$$V17_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} NigWenNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{17}$$

- $V18_s$ ($AveAllNigWenPer_s$): average weekly nights worked for all nurses on weekend nights (Friday, Saturday, or Sunday) during the planning period.

$$V18_s = \frac{\sum_{i=1}^{I} V17_{i,s}}{V39_{s,7}}, \qquad \forall s \tag{18}$$

- $V19_{isk}$ ($Ave\text{Nur}NigPer_{isk}$): average daily nights worked by nurse i during the planning period. $NigNur_{isk}$ is a variable that takes the value 1 if nurse i is assigned night on day (s,k); 0 otherwise.

$$V19_{i,CurWee,CurDay} = \frac{\sum_{s=1}^{CurWee} \sum_{k=1}^{CurDay} NigNur_{isk}}{7 * (CurWee - 1) + CurDay}, \qquad \forall i, \forall CurWee, \forall Curday \tag{19}$$

- $V20_{sk}$ ($AveAllNigPer_{sk}$): average daily nights worked by all nurses during the planning period.

$$V20_{s,k} = \frac{\sum_{i=1}^{I} V19_{i,s,k}}{V39_{s,7}}, \qquad \forall s, \forall k \tag{20}$$

- $V21_{is}$ ($AveNurNigSatPer_{is}$): average weekly saturday nights worked by nurse i during the planning period. $NigSatNur_{is}$ is a variable that takes the value 1 if nurse i is assigned night on saturday of week s; 0 otherwise.

$$V21_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} NigSatNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{21}$$

- $V22_s$ ($AveAllNigSatPer_s$): average weekly Saturday nights worked by all nurses during the planning period.

$$V22_s = \frac{\sum_{i=1}^{I} V21_{i,s}}{V39_{s,7}}, \qquad \forall s \tag{22}$$

- $V23_{is}$ ($AveNurNigSunPer_{is}$): average weekly sunday nights worked by nurse i during the planning period. $NigSunNur_{is}$ is defined, which takes the value of 1 if nurse i is assigned night on sunday of week s; 0 otherwise.

$$V23_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} NigSunNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{23}$$

- $V24_s$ ($AveAllNigSunPer_s$): average weekly sunday nights worked by all nurses during the planning period, similar to $V16_s$ but adding the variable $V23_{is}$.

$$V24_s = \frac{\sum_{i=1}^{I} V23_{i,s}}{V39_{s,7}}, \qquad \forall s \tag{24}$$

- $V25_{isk}$($Ave\text{Nur}\text{R}esNwePer_{isk}$): average daily rests on non-weekend days for nurse i during the planning period. $DesNweNur_{isk}$ is a variable that takes the value 1 if nurse i rests on day (s, k) other than saturday and sunday; 0 otherwise.

$$V25_{i,CurWee,CurDay} = \frac{\sum_{s=1}^{CurWee} \sum_{k=1}^{CurDay} ResNweNur_{isk}}{7 * (CurWee - 1) + CurDay}, \qquad \forall i, \forall CurWee, \forall CurDay \tag{25}$$

- $V26_{sk}$ ($AveAll\text{R}esNwePer_{sk}$): average daily rests on non-weekend days for all nurses during the planning period.

$$V26_{sk} = \frac{\sum_{i=1}^{I} V25_{i,s,k}}{V39_{s,7}}, \qquad \forall s, \forall k \tag{26}$$

- $V27_{is}$ ($Ave\text{Nur}\text{R}esSatPer_{is}$): average weekly Saturday rests for nurse i during the planning period. $ResSatNur_{is}$ is a variable that takes the value 1 if nurse i rests on Saturday of week s; 0 otherwise.

$$V27_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} ResSatNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{27}$$

- $V28_s$ ($AveAllResSatPer_s$): average saturday rests for all nurses during the planning period.

$$V28_{CurWee} = \frac{\sum_{i=1}^{I} V27_{i,CurWee}}{V39_{CurWee,7}}, \qquad \forall CurWee \tag{28}$$

- $V29_{is}$ ($Ave\text{Nur}ResSunPer_{is}$): average weekly Sunday rests for nurse i during the planning period. $DesSunNur_{is}$ is a variable that takes the value 1 if nurse i rests on Sunday of week s; 0 otherwise.

$$V29_{i,CurWee} = \frac{\sum_{s=1}^{CurWee} \text{Res}SunNur_{is}}{CurWee}, \qquad \forall i, \forall CurWee \tag{29}$$

- $V30_s$ ($AveAllResSunPer_s$): average sunday rests for all nurses during the planning period.

$$V30_{CurWee} = \frac{\sum_{i=1}^{I} V29_{i,CurWee}}{V39_{CurWee,7}}, \qquad \forall CurWee \tag{30}$$

- $V31_i$ ($DedNur_i$): weekly work dedication of nurse i in hours. The usual dedication of 48 weekly hours is assumed by default. For halftiime nurses, this value is 24, but another value may be used according to the signed contract with the nurse.

Observation: some variables mentioned above are calculated only on Sundays: $V7_{is}$ - $V12_s$, $V15_{is}$ - $V18_s$, $V20_{sk}$, $V22_s$ - $V24_s$, and $V27_{is}$ - $V30_s$.

3.1.1 Nurses' "Work Well-being" Criteria

Table 4 defines the 20 proposed criteria in this work to form the nurses' preference structure in various hospital centers where this methodology has been implemented to estimate a multi-criteria function of their work well-being. These criteria can be modified or adjusted according to the context and particular conditions of each institution.

Table 4
Nurses' Work Well-being Criteria and Related Variables.

Symbol	**Criteria and Related Variables**
$C1$	Balance the number of hours worked by all nurses during the week ($V3_{isk}$, $V4_{sk}$).
$C2$	Balance the number of hours worked by all nurses during the planning period ($V3_{isk}$).
$C3$	Balance rests on holidays for all nurses ($V7_{isk}$, $V8_{sk}$).
$C4$	Meet personal needs of nurses by avoiding certain shifts ($V38_{ijsk}$).
$C5$	Penalize excess weekly dedication of a nurse ($V3_{isk}$).
$C6$	Diversity and variation in shift assignment. Some institutions may desire the opposite situation, assigning blocks of consecutive days doing the same shift. In this case, as indicated later, it is possible to assign zero score to shift parameters, thus nullifying the criterion's cost ($V38_{ijsk}$).
$C7$	Assign the entire weekend as two continuous days (Saturday and Sunday) either both work or both rest. It is considered better to rest a full weekend and then work another full weekend rather than resting twice in two weekends only on Saturday or Sunday, as two continuous rest days allow greater flexibility to enjoy the weekend ($V38_{ijsk}$).
$C8$	Rest after a night shift ($V38_{ijsk}$).
$C9$	Balance sunday rests for all nurses ($V11_{isk}$, $V12_{sk}$).
$C10$	Penalize excess weekly dedication of a nurse plus a margin. This criterion is similar to C5 but uses an additional larger penalty when not only the nurse's work dedication is exceeded but also her dedication plus a margin ($V3_{isk}$).
$C11$	Balance the number of non-holiday daily rests for all nurses during the planning period ($V13_{isk}$, $V14_{sk}$).
$C12$	Balance the number of holiday daily rests for all nurses during the planning period. Similar to $C11$, but for holidays ($V15_{isk}$, $V16_{sk}$).
$C13$	Balance the number of nights worked during weekends for all nurses. Weekend days are considered Fridays, Saturdays, and Sundays, as Friday night extends to Saturday morning, and it is assumed the nurse cannot use that Saturday since she must rest ($V17_{isk}$, $V18_{sk}$).
$C14$	Balance the total number of nights for all nurses ($V19_{isk}$, $V20_{sk}$).
$C15$	Balance the number of Saturday nights worked for all nurses. A night shift on a Saturday is considered less desirable than a night shift on a weekday ($V21_{isk}$ $V22_{sk}$).
$C16$	Balance the number of Sunday nights worked for all nurses ($V23_{isk}$, $V24_{sk}$).
$C17$	Penalize the Friday night shift if Saturday and Sunday are rest ($V38_{isk}$).
$C18$	Assign at least one rest day per week to all nurses ($V38_{isk}$).
$C19$	Balance Saturday rests for all nurses ($V27_{isk}$, $V28_{jsk}$).
$C20$	Balance Sunday rests for all nurses ($V29_{isk}$, $V30_{sk}$).

3.1.2 Calculation of Coefficient Matrix for Each Criterion

Quantifying the value of a well-being coefficient directly is not easy. It may be more convenient to estimate it through indirect, objective, and impartial methods than leaving it in the hands of the person assigning shifts subjectively. Instead of maximizing nurses' work well-being, this work estimates their level of "aversion" to certain shifts by assigning cost or penalty coefficients for undesirable situations and then minimizing the global aversion level. The calculation of the coefficient matrix considering this proposal is detailed below, applied to a real case.

Let (M_{sk}) be the assignment coefficient matrix for day k of week s. Its rows (i) represent the nurses, and its columns (j) represent the shifts; its elements are represented by m_{ijsk}, if shift j were assigned to nurse i on day (s,k). Let CR_{skh} be the h criterion matrix, such that each element (i,j) is represented by cr_{ijskh}, if shift j were assigned to nurse i on day (s,k) for criterion h. To calculate M_{sk} for a particular day k of a week s, the calculation of CR_{skh} is required based on all previous days and weeks up to s and k dynamically. For example, to calculate matrix $M_{2,3}$, previous assignments up to day 3 of week 2 for each criterion h are required. This way, criteria values are re-estimated, and the new $CR_{2,3,h}$ is built. Finally, the calculation of M_{sk} is shown in equation (31). Note that both matrices M and CR are square matrices $I * J$.

$$M_{sk} = \sum_{h=1}^{H} CR_{skh}, \quad \forall s,k \quad \text{or equivalently} \quad m_{ijsk} = \sum_{h=1}^{H} cr_{ijskh}, \quad \forall \text{s, k}, i, j \tag{31}$$

Each element cr_{ijskh} of nurse i, in shift j, on day (s,k) criterion h has an associated coefficient w_h representing an adjustment value to the criterion according to the shift assignment expert. This value is detailed later in this section. Below is the estimation of the 20 criteria defined earlier.

- Criterion 1 cr_{ijsk1}: cost of assigning a long shift (longer than the average of all nurses) to a nurse if the nurse has a higher average daily hour than all nurses. The cost increases by summing two differences corresponding to the two previous conditions: $V1_j – V2_{sk}$, and $V3_{isk}$ - $V4_{sk}$. This cost is calculated if $V1_j > V2_{sk}$ and $V3_{isk} > V4_{sk}$.

$$cr_{ijsk1} = w_1 * \left(V1_j - V2_{sk} + V3_{isk} - V4_{sk}\right), \qquad \forall\, s,k,i,j \tag{32}$$

- Criterion 2 cr_{ijsk2}: similar to criterion 1 but taking the averages for the entire period $V5_{isk}$ and $V6_{sk}$ until the assignment time. It is calculated if $V1_j > V2_{sk}$ y $V5_{isk} > V6_{sk}$.

$$cr_{ijsk2} = w_2 * \left(V1_j - V2_{sk} + V5_{isk} - V6_{sk}\right), \qquad \forall\, i,j,s,k \tag{33}$$

- Criterion 3 cr_{ijsk3} (equations (34) and (35)): cost of assigning a different shift than rest on a holiday to a nurse if her accumulated rests on holidays are less than the average of all nurses. Equations (34), if j is a rest shift, and (35), if j is different from rest.

$$cr_{ijsk3} = w_3 * (V7_{isk} - V8_{sk}), \qquad if\ V7_{isk} > V8_{sk}, \ \forall\, i,j,s,k \tag{34}$$

$$cr_{ijsk3} = w_3 * (V8_{sk} - V7_{isk}), \qquad if\ V7_{isk} < V8_{sk}, \ \forall\, i,j,s,k \tag{35}$$

This criterion has a dual effect: penalizing the rest shift for a nurse if her accumulated rests are higher than the average of all nurses (34) and penalizing shifts different from rest for a nurse if her accumulated rests are lower than the average of all nurses (35).

- Criterion 4 cr_{ijsk4}: cost or "aversion" for assigning an undesirable shift. The criterion value is taken equal to the same weight (w_4), meaning a very high value. For example, if a nurse studies on tuesday mornings, she would not want the morning or day shift assigned on those days. Possible imbalances in these days' assignments could be corrected in subsequent days since other balance criteria operate permanently.

- Criterion 5 cr_{ijsk5}: cost of exceeding the weekly hours for a nurse according to her dedication.

$$cr_{ijsk5} = \max\left\{w_5 * \left(V3_{isk} * k + V1_j - V31_i\right), 0\right\}, \qquad \forall\, i,j,s,k \tag{36}$$

  This cost is only assigned if its value is greater than zero. This criterion has practically no effect during the first 4 days of the week for persons with a 48-hour dedication if the maximum shift duration is 12 hours.

- Criterion 6 $cr_{ijsk6} = w_6$: cost of repeating the previous shift. If nurse i is assigned to shift j on day (s,k) means $V38_{ijsk} = j$ and it is intended to reassign shift j the next day. This criterion is used to force variation in the assignment.

- Criterion 7 cr_{ijsk7}: cost of not assigning rest on both saturday and sunday. Eq. (37) and Eq. (38).

$cr_{ijsk7} =$

$$w_7{}^1, \text{ if the previous day (Saturday) was not rest and the current day is rest,} \tag{37}$$

$$w_7{}^2, \text{ if the previous day (Saturday) was rest and the current day is not rest, for both } \forall\, i,j,s,k \tag{38}$$

This criterion seeks to have the weekend complete in rest instead of having a rest on two consecutive Saturdays.

- Criterion 8 $cr_{ijsk8} = w_8$: cost of assigning a shift other than rest or night after night. A nurse should have rest or another night shift after working a night shift; in the latter case, she would have the entire day to rest. This criterion is to avoid working more than 12 hours continuously.

- Criterion 9 cr_{ijsk9}: cost of assigning rest on a sunday to a nurse whose accumulated sunday rests are higher than all nurses' average. The goal is to balance sunday rests for all nurses. If $V11_{is} > V12_s, j = 5$ and $k = 7$, then $cr_{ijsk9} = w_9$.

- Criterion 10 $cr_{ijsk(10)}$: similar to criterion 5, it is the cost of exceeding a nurse's weekly dedication above a margin (ex). This generates two cases: when $ex = 0$, equation (39) and when $ex = holgura$ equation (40).

$$cr_{ijsk(10)}{}^1 = \max\{w_{(10)}{}^1 * (V3_{isk} * k + (V1_j) - V31_i), 0\}, \qquad \forall\, i, j, s, k \tag{39}$$

$$cr_{ijsk(10)}{}^2 = \max\{w_{(10)}{}^2 * (V3_{isk} * k + (V1_j + ex) - V31_i), 0\}, \qquad \forall\, i, j, s, k \tag{40}$$

- Criterion 11 $cr_{ijsk(11)}$: cost to enforce rest on Monday to Friday during the planning period for the nurse if her accumulated rests is lower than the accumulated average of all nurses during the period. If $V13_{isk} < V14_{sk}, j \neq 5$ and $k = (1, \ldots, 5)$, then $cr_{ijsk(11)} = w_{(11)}$.

- Criterion 12 $cr_{ijsk(12)}$: cost to enforce weekend rest during the planning period for the nurse if her corresponding accumulated rest is lower than the accumulated average of all nurses during the period. If $V15_{is} < V16_s,\ j \neq 5$ and ($k = 6\ o\ k = 7$), then $cr_{ijsk(12)} = w_{(12)}$.

- Criterion 13 $cr_{ijsk(13)}$: cost to balance weekend nights. Penalizes the weekend night shift for nurses with an accumulated (weekend night) higher than the accumulated average for all nurses. If $V17_{is} > V18_s, j = 2$ and ($k = 6\ o\ k = 7$), then $cr_{ijsk(13)} = w_{(13)}$.

- Criterion 14 $cr_{ijsk(14)}$: cost to balance the total number of nights during the planning period. To seek equity, penalizes $cr_{ijsk(14)} = w_{(14)}$ in any of two cases: $V19_{isk} < V20_{sk}$ y $j \neq 2$; or, $V19_{isk} > V20_{sk}$ and $j = 2$.

- Criterion 15 $cr_{ijsk(15)}$: cost to balance the total number of Saturday nights during the planning period. If $V21_{is} > V22_s$, $j = 2$ and $k = 6$, then $cr_{ijsk(15)} = w_{(15)}$.

- Criterion 16 $cr_{ijsk(16)}$: cost to balance the total number of Sunday nights during the planning period. If $V23_{is} > V24_s$, $j = 2$ and $k = 7$, then $cr_{ijsk(16)} = w_{(16)}$.

- Criterion 17 $cr_{ijsk(17)} = w_{(17)}$: penalize if the Friday shift was different from night and Saturday was rest, and Sunday is different from rest. If $V38_{is5} \neq 2$, $V38_{is6} = 5$ and $V38_{is7} \neq 5$. If the nurse did not work night on Friday and rested on Saturday, she should also rest on Sunday to enjoy the full weekend.

- Criterion 18 $cr_{ijsk(18)} = w_{(18)}$: assign at least one rest day per week to each nurse. If $V25_{is1} + V25_{is2} + V25_{is3} + V25_{is4} + V25_{is5} + V27_{is} + V29_{is} = 0$ and j diferent 5. See definition of variables before.

- Criterion 19 $cr_{ijsk(19) = w_{(19)}}$: balance the number of Saturday rests for each nurse during the planning period. If $V27_{is} > V28_s, j = 5$ and $k = 6$.

- Criterion 20 $cr_{ijsk(20)} = w_{(20)}$: balance the number of Sunday rests for each nurse during the planning period. If $V29_{is} > V30_s, j = 5$ and $k = 7$.

3.2. Mathematical Model Formulation

Parameters:
- m_{ijsk}: cost or penalty coefficients for the day (s, k) (calculated in section 3.1.2).

Decision variables:

- $x_{ijsk} = \begin{cases} 1, \text{if the nurse } i\ takes\ shift\ j\ on\ day\ (s,k) \\ 0, \text{otherwise} \end{cases}$

Mathematical model for each day (s, k):

$$\text{minimize } z_{sk} = \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijsk} * m_{ijsk} \tag{41}$$

s.t.

$$\sum_{i=1}^{I} x_{ijsk} = 1, \qquad \forall j \in J \tag{42}$$

$$\sum_{j=1}^{J} x_{ijsk} = 1, \qquad \forall i \in I \tag{43}$$

$$x_{ijsk} \in \{0,1\}, \qquad \forall i \in I, \forall j \in J \tag{44}$$

The objective function equation (41) minimizes the penalty coefficients depending on the nurse assignments. Constraints (42) and (43) ensure respectively that each shift is assigned to a single nurse and each nurse is assigned to a single shift. Constraints (44) define the binary nature of the variables.

3.3 Parameter Calibration

The model parameters are adjusted, day after day, based on the previously daily assignment results. A reinforcement learning method was implemented based on the perception of each assignment by an expert, who could be the person responsible for shift assignment in the hospital center, dynamically adjusting the weights of each w_h in each daily iteration.

This section shows 20 cost coefficients taken from a particular case of a colombian hospital center and describes the most relevant ones. Values range from 2 to 68,000, a scale adjusted based on data training. This scale may vary for each institution; the important thing is that the relationship between their values allows adjusting an indicator according to dynamically obtained results.

The weights assigned to criteria as described ahead look subjective, they are initially chosen assigning greater values for the criteria considered more important. Once the weights assigned to criteria are completed, the problem is solved, and its solution is shown to the person in charge of the assignment process. In this way a stage of readjusting the values of weights is made up to the point when the person agrees with the solution. At this point the weights are considered ready to be applied to the real hospital. It is not discarded that eventually could be further corrections in the value of the weights as the conditions change. Although the definition of the initial weights can be made using a subjective logical criterion, as the model undergoes the adjustment phase, the algorithm converges to the same optimal solution. Therefore, the scale of the weights is irrelevant, as is the value of the objective function. What matters in the solution is the assignment of shifts to the nurses (decision variable values), minimizing their total level of aversion (objective function) regardless of its value.

- $w_h = 2$, arbitrarily selected minimum value. This criterion is considered one of the least important in the assignment process.
- $w_2 = 40$ balancing the entire planning period is considered more important than balancing a week. An unbalanced week could be balanced inversely in the following week.
- $w_3 = 3$ this criterion is considered only slightly more important than the first.
- $w_4 = 60.000$ a very high value to try to avoid that shift. Preferences can be made for any number of shifts.
- $w_5 = 2.000$ cost for each hour exceeding the weekly dedication value.
- ${w_6}^1 = 1.000$ and ${w_6}^2 = 2.000$ doing two nights is considered less desirable than repeating any other shift for two consecutive days. The Nig-Res-Nig sequence is equivalent to repeating the night shift since a night must be followed by rest. Assigning night on both days is considered a repetition of the night shift. The Nig-Nig (two consecutive nights) sequence is considered in the parameter w_4.
- ${w_7}^1 = 2, {w_7}^2 = 7$.
- $w_8 = 68.000$ working during the day after a night ending at 7:00 AM would be assigning more than 12 working hours in a 24-hour period, which is very undesirable.
- $w_9 = 3.600$.
- ${w_{(10)}}^1 = 2.000$ y ${w_{(10)}}^2 = 5.000$ if the shift exceeds 52 hours (48-hour work dedication plus a 4-hour margin), an additional penalty is incurred. Therefore, two values exist.
- $w_{11} = 5$ penalizes shifts other than rest for nurses.
- $w_{12} = 5, w_{13} = 8, w_{14} = 8$.
- $w_{15} = 1.600$ penalizes the Saturday night shift for nurses with accumulated Saturday nights higher than the average for all nurses.

- $w_{16} = 1.600$ penalizes the Sunday night shift for nurses with accumulated Sunday nights higher than the average for all nurses.
- $w_{17} = 1.600$.
- $w_{18} = 800$ penalizes a shift other than rest for a nurse who has not had any rest during the week.
- $w_{19} = 3$ penalizes the Saturday rest shift for nurses with accumulated Saturday rests higher than the corresponding average for all nurses during the planning period.
- $w_{20} = 4$ penalizes the Sunday rest shift for nurses with accumulated Sunday rests higher than the corresponding average for all nurses during the planning period.

4. Computational Results

The proposed dynamic multi-criteria optimization model for solving the NSP incorporated the Hungarian method for the daily scheduling. Additionally, an interface was created to interact with the program. Before starting the shift scheduling, the software identifies Sundays and fixed and movable holidays according to Colombian legislation, i.e., moving movable holidays to the following Monday.

The proposed methodology was implemented in more than 20 hospital centers in Colombia with highly efficient results, showing improvements in the following aspects:

- Equity in shift assignment: A more equitable distribution of nurses' shifts is guaranteed, as although the optimization criteria are subjective and specific to the institution, overloads or underutilization of personnel are avoided. This results in better shift rotation among personnel, contributing to a fairer distribution of workloads and greater employee satisfaction.
- Positive impact on service quality: Greater patient satisfaction and reduced waiting times are evidenced as this solution allows for better human resource planning, resulting in more time dedicated to patient care and more timely service.
- Resource optimization: The model contributed to better utilization of available human resources (nurses), reducing the need for overtime and minimizing personnel underutilization as it balances supply with demand for resources more objectively.
- Model flexibility: The dynamic model's ability to adapt it to changes in work demand and personnel preferences was demonstrated. The model allowed for quick and precise adjustments to shift assignments in unforeseen situations, as its execution time was less than 30 seconds in case of sudden changes.
- Resource release: Routine work for personnel responsible for nurse shift assignment, a highly tedious and time-consuming activity due to the numerous conditions and requirements to be programmed for their needs' satisfaction, was released. Thus, the institution can use the released resources to focus on their primary task of patient care and attention in the hospital center rather than the shift scheduling.

Although there is not a detailed comparison between the results obtained in the manual and the automated process, since for the manual process there were not a measure for the value of the objective function, the satisfaction level shown by all the institutions was very high with the application of this proposed approach.

5. Conclusions and contributions

This work emphasizes developing and formulating a "work well-being" matrix for nurses, i.e., a proposal for criteria to estimate their preference structure, experimental and dynamic heuristic estimation of the planning process considering numerous changing factors over time, and the inclusion of non-linear criteria since some factors may be much less desirable if their values deviate significantly from given parameters such as averages. Estimating the "work well-being" matrix could be considered an Artificial Intelligence (AI) application since as time progresses, the software "trains" or "learns" to make more accurate decisions for satisfying the optimization criteria according to the system's dynamics, as its values are accumulated or averaged daily to maximize the total well-being of all nurses throughout the planning period. An outstanding contribution of this work is the management of a very large number of criteria used to balance the "work well-being". For the case presented in this paper 20 criteria have been used, but this number could be much greater, since the difficulty of the proposed model rises in a liner way with the number of criteria. To the best of our knowledge, such a large number of criteria have not been used in the literature for this problem.

For the daily model solution, the hungarian method is used with a square matrix guaranteeing the existence of a feasible solution. In this case, any optimization package can be used; however, in this work, the optimization algorithm was developed, programmed, and incorporated into the software used by the hospital center for automatic nurse shift assignment to use the institution's own resources. This solution procedure was implemented in more than 20 hospitals in Colombia, obtaining satisfactory and efficient results.

The methodology presented in this work could be used to generally solve the shift assignment problem for nurses, medical and paramedical personnel in any hospital center. Moreover, this is a flexible and dynamic methodology that could be applied to the particular conditions of any work area, such as manufacturing or services. The results obtained demonstrate the value and effectiveness of the dynamic multi-criteria optimization model for nurse shift assignment in health institutions, contributing to improving the staff well-being, operational efficiency, and the quality of service provided to patients.

This work could be a starting point for exploring new applications of Operations Research to real systems, helping to close the gap between scientific development and its practical implementation.

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